Ph 406 Final Exam

Princeton University
1:30-4:30 p.m., May 21, 1992
Palmer 222

- Do not begin the exam until told to do so.
- The exam consists of 4 problems, each worth 10 points.
- Do all work you wish graded in the exam booklets provided.
- The exam is closed book, closed notes; two 'crib sheets' are included as part of the exam material.

1. In an alternate universe there are three charge states of the spin-$\frac{1}{2}$ nucleons, +, 0, and −. These all have masses roughly that of the proton, and experience strong, electromagnetic, and weak interactions of strengths similar to those in our universe.

   (a) Indicate the form of a semi-empirical mass formula for nuclei made from these nucleons, in terms of the nucleon numbers $A = N_+ + N_0 + N_-$, $Z = N_+ - N_-$, and N_0. You may ignore any pairing term.

   (b) Deduce the trend of Z and N_0 *vs.* A for beta-stable nuclei. Are heavier nuclei more likely to decay via fission or 'α'-decay (where an 'α' particle in this universe has $N_+ = N_0 = N_- = 2$)?

   Would you like to live in this universe?

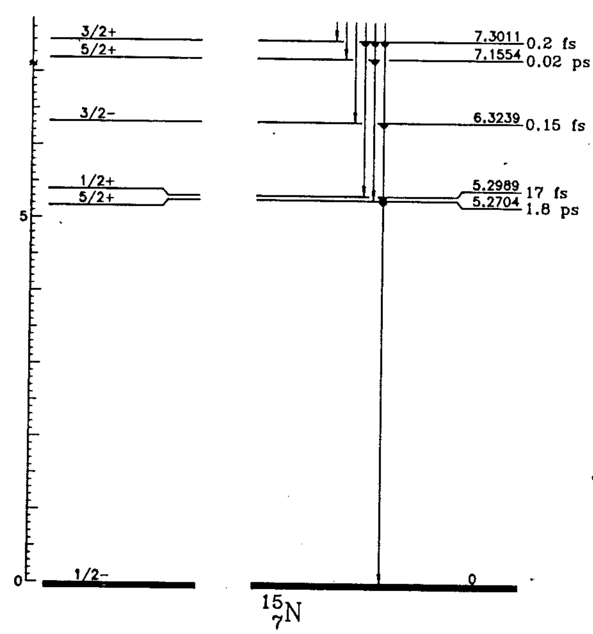
2. Excited nuclei:

   (a) The spin-parity J^P and excitation energy E of the first few states of ${}^{170}_{72}\text{Hf}$ are

J^P	0^+	2^+	4^+	6^+	8^+
E(keV)	0	100	321	641	1041

   Explain how these states are indicative of rotational excitations, and deduce the moment of inertia of the Hf nucleus. Compare this with the moment of inertia supposing the Hf nucleus is a sphere ($I = \frac{2}{5}MR^2$).

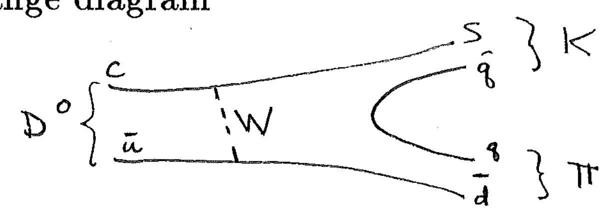
   (b) Give the shell-model configurations for the ground state and first five excited states of ${}^{15}_{7}\text{N}$ shown below, supposing all states can be accounted for by single-particle excitations.



3. Elementary particle decays:

   (a) The $\psi'(3685)$ vector meson can decay to $\chi(3415)+\gamma$. The χ particle is believed to be a $^3P_0$ $c\bar{c}$ state. If so, predict the angular distribution of the γ relative to the direction of the electron supposing the ψ' is produced in a colliding-beam experiment $e^+e^- \to \psi' \to \chi\gamma$. Recall that at high energies the one-photon annihilation of e^+e^- proceeds entirely via tranversely polarized photons ($S_z = \pm 1$).

   (b) The charmed meson D^0 can decay to $K\pi$ via the Cabbibo-favored W-exchange diagram



   If this were the only possible diagram, predict the ratio of branching ratios:

$$\frac{\Gamma(D^0 \to K^-\pi^+)}{\Gamma(D^0 \to \bar{K}^0\pi^0)}.$$

   You may assume the $K\pi$ system is in an isospin-$\frac{1}{2}$ state.

