PRINCETON UNIVERSITY   **PHYSICS 104 LAB**
Physics Department   **Week #10**

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EXPERIMENT IX
PHYSICAL OPTICS: Interference and Diffraction

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This is the second week of experiments on the behavior of light. Last week we adopted the simple but useful assumptions of ray optics. This week you will explore the consequences that light is a wave phenomenon. The challenge is that the wavelength of light is too small to see.

In this Lab you will study various examples of interference and diffraction of light:

- Perform Young's double slit experiment (which provided the first compelling evidence that light does behave like a wave) and get a rough measurement of the wavelength of the laser light.
- Explore the behavior of light waves passing through a number of double slits each with a different distance between the slits.
- Explore the behavior of light waves passing through a number of single slits, each with a different width, and through a circular aperture.
- Explore the behavior of light waves that pass through several multi-slit gratings, each with a different number of slits.
- Use a high-quality multi-slit grating with ≈ 1000 slits per mm to make a rather precise determination of the wavelengths of the three visible spectral lines in hydrogen (the Balmer series, the lines on which Bohr based his theory of atom).

Because of the very small wavelength of light, diffraction and interference are easily seen only by use of devices with very small apertures. To obtain a detectable amount of light passing through these small aperutures, you will be using a laser as the light source.

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CAUTION : Never Look Directly At The Light From A Laser

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Laser light is very bright and focuses to a very small spot on the retina, possibly causing permanent damage.

Diffraction as a Consequence of Faraday's Law

The work of Thomas Young and others around 1800 on the interference and diffraction of light led to the conclusion that light was a wave phenomenon, some 60 years before Maxwell provided the understanding that light consists of waves of electricity and magnetism. In retrospect, we can understand how Faraday's law applied to electromagnetic waves implies the basic features of diffraction = the spreading of a beam of light after it passes through a small aperture.

Consider a linearly polarized plane wave with electric field $\boldsymbol{E} = \hat{\boldsymbol{x}}E_x e^{i(kz-\omega t)}$ of angular frequency ω incident on a perfectly absorbing screen in the plane $z = 0$ that has a square aperture of edge a centered on the origin. We apply the integral form of Faraday's Law to a semicircular loop with its straight edge bisecting the aperture and parallel to the transverse electric field E_x, as shown in the figure.
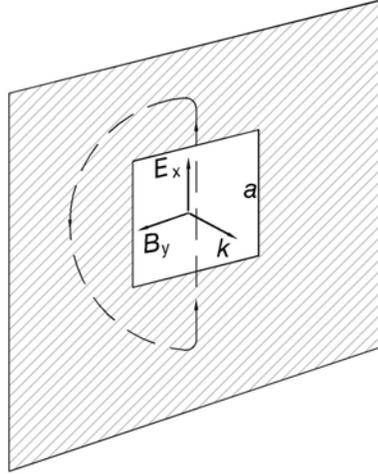
The electric field is essentially zero close to the screen on the side away from the source. Then, at time $t = 0$, the electric field in the aperture has strength E_x, so that integrating around the loop we have

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = E_x a \neq 0.$$

(If the loop were on the source side of the screen, the integral would vanish.)

Faraday's Law tells us immediately that the time derivative of the magnetic flux through the loop is nonzero. Hence, there must be a nonzero longitudinal component, B_z, to the magnetic field, once the wave has passed through the aperture. In detail, since for a wave, $cB_y = E_x$, Faraday's Law leads to

$$cB_y a = E_x a = \oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d}{dt} \int \boldsymbol{B} \cdot d\mathbf{Area} \approx -\frac{dB_z}{dt}\frac{a^2}{2},$$

where B_z is a characteristic value of the longitudinal component of the magnetic field over that half of the aperture enclosed by the loop. The longitudinal magnetic field is caused by the incident wave, and so must have time dependence of the form $e^{-i\omega t}$. Hence, $dB_z/dt = -i\omega B_z = -2\pi i c B_z / \lambda$. Plugging this into the above version of Faraday's Law, we find

$$\frac{B_z}{B_y} \approx -i\frac{\lambda}{\pi a}.$$

Because light waves propagate in a direction perpendicular to the magnetic field, we see that the wave is no longer directed purely along the z axis after passing through the aperture (since $B_z \neq 0$), and we say that it has been **diffracted** as a consequence of Faraday's Law.

