Princeton University

Ph304 Problem Set 10

Electrodynamics

(Due in class, Wednesday Apr. 23, 2003)

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Problem sessions: Sundays, 7 pm, Jadwin 303

Text: *Introduction to Electrodynamics, 3rd ed.*
by D.J. Griffiths (Prentice Hall, ISBN 0-13-805326-X, now in 6th printing)
Errata at http://academic.reed.edu/physics/faculty/griffiths.html

Reading: Griffiths secs. 9.4-9.5.

1. Griffiths' prob. 9.19.

2. Griffiths' prob. 9.22. In part a), use dimensional analysis to determine the functional form of the velocity of deep-water waves, which might depend on the wavelength λ, the density ρ of water, and the acceleration g of gravity (which provides the restoring force against wavy deformations), but which does not depend on the depth of the water.

3. Variant of Griffiths' prob. 9.25. Consider a medium that is sufficiently well described by a single resonance of angular frequency ω_0, and with a nonzero absorption coefficient γ. The oscillator strength may be taken as unity. Show that at the central frequency, $\omega = \omega_0$, the group velocity is given by

$$v_g = \frac{1}{dk/d\omega} = \frac{c}{1 - \omega_p^2/\gamma^2},$$

where ω_p is the so-called plasma frequency,

$$\omega_p^2 = \frac{Nq^2}{m\epsilon_0}.$$

If it happens that $\gamma < \omega_p$, then the group velocity is negative! However, since this bizarre effect occurs for frequencies where absorption is very strong, it was thought to be largely irrelevant. But recent experiments involving a pair of closely spaced absorption lines that are pumped into inverted populations has been able to demonstrate negative group velocity without attenuation.

The concept of negative group velocity requires careful thought, as one can be misled into thinking it implies faster-than-light effects. See,
http://kirkmcd.princeton.edu/examples/negativegroupvelocity.pdf

A related, but less controversial, phenomenon attainable with pumped pairs of absorption lines is "slow light": http://kirkmcd.princeton.ed/examples/slowlight.pdf

4. Griffiths' prob. 9.29. You are asked to compute the so-called energy velocity,

$$v_E = \frac{\int \langle \mathbf{S} \rangle \cdot d\mathbf{a}}{\int \langle u \rangle \ da},$$

and compare this with the group velocity v_g.

5. Griffiths' prob. 9.30.

6. Griffiths' prob. 9.38.