Princeton University
Ph304 Problem Set 4
Electrodynamics

(Due in class, Friday, March 7)

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Problem sessions: Sundays, 8 pm, Jadwin 303

Text: *Introduction to Electrodynamics, 3rd ed.*
by D.J. Griffiths (Prentice Hall, ISBN 0-13-805326-X, now in 6th printing)
Errata at http://academic.reed.edu/physics/faculty/griffiths.html

Reading: Griffiths secs. 4.3-4.4, 7.1.1-7.1.2

1. Griffiths' probs. 4.4 and 4.7. For problem 4.4, evaluate both the force on the charge due to the (induced, *i.e.*, not permanent) dipole field, and the force on the dipole due to the charge.

   One way to approach prob. 4.7 (in which "ideal dipole" means "tiny, permanent dipole") is to recast eq. (4.5) as the gradient of some quantity, using an appropriate vector calculus identity. Then, invoking the mechanical relation $\mathbf{F} = -\nabla U$, you can identify the energy U. Note that this transformation holds only if $\nabla \times \mathbf{E} = 0$, *i.e.*, it holds only for electrostatics.

   Counterexample: if $\mathbf{p} = p\hat{\mathbf{x}}$ and $\mathbf{E} = kx\hat{\mathbf{y}}$, then $\mathbf{p} \cdot \mathbf{E} = 0$, but the force on the dipole is nozero according to eq. (4.5). However, in this case $\nabla \times \mathbf{E} \neq 0$, so there must be a time-dependent magnetic field present, and the problem is not one of electrostatics.

   You should find that $U = -\mathbf{p} \cdot \mathbf{E}$ for a dipole of fixed magnitude p, but $U = -\frac{1}{2}\mathbf{p} \cdot \mathbf{E}$ for an induced dipole that obeys $\mathbf{p} = \alpha\mathbf{E}$. The difference arises because there is a positive energy of deformation of the induced dipole – as could be confirmed by a model for polarizable molecules. Use $\mathbf{F} = -\nabla U$ to calculate the force on the dipole in prob. 4.4, and compare with the result obtained using eq. (4.5).

   This illustrates that to deduce the force from the energy U, the total energy must be used. We note the famous paradox hinted in sec. 4.4.4, that if $U = U_{\text{electrostatic}}$, then $\mathbf{F} = -\nabla U_{\text{el}}$ is ok if the external field is maintained by a set of fixed charges (as in prob. 4.4), but when the external fields are maintained by a set of conductors held at fixed potentials by batteries, then $\mathbf{F} = -\nabla U_{\text{tot}} = -\nabla(U_{\text{batt}} + U_{\text{el}}) = +\nabla U_{\text{el}}$, because $\nabla U_{\text{battery}} = -2\nabla U_{\text{el}}$.

2. Griffiths' prob. 4.28.

3. Griffiths' prob. 4.34. By now you may appreciate such shortcuts as noting that since the dipole is the ultimate source of the all of the electric potential in this problem, and that the dipole potential varies as $P_1(\cos\theta)$, so only terms in the Legendre polynomial expansion will be those in P_1. Hence, you could write

$$V(r < R) = \left(\frac{A_1}{r^2} + B_1 r\right) P_1(\cos\theta),$$

$$V(r > R) = \left(\frac{C_1}{r^2} + D_1 r\right) P_1(\cos\theta).$$

   To determine the four unknowns A_1, B_1, C_1, and D_1, you need four equations. Of course, the potential at infinity can be set to zero in this problem, so $D_1 = 0$. However, we cannot set A_1 to zero, because of the dipole at the origin; indeed, the term $A_1 P_1/r^2$ is the potential (for $r < R$) due to dipole p when it is embedded inside a dielectric of constant ϵ_r. A way to get at this term is to consider the electric displacement $\mathbf{D}$, which is due only to the external charges in the problem, *i.e.*, due only to the dipole at the origin. In general, the field $\mathbf{D}$ due to a specified external charge distribution is the same as the field $\mathbf{E}$ due to that charge distribution in vacuum, but multiplied by ϵ_0. Thus, from Griffiths' eq. (3.102), we know that

$$\mathbf{D}_{\rm dipole} = -\nabla \frac{pP_1}{4\pi r^2}.$$

But, we also know that

$$\mathbf{D}_{\rm dipole} = \epsilon_r \epsilon_0 \mathbf{E}_{\rm dipole\ inside\ dielectric} = -\epsilon_r \epsilon_0 \nabla V_{\rm dipole\ inside\ dielectric} = -\epsilon_r \epsilon_0 \nabla \left(\frac{A_1 P_1}{r^2} \right).$$

etc.

Show also that the results of this problem satisfy Griffiths' eq. (3.105), when you take into account that the complete field of a point dipole in vacuum is given by Griffiths' eq. (3.106). (What is the expression for the field of a point dipole embedded in a medium of relative dielectric constant ϵ_r?) The total dipole moment of the system is the sum of the original point dipole $\mathbf{p}$ and the integral of the induced polarization density $\mathbf{P} = (\epsilon_r - 1)\epsilon_0 \mathbf{E}$.

4. Griffiths' prob. 4.38.

5. Griffiths' prob. 7.1.

6. Griffiths' prob. 7.41. The premise of this problem is that there will be a steady current that travels distance $2\pi a$ inside the (curved but otherwise uniform) sheet. Since the voltage drop around the circumference of the sheet is V_0, the electric field strength inside the sheet should be $V_0/2\pi a$ (what is its direction?). Verify that an appropriate derivative of the potential $V(s, \phi)$ found in part (a) leads to this result.

   The surface charge found in part (b) is needed to shape the electric field to the desired uniform value on the surface. It should therefore be no surprise that a (small) surface charge is required to maintain the electric field that drives a steady current inside an ordinary wire....

   The total charge on the surface is zero. [Can you give a simple argument for this, before solving for the potential?] This fact means that you can neglect the logarithmic term that appears in the general (exterior) solution for the potential in cylindrical coordinates. Can you give another argument as to why the logarithmic term is not present?