The magnitude of the ratio B_z / B_y found above is a measure of the spread of angles of the magnetic field vector caused by the diffraction. And, since the magnetic field $\boldsymbol{B}$ is always perpendicular to the wave vector $\boldsymbol{k}$ that points along the local direction of the wave, we infer that far from the screen the wave vectors occupy a cone of characteristic angle

$$\theta \approx \left| \frac{B_z}{B_y} \right| = \frac{\lambda}{\pi a}.$$

which is representative of the diffraction angle for an aperture of size a.

Using the fourth Maxwell equation including the displacement current, we could make an argument for diffraction of the electric field similar to that given above for the magnetic field. We would find a spread in the directions of the electric field vectors by amount $\theta \approx \lambda / \pi a$, and hence an angular spread in the direction of the rays of the wave also given by $\theta \approx \lambda / \pi a$.

The presence of a factor i in the ratio B_z / B_y implies a very subtle effect in which the phase of the waves shifts by $90°$ between the screen and a distant observer. There is no easy way to detect this phase shift.

1. Young's Double Slit Experiment

In this section, you will make a double slit and use it to measure the wavelength of light from a laser.

On your Lab bench is a small piece of exposed photographic plate. Look at the reflection from each side and find the side with the black emulsion on it. You also have a pair of razor blades taped together, and a straight-edge.

Place the straight-edge on the emulsion side of the glass plate, and scratch a pair of lines about 1/2 inch long into the emulsion by drawing the pair of razor blades along the straight edge. Use your fingers to press on the dull side of the razor blades during this operation! Look at the emulsion with a magnifier to see if there is a clean double line scratched in the emulsion. If it is less than ideal, try again in another spot at least ½" away.

The spacing between the lines can be inferred by measuring the thickness of the razor blades with a micrometer caliper. Assuming that the ground edges on the razor blades are alike, the spacing *d* of the lines is just the thickness of one blade.

Mount your double slit in a spring clip in front of the helium-neon laser, with the slits horizontal. Mount the metal plate screen with the millimeter scale at the other end of the optical bench. Carefully adjust the height of the slits until the laser beam is centered on them, producing an interference pattern on the metal plate screen.

Measure the spacing *A* between maxima on the scale. You should try to include a large number of maxima in the measurement so that you can measure the distance between them and divide by the number of intervals to get better accuracy. Also measure the distance *D* from the slits to the screen.

The angle between adjacent maxima in the interference pattern is given by

$$\sin \theta = \frac{\lambda}{d} = \frac{A}{\sqrt{A^2 + D^2}},$$

where d is the separation of the slits.

Deduce the wavelength of the laser from your measurements, and compare your result with the nominal value of the wave-length of the laser, $\lambda = 632.8$ nm. There may be as much as a 10% error due to asymmetric grinding of the edges of the razor blades. Write your result in the space provided on the blackboard. We shall arrive at a class value and an uncertainty bar by finding the average and the standard deviation of the measurements of the individual groups.

As was shown experimentally in 1909, the form of Young's double slit interference pattern is unchanged even when the intensity of light is so low that there is never more than one photon in the apparatus at a time. At the quantum level, each photon interferes with itself. Interference occurs in Young's experiment only if the light travels along both rays. Hence, we cannot say which slit a photon went through – if Young's interference pattern is observed.

