Princeton University

Ph304 Problem Set 7
Electrodynamics

(Due in class, Wed. Apr. 2, 2003)

Instructor: Kirk T. McDonald, Jadwin 309/361, x6608/4398
kirkmcd@princeton.edu
http://puhep1.princeton.edu/~mcdonald/examples/

AI: Matthew Sullivan, 303 Bowen Hall, x8-2123
mtsulliv@princeton.edu

Problem sessions: Sundays, 7 pm, Jadwin 303

Text: *Introduction to Electrodynamics, 3rd ed.*
by D.J. Griffiths (Prentice Hall, ISBN 0-13-805326-X, now in 6th printing)
Errata at http://academic.reed.edu/physics/faculty/griffiths.html

Reading: Griffiths chap 7.

1. Griffiths' prob. 6.25. For a more dramatic version of this problem, see http://puhep1.princeton.edu/~mcdonald/examples/diamagnetic.pdf

   Other magnetic leviation websites include http://www.psfc.mit.edu/ldx/levcam.html

2. Griffiths' prob. 6.27. Griffths has in mind a solution for the magnetic field $\mathbf{B}$ via considerations of free and bound currents. This is tricky enough that he awards the problem a **!** However, as is typical for problems involving magnetic media, this problem is susceptible to a more straightforward solution that emphasizes the field $\mathbf{H}$, since away from the origin, $\nabla \times \mathbf{H} = 0$, so you can write $\mathbf{H} = -\nabla W$ where the scalar potential W obeys Laplace's equation in the two regions $0 < r < R$ and $R < r$.

   Whether your solution emphasizes $\mathbf{B}$ or $\mathbf{H}$, to get started you need to understand the character of the field (or potential) for small r (inside the medium). One view is that since $\nabla \times \mathbf{H} = \mathbf{J}_{\text{free}}$ and $\mathbf{J}_{\text{free}}$ is entirely due to the point dipole, the field $\mathbf{H}$ (and the potential W) near the dipole is essentially that of the point dipole alone, as if the medium weren't there. If you prefer to consider the field $\mathbf{B}$, which is due to both free and bound currents, recall eq. (6.33).

   Not for credit in 2003: Show also that the magnetic field $\mathbf{B}$ of this problem obeys Griffths' eq. (5.89) if you use the full form (5.90) for the field of a point dipole in vacuum (suitably modified for a point dipole embedded inside a magnetic medium). The total dipole moment of the system is the sum of the original point dipole $\mathbf{m}$ and the integral of the induced magneitzation $\mathbf{M} = \chi_m \mathbf{H} = (\mu - \mu_0)\mathbf{B}/\mu\mu_0$.

3. Griffiths' prob. 7.44.

4. Variant of Griffiths' prob. 7.52. a) For even greater simplicity, suppose that both radii a and b are small compared to the height z. b) The real interest in this problem is, however, when $z \ll a \approx b$, because this calculation gives us insight into the question of the self inductance of a single loop of wire (a torus) with major radius a and minor radius z. Show that eq. (7.22) leads to

$$M_{12} \approx \mu_0 a \int_0^{\pi/2} \frac{\cos 2\alpha \; d\alpha}{\sqrt{\sin^2 \alpha + c^2/4a^2}} \,,$$

   where $c^2 = (a-b)^2 + z^2$, and $\alpha = \phi/2$ with ϕ as the azimuthal angle between a point on loop a and one on loop b. Evaluate this integrate by splitting it into 2 parts, $0 < \alpha < \epsilon$ and $\epsilon < \alpha < \pi/2$, where $c/2a \ll \epsilon \ll 1$. The result should be independent of ϵ:

$$M_{12} \approx \mu_0 a \left(\ln \frac{8a}{c} - 2 \right).$$

   It can then be shown that a wire of radius c formed into a circular loop of radius a has self inductance given by the above expression, but with the 2 changed to 7/4. This important result is not, however, very evident from the series expansion for M_{12} proposed by Griffiths.

5. Griffiths' prob. 7.58.

6. Griffiths' prob. 7.59.