Princeton University

Ph304 Problem Set 12

Electrodynamics

(Due 5 pm, Wednesday May 14, 2003; turn in to me, or to my mail box)

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Last problem session: Sunday, May 11, 7 pm, Jadwin 303 (no session May 4)

Text: *Introduction to Electrodynamics, 3rd ed.*
by D.J. Griffiths (Prentice Hall, ISBN 0-13-805326-X, now in 6th printing)
Errata at http://academic.reed.edu/physics/faculty/griffiths.html

Reading: Griffiths chap. 11.

1. Griffiths' prob. 11.6.

2. Griffiths' prob. 11.11.

3. Griffiths' prob. 11.12. This problem reaffirms that a steady current loop does not radiate. Yet, in the microscopic view, the steady circular current consists of a collection of charges all of which are accelerating. There must be some kind of interference effect that suppresses the radiation. A hint of this can be gleaned from prob. 11.11, where the radiated power of an oscillating quadrupole varies as ω^6, compared to ω^4 for an oscillating dipole. Since the velocity of an electron moving in a ring of radius r is ωr, we see that the dipole radiation from a single circulating electron varies as v^4 (strictly speaking, only if $v \ll c$), while the quadrupole radiation from two circulating electrons at opposite ends of a diameter varies as v^6. We infer that the power radiated by n electrons evenly spaced around a ring varies as v^{2n+2}, which is very small for large n.

   If you like technical details, see Ph501 Prob. 7, Set 8.

   The problem of n electrons around a ring was first posed (and solved via series expansions without explicit mention of Bessel functions) by J.J. Thomson, Phil. Mag. **6**, 673 (1903). He knew that atoms (in what we now call their ground state) don't radiate, and used this argument to support his model that the electric charge in an atom must be smoothly distributed. This was a classical precursor to the view of a continuous probability distribution for the electron's position in an atom.

   Thomson's work was followed shortly by an extensive treatise by G.A. Schott, *Electromagnetic Radiation* (Cambridge U.P., 1912), that included analyses in term of Bessel functions correct for any value of v/c.

   These pioneering works were largely forgotten during the following era of nonrelativistic quantum mechanics, and were reinvented around 1945 when interest emerged in relativistic particle accelerators. See, for example, the famous "unpublished" paper of Schwinger, http://puhep1.princeton.edu/~mcdonald/accel/schwinger.pdf

4. Griffiths' prob. 11.14. Analyze the time dependence of the total energy of the electron in the approximation that the radius of the electron's orbit changes very little per revolution to show that the electron would spiral into the origin in time

$$t_0 = \frac{a_0^3}{4r_0^2 c} = \frac{1}{4\alpha^5}\frac{\lambda_C}{c} \approx 10^{-11} \text{ s},$$

   where $r_0 = \alpha\lambda_C$ is the classical electron radius, $\lambda_C = \hbar/mc = 3.9 \times 10^{-13}$ m is the (reduced) Compton wavelength of the electron, $a_0 = \lambda_C/\alpha = 5 \times 10^{-11}$ m is the Bohr radius, and $\alpha = e^2/4\pi\epsilon_0\hbar c$ is the fine structure constant. The time t_0 is about α times the lifetime of the $2P_{1/2}$ state. For "circular" orbits in quantum mechanics, $l = n-1$ and the radius of the orbit scales as $n^{3/2}$. Replacing a_0 in the above by $n^{3/2}a_0$ predicts that the lifetime of "circular" orbits scales as $n^{4.5}$, in agreement with quantum mechanics (see p. 269 of the book by Bethe and Salpeter).

5. Griffiths' prob. 11.16.

6. Griffiths' prob. 11.23.