One way to derive the pattern of light on the screen after the laser beam has passed through a double slit is as follows. Consider a plane wave incident on the double slit from the left, as shown in the figure below. The electric field strength is then equal at the openings of the two slits, say with value E_0. The wave that travels along ray 0 reaches the screen with field strength $E_0 e^{i(kr_0 - \omega t)}$, where ω is the angular frequency of the light, and $k = 2\pi / \lambda = \omega / c$. Similarly, the wave that travels along ray 1 arrives at the screen with field strength $E_0 e^{i(kr_1 - \omega t)}$.
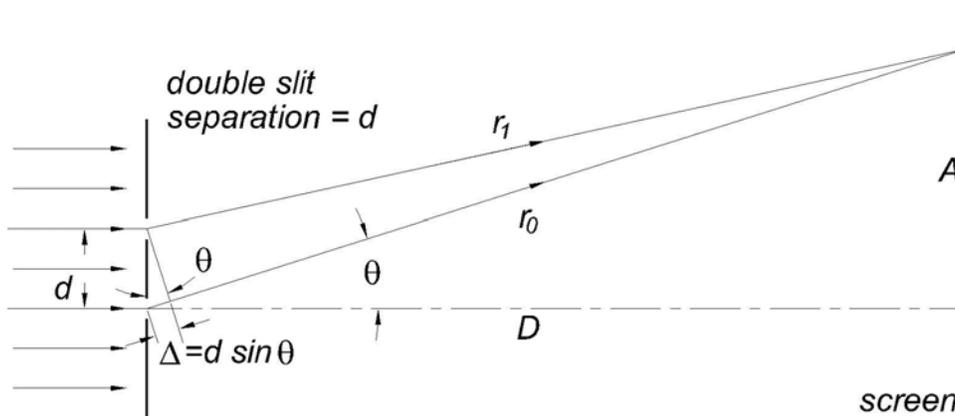
From the geometry of the similar triangles in the figure (and the assumption that $A \gg d$ so that the rays are essentially parallel), we see that the length of ray 0 is longer than ray 1 by $\Delta = d \sin \theta$, so that $r_1 = r_0 - \Delta = r_0 - d \sin \theta$. Using this, the combined field strength at the point on the screen is

$$E = E_0 e^{i(kr_0 - \omega t)} + E_0 e^{i(kr_1 - \omega t)} = E_0 \left(e^{i(kr_0 - \omega t)} + e^{i(kr_0 - \omega t)} e^{-ikd \sin \theta} \right) = E_0 e^{i(kr_0 - \omega t)} \left(1 + e^{-ikd \sin \theta} \right)$$

$$= E_0 e^{i(kr_0 - \omega t)} e^{-i\frac{kd}{2} \sin \theta} \left(e^{i\frac{kd}{2} \sin \theta} + e^{-i\frac{kd}{2} \sin \theta} \right) = 2 E_0 e^{i(kr_0 - \omega t)} e^{-i\frac{kd}{2} \sin \theta} \cos\left(\frac{kd}{2} \sin \theta \right).$$

The intensity I on the screen depends on the (absolute) square of the electric field, so we have

$$I \propto \cos^2\left(\frac{kd}{2} \sin \theta \right) = \cos^2\left(\frac{\pi d}{\lambda} \sin \theta \right).$$

The intensity is maximal whenever

$$\frac{\pi d}{\lambda}\sin\theta = m\pi, \quad \text{or} \quad \sin\theta = \frac{m\lambda}{d}, \quad m = 1,2,3,....$$

See Tipler and Mosca for a more geometric derivation using phasors.

2. Young's Experiment, continued

Mount the helium-neon laser ($\lambda = 632.8$ nm), the slit-film mask, and the gridded screen (to catch the diffracted light) on your optical bench. The mask has many different patterns, as shown schematically in the figure to the right.

The code at the side of each set of slits indicates the pattern.

Top # = # of slits.
Center # = width of slit ($\times 4.4\times10^{-3}$ cm).
Bottom # = slit separation ($\times 4.4\times10^{-3}$ cm).

For example, pattern 3A is labeled (**15, 1, 3**), which implies 15 slits each of width 4.4×10^{-3} cm, and separation of 13.2×10^{-3} cm. between slits. The widths are only approximate. Believe your data.

