Princeton University

Ph304 Problem Set 5
Electrodynamics

(Due in class, Friday, March 14)

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Text: *Introduction to Electrodynamics, 3rd ed.*
by D.J. Griffiths (Prentice Hall, ISBN 0-13-805326-X, now in 6th printing)
Errata at http://academic.reed.edu/physics/faculty/griffiths.html

Reading: Griffiths secs. 5.1 – 5.4.

1. Griffiths' prob. 5.11.

2. Griffiths' prob. 5.38. Solve this problem both using Ohm's law (7.2) noting that $J_\perp = 0$, and using special relativity by going to the rest frame of the moving negative charge.

3. Griffiths' prob. 5.39. d) Hall also realized that his effect could be used to determine the speed v of the charges by extending the analysis of part b). The current density J is readily calculated from knowledge of the size of the sample and the current I. Express the Hall voltage in terms of J, B and a (measurable) constant R_H (the Hall coefficient). Then use Ohm's law in the form $J = \sigma E_0$, where E_0 is the electric field (due to a battery) along the direction of the current to find an expression for the speed v. Estimate a numerical value for v, noting that for copper, $\sigma \approx 10^8$ SI units, $R_H \approx 10^{-10}$ SI units, and the longitudinal field E_0 in Hall's experiment was about 1 V/m.

   Edwin Hall was a graduate student at Johns Hopkins in 1879 when he performed his experiment, at the suggestion of Rowland. It established the sign of the charge carriers in metallic conduction as negative, and their speed as astonishingly small. The experiment was motivated by their intuition that Maxwell (who died in 1879) was wrong when he claimed (sec. 501 of his Treatise) that currents are not affected by magnetic fields, but only the conductors experience the Biot-Savart force.

   A useful discussion of how the magnetic force on the conduction electrons results in a force on the positive ion lattice of the conductor is given in Am. J. Phys. **49**, 493 (1981).

   Note that our concept of a conductor is extended in two ways by the above discussion. First, when currents flow inside an Ohmic conductor, there must be an electric field to drive the currents. Second, when conductors are inside magnetic fields and charges are in motion, then additional electric fields may arise.

4. Griffiths' prob. 5.41.

5. Griffiths' prob. 5.42. Show also that the same result is obtained via the integral

$$\frac{1}{2\mu_0}\int B_z^2\, d\text{Area}$$

   over the equatorial plane of the sphere (both inside and out). This hints that the Maxwell stress tensor for magnetic fields can be written

$$T_{ij} = \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right),$$

   so that one can think of magnetic field lines as exerting a tension $B^2/2\mu_0$ along their direction (and a pressure $B^2/2\mu_0$ transverse to their direction), similar to that for the electric field. See our comment to Griffiths' prob. 2.38 (Set 2).

6. Griffiths' prob. 5.46. The magnetic field of the Helmholz coil is circularly symmetric: $\mathbf{B} = B_r(r, z)\hat{\mathbf{r}} + B_z(r, z)\hat{\mathbf{z}}$, where $\mathbf{r}$ is the radius vector perpendicular to the z axis. Use the Maxwell equation $\nabla \cdot \mathbf{B} = 0$ and a Gaussian pillbox of radius r and thickness dz with axis along the z axis to show that for small r,

$$B_r(r, z) \approx -\frac{r}{2}\frac{\partial B_z(0, z)}{\partial z}\,.$$

Then use $\nabla \times \mathbf{B} = 0$ (in cylindrical coordinates) to show that

$$B_z(r, z) \approx B_z(0, z) - \frac{r^2}{4}\frac{\partial^2 B_z(0, z)}{\partial z^2}\,.$$

Clearly, these are the leading terms in a series expansion.

Use these relations to evalulate B_r and B_z near the axis of the Helmholtz coil, in terms of $B_0 = B_z(0, 0)$.