	1	2	3	4	5
A	1 32 -	1 2 -	15 1 3	1 1 -	1 2 -
B	1 16 -		30 1 1	2 1 2	2 2 2
C	1 8 -	16 -	80 .25 .5	3 1 2	2 2 6
D	1 4 -	1 1	40 .5 1	4 1 2	2 2 14
E	1 2 -	-	20 1 2	10 1 2	2 2 30
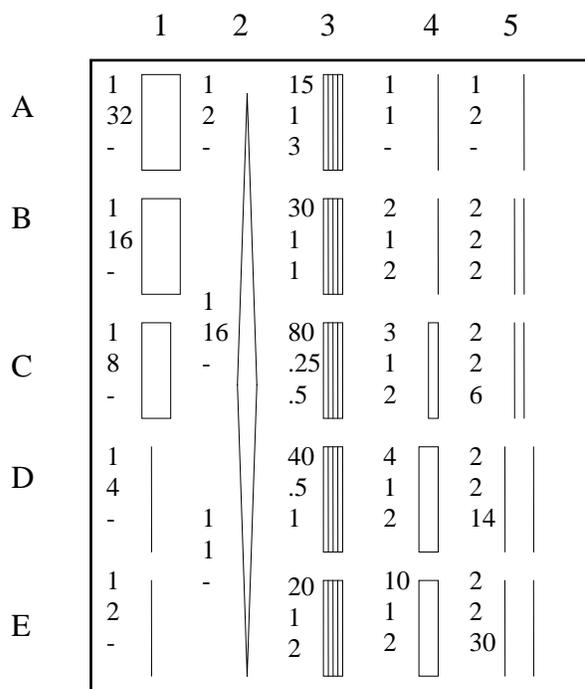
Complete your look at Young's double slit pattern by observing the patterns gotten by shining the laser onto the screen through a series of double slits, 5B, 5C, 5D, 5E, with increasing distance *d* between the slits. Sketch the interference patterns in your notebooks, and write down your qualitative observations.

3. Diffraction by a Single Slit

Even a single slit of width *a*, has an interference pattern on a distant screen, because of interference between light coming from one part of the slit and light coming from another. This "self-interference" is termed diffraction. In this pattern, **minima** appear at angles θ such that

$$\sin\theta = \frac{m\lambda}{a}, \quad m = 1,2,3,...$$

Look at the single slit diffraction pattern by shining laser through a series of single slits of decreasing width --- 1C, 1D, 1E, and observing the pattern on the screen --- What happens to the pattern on the screen as you move from 1C to 1E? Once again draw the patterns in your notebook and write down your observations.

Reconsider your results for experiment 2. If the double slit separation obeys $d = n a$ for some integer *n*, then at angle θ such that $\sin\theta = n\lambda/d$, there should be a maximum in the double slit pattern; however, this angle also obeys $\sin\theta = \lambda/a$, corresponding to a minimum in the single

slit pattern. Since the light from each slit interferes destructively, there cannot be any light in the double slit pattern --- and the nth maximum of that pattern is "missing". Check your data to confirm this result, and/or repeat experiment 2 carefully observing the higher-order maxima.

A derivation of the single slit diffraction pattern is very similar to that given above for a double slit. Consider a plane wave incident from the left onto a single slit of width a, as shown in the figure below. The electric field strength is uniform over the slit, say with value E_0. The pattern observed on the screen is made up of contributions from subregions of the slit. For example, we consider the ray that emanates from a region of width dy at height y above the bottom of the slit. The wave that travels along this ray, whose length is r, reaches the screen with field strength $E_0 e^{i(kr-\omega t)}$.
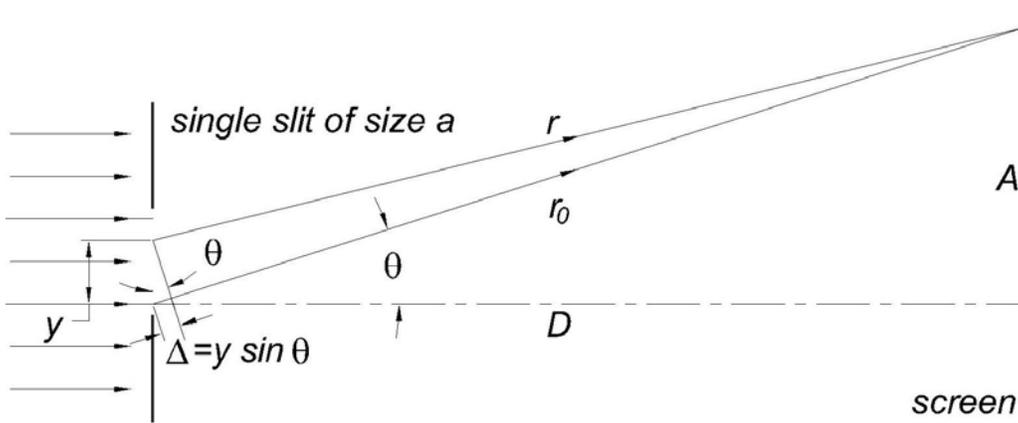
From the geometry of the similar triangles in the figure, we see that the length of ray r_0 is related to that of ray r by $r = r_0 - \Delta = r_0 - y\sin\theta$. Using this, the combined field strength at the point on the screen is

$$E \propto \int_0^a E_0 e^{i(kr-\omega t)} dy = \int_0^a E_0 e^{i(kr_0 - k\Delta - \omega t)} dy = E_0 e^{i(kr_0 - \omega t)} \int_0^a e^{-iky\sin\theta} dy = E_0 e^{i(kr_0 - \omega t)} \frac{e^{-ika\sin\theta} - 1}{-ik\sin\theta}$$

$$= E_0 e^{i(kr_0 - \omega t)} e^{-i\frac{ka}{2}\sin\theta} \frac{e^{-i\frac{ka}{2}\sin\theta} - e^{i\frac{ka}{2}\sin\theta}}{-ik\sin\theta} = 4aE_0 e^{i(kr_0 - \omega t)} e^{-i\frac{ka}{2}\sin\theta} \frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta}.$$

The intensity I on the screen depends on the (absolute) square of the electric field, so we have

$$I \propto \frac{\sin^2\left(\frac{ka}{2}\sin\theta\right)}{\left(\frac{ka}{2}\sin\theta\right)^2} = \frac{\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2},$$

recalling that $k = 2\pi/\lambda$. The function $(\sin x / x)^2$ takes on its greatest value at x = 0 (where the value is 1), and has an infinite set of secondary maxima whose intensities are small. There is also an infinite set of minima at

$$\frac{\pi a}{\lambda}\sin\theta = m\pi, \quad \text{or} \quad \sin\theta = \frac{m\lambda}{a}, \quad m = 1,2,3,...$$

3a. Diffraction by a Circular Aperture

Make a tiny, neat, round pinhole in a piece of aluminum foil by smoothing it over a piece of Lucite (or the bench) and firmly pressing a sharp sewing needle down on it. Remove the needle before lifting the foil from the Lucite.

Place the pinhole close in front of the laser and observe the diffraction pattern on the screen. Make a measurement of the diameter of the first dark ring and calculate the diameter of the pinhole, knowing the wavelength. The relationship for circular geometry is

$$\sin\theta = 1.22\frac{\lambda}{a},$$

where a is the diameter of the hole, and θ is the angle from the center of the diffraction pattern to the first dark ring (*i.e.*, the radius of the spot's central maximum).

The diffraction pattern for a circular aperture of diameter a can be deduced from the geometry of the figure below.
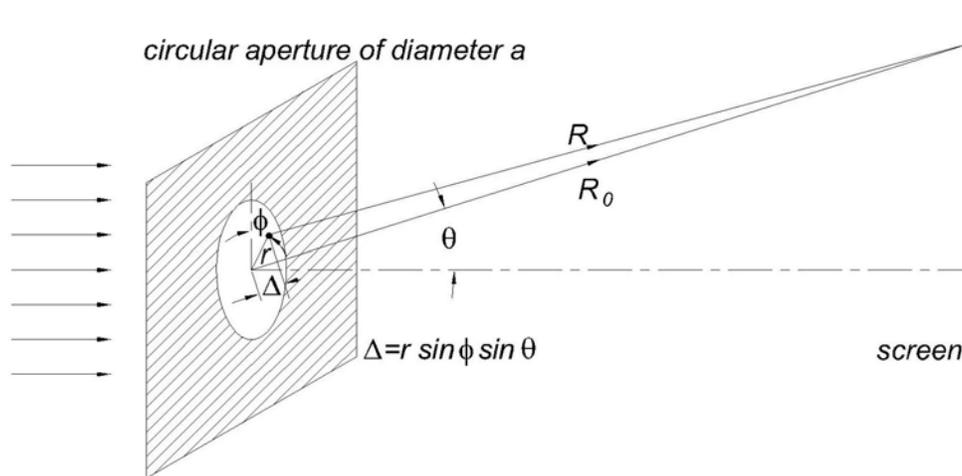
The key is to determine the path difference Δ between the central ray, R_0, which makes angle θ to the axis of the aperture, and an arbitrary ray R that emanates from the point (r,ϕ) in the aperture. The azimuthal angle ϕ is measured with respect to the intersection of the plane of the aperture with the plane containing the ray R_0 and the axis of the aperture. If the observation point on the screen is directly above the axis, as shown in the figure, then just as for the case of the single slit the path difference is $\Delta = y\sin\theta$, where $y = r\sin\phi$ is the height of the point (r,ϕ). That is, $\Delta = r\sin\phi\sin\theta$. Using this, the sum of the electric field at the screen from all rays emanating from the circular aperture (or radius $a/2$) is

$$E \propto \int_0^{a/2} r\,dr \int_0^{2\pi} d\phi\, e^{-ik\Delta} = \int_0^{a/2} r\,dr \int_0^{2\pi} d\phi\, e^{-ikr\sin\phi\sin\theta} = 2\pi \int_0^{a/2} r\,dr\, J_0(kr\sin\theta) = \frac{\pi a^2}{2}\frac{J_1\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta},$$

where your integral tables will tell you that $J_0(x)$ and $J_1(x)$ are so-called Bessel functions, which behave somewhat like $\cos x$ and $\sin x$. In particular, since $J_1(x)$ is similar to $\sin x$, the diffraction pattern from a circular aperture is quite similar to that of a single slit. The first minimum of the diffraction pattern occurs when $J_1\left((ka/2)\sin\theta\right) = 0$. From numerical tables of the zeroes of Bessel functions, we learn that this implies $(ka/2)\sin\theta = (\pi/\lambda)a\sin\theta = 3.823$, and hence

$$\sin\theta = 1.22\frac{\lambda}{a}.$$

4. Multi-Slit Diffraction

A screen with multiple slit, *i.e., n* identical slits, each separated from the next by distance *d* has the remarkable property that as the number *n* of slits increases, the maxima occur exactly at the same angles as for a double slit of separation *d*, except that the maxima become brighter and brighter, and narrower and narrower. These maxima are called the primary maxima :

For multi-slit diffraction, the primary maxima appear at angles θ such that $\sin\theta = m\lambda/d$, where *m* = 1, 2, 3, ...

More and more secondary maxima appear between the principal maxima as n goes up, but they have less and less importance as the number of slits is increased.

Observe the diffraction patterns by shining the laser beam through slits 4B, 4C, 4D, 4E, 3E, in which the number of slits increase from 2 to 20, always with the same slit width and separation. Draw the patterns observed on the screen in your notebook.

The multi-slit diffraction pattern can be calculated following the preceding arguments, noting that the path difference for between rays that emanate at angle θ from adjacent slits of separation *d* is $\Delta = d\sin\theta$, just as in the case of a double slit. Thus slit *m* has path length difference from the first ray given by $\Delta_m = md\sin\theta$. For an array of *n* slits, we sum up the *n* contributions to the electric field on the screen, and note that the mth term differs from the first only by the phase factor $e^{-ik\Delta_m}$. Hence, referring all path differences and phase factors to the first ray, the sum can be written

$$E \propto 1 + e^{-ik\Delta} + e^{-2ik\Delta} + ... + e^{-(n-1)nik\Delta} = \frac{1-e^{-nik\Delta}}{1-e^{-ik\Delta}} = \frac{e^{-ni\frac{k\Delta}{2}}}{e^{-i\frac{k\Delta}{2}}}\frac{e^{ni\frac{k\Delta}{2}}-e^{-ni\frac{k\Delta}{2}}}{e^{i\frac{k\Delta}{2}}-e^{-i\frac{k\Delta}{2}}} = e^{-(n-1)i\frac{k\Delta}{2}}\frac{\sin\left[\dfrac{nk\Delta}{2}\right]}{\sin\left(\dfrac{k\Delta}{2}\right)}.$$

Whenever the denominator is tiny, *i.e.*, when $k\Delta/2 = (\pi d/\lambda)\sin\theta = m\pi, \ m = 1,2,3,...$, the magnitude of the electric field on the screen is *n* times that of due to a single slit (so the intensity is n^2 that of a single slit), and we call this a primary maximum. The pattern of intensity is

$$I \propto |E|^2 \propto \frac{\sin^2\left[\dfrac{nk\Delta}{2}\right]}{\sin^2\left(\dfrac{k\Delta}{2}\right)} = \frac{\sin^2\left[\dfrac{n\pi d\sin\theta}{\lambda}\right]}{\sin^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)}.$$

5. Diffraction Grating

5a. Calibrate the Grating

The grating to be used in this part is a 35 mm slide labeled 1000 lines/mm. To obtain good accuracy in the measurement of the spectral lines (part 5b), you should first calibrate the grating by determining the line spacing d using the technique of parts 1-4, now assuming that the wavelength of the helium-neon laser is 632.8 nm.

5b. Measurement of Spectral Lines of Mercury and Hydrogen

Place your calibrated grating in the apparatus sketched on the right. Begin by measuring the wavelength of two of the brighter visible mercury lines. Then determine the wavelengths of the three visible Balmer spectral lines of hydrogen. Record these in your Lab notebook, and also write them on the board in the appropriate column.

As stated before, the relationship that governs the first primary maximum is: $d \sin \theta = \lambda$. Because the angles are large in this experiment (including the calibration), be sure to measure $\sin \theta$ and not $\tan \theta$.

View of the apparatus looking down from above

meter stick

grating and holder

slit

θ

θ

light source

your eye

Spectral lines
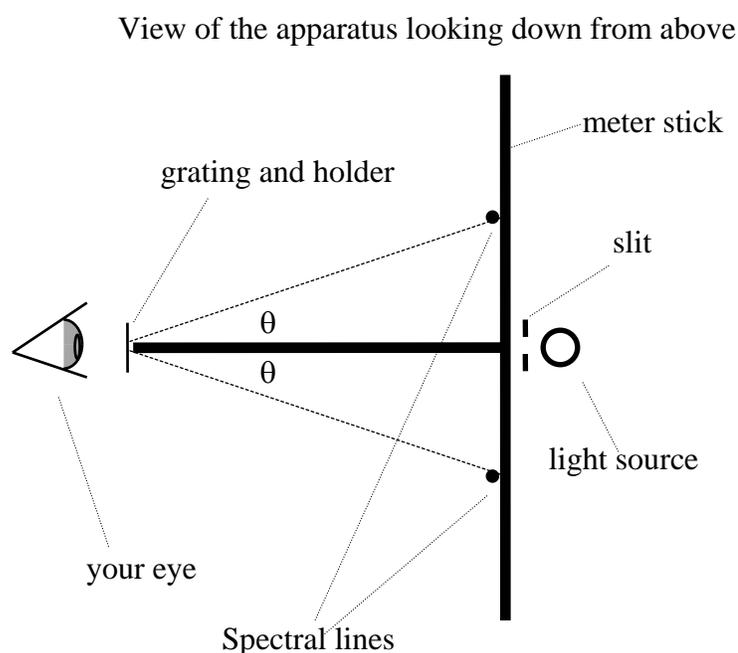
Do NOT use a laser with the apparatus shown in the above figure!

The main difference from what you have been doing earlier in this Lab (passing light through holes of various shapes to form a pattern on a screen) is that now you look through the grating, as shown in the illustration. The nominal values of the mercury and hydrogen spectral lines are given in the table below.

Mercury Lines (in the visible)	Wavelength (nm)	Hydrogen Lines (in the visible)	Wavelength (nm)
yellow doublet	577 / 579	red	656
green	546	aqua	486
blue-violet	436	blue-violet	434
violet	405		

6. OPTIONAL: Width of a Human Hair

Cut a hole in an index card and tape one of your hairs to it, centered on the hole. To improve the alignment, cut small notches on two edges of the card, and lay the hair in the notches, applying the tape to the hair on the back side of the card.

Shine the helium-neon laser on the hair, projection the resulting diffraction pattern onto a screen behind. Describe this diffraction pattern and deduce the width of your hair from it.

The procedure is the complement of shining the laser through a small single slit. For comparison, scratch a single slit into the emulsion on the glass plate (as in part 1 of this Lab), and shine the laser through this slit onto the screen. Again, deduce the width the slit from your observations.

These two experiments illustrate Babinet's principle of complementary screens. Suppose a wave with wave function Ψ_0 is incident on a screen with an aperture such that the transmitted wave is described by function Ψ_1. We also consider the case of a second screen whose shape is the complement of the first; that is, the combination of the first and second screens is a completely opaque screen. Then the sum of the wave function Ψ_2 observed on the far side of the second screen and the wave function Ψ_1 on the far side of the first screen is simply to total incident wave function Ψ_0. Therefore, $\Psi_2 = \Psi_0 - \Psi_1$.
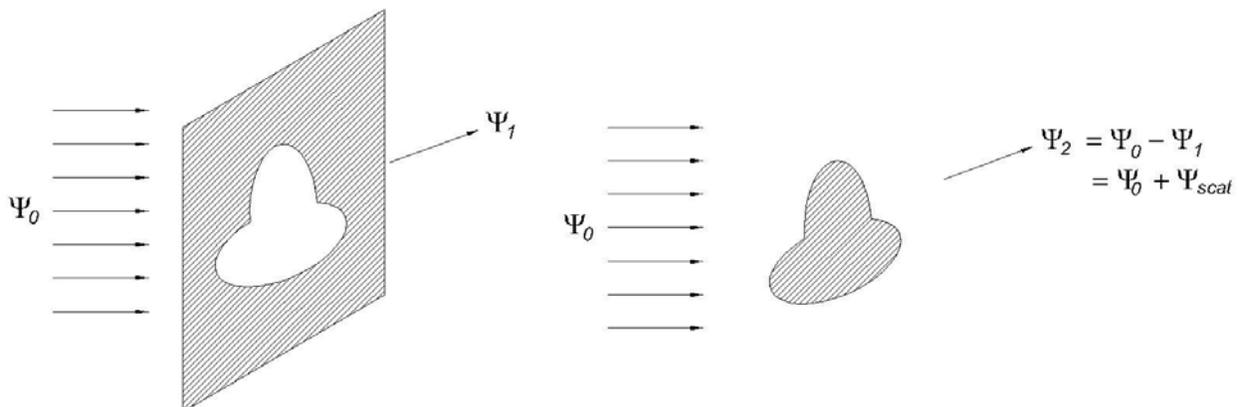
When the second screen is a small object, we may prefer to think of its effect as scattering of the incident wave, in which case we write $\Psi_2 = \Psi_0 + \Psi_{scat}$. Comparing with the previous expression for Ψ_2, we find that $\Psi_{scat} = -\Psi_1$. Then, since the intensity of the pattern on the screen depends on $|\Psi|^2$, we have that $I_{scat} = |\Psi_{scat}|^2 = |\Psi_1|^2 = I_1$. Thus, Babinet's principle is that the intensity of the diffraction patterns from complementary screens in the same! *This conclusion depends on there being no confusion between the scattered wave Ψ_{scat} and the unscattered wave Ψ_0, which is not strictly true in all cases.* How well does your study of your hair and a single slit support Babinet's principle?

PRINCETON UNIVERSITY   **PHYSICS 104 LAB**
Physics Department   **Week #10**
Name:_____   **Date/Time of Lab:**_____

EXPERIMENT IX   PRELAB PROBLEM SET

1. Light with a wavelength of 500 nm from a very distant monochromatic point object enters a pinhole camera through a small hole with a diameter of 0.3 mm. What is the smallest diameter spot that will form on the screen of the camera 12.5 cm from the hole? Will the extended size of this spot be visible to the naked eye? If not, then experiment 3a on round-aperture diffraction will require a hole even smaller than 0.3 mm!

2a. For the diffraction grating that you will use in experiment 5, and for the shortest-wavelength mercury line, how far apart will the two first-order lines appear on the screen? (One will be to the left of center, the other to the right.) Suppose that the screen and grating are 70 cm apart.

2b. How wide must the screen be to contain both first-order red Balmer lines at once?

Continued on reverse.

3. How far apart will neighboring minima be in the single-slit diffraction pattern produced by mask 1E with the helium-neon laser, if viewed on a screen at 70 cm?