   Draw any other Cabbibo-favored diagrams for these decays.

4. The elastic scattering reactions $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ can only take place via Z^0 exchange. Estimate the total cross sections for these reaction at high energies.

   Recall that the piece of the gauge-covariant derivative relevant to the Z^0 can be written

$$i\frac{g}{\cos\theta_W}Z(I_3 - \sin^2\theta_W Q),$$

   where $g^2/\cos^2\theta_W M_Z^2 = G/\sqrt{2}$, I_3 refers to weak isospin, and Q is electric charge in units of $+e$. Also, the total cross section is the integral over the angular distribution, which should be examined separately for left- and right-handed electrons (in the center-of-mass frame, of course)....

Table 5.1

	x_{nl}	Neutron		Proton		
1s	3.14	$1s_{\frac{1}{2}}$	2	2	$1s_{\frac{1}{2}}$	1s
1p	4.49	$1p_{\frac{3}{2}}$	6	6	$1p_{\frac{3}{2}}$	1p
		$1p_{\frac{1}{2}}$	8	8	$1p_{\frac{1}{2}}$	
1d	5.76	$1d_{\frac{5}{2}}$	14	14	$1d_{\frac{5}{2}}$	1d
		$2s_{\frac{1}{2}}$	16	16	$2s_{\frac{1}{2}}$	
2s	6.28	$1d_{\frac{3}{2}}$	20			2s
				20	$1d_{\frac{3}{2}}$	
1f	6.99	$1f_{\frac{7}{2}}$	28			
		$2p_{\frac{3}{2}}$	32	28	$1f_{\frac{7}{2}}$	
2p	7.73	$1f_{\frac{5}{2}}$	38			1f
		$2p_{\frac{1}{2}}$	40	32	$2p_{\frac{3}{2}}$	
1g	8.18	$1g_{\frac{9}{2}}$	50	38	$1f_{\frac{5}{2}}$	2p
		$2d_{\frac{5}{2}}$	56	40	$2p_{\frac{1}{2}}$	
2d	9.10	$1g_{\frac{7}{2}}$	64			
		$1h_{\frac{11}{2}}$	76	50	$1g_{\frac{9}{2}}$	
1h	9.36	$3s_{\frac{1}{2}}$	78			1g
		$2d_{\frac{3}{2}}$	82	58	$1g_{\frac{7}{2}}$	
3s	9.42	$2f_{\frac{7}{2}}$	90	64	$2d_{\frac{5}{2}}$	
		$1h_{\frac{9}{2}}$	100			
2f	10.42	$3p_{\frac{3}{2}}$	104			2d
1i	10.51	$1i_{\frac{13}{2}}$	118	76	$1h_{\frac{11}{2}}$	
		$2f_{\frac{5}{2}}$	124	80	$2d_{\frac{3}{2}}$	1h
3p	10.90	$3p_{\frac{1}{2}}$	126	82	$3s_{\frac{1}{2}}$	
						3s
		$2g_{\frac{9}{2}}$	136			
1j	11.66	$1i_{\frac{11}{2}}$	148	92	$1h_{\frac{9}{2}}$	
		$2g_{\frac{7}{2}}$	156	100	$2f_{\frac{7}{2}}$	
2g	11.70					

The first and last columns give the sequence of energy levels in a spherical well with infinite walls. The second column gives the corresponding values of $x_{nl} = k_{nl}R$. The third column gives the observed sequence of spin-orbit coupled levels for neutrons, and the fourth the cumulative number of available states in these levels. The remaining two columns give the levels and number of states for protons. The spacings are chosen so that the filling of the neutron and proton shells for stable nuclei is approximately in step down the columns. Lines are drawn at the 'magic numbers'.

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

m_1	m_2	J M	J M	...
m_1	m_2	Coefficients		
.	.			
.	.			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^{\,\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^{\,j}_{m',m} = (-1)^{m-m'} d^{\,j}_{m,m'} = d^{\,j}_{-m,-m'}$$

1/2×1/2

m_1	m_2	$J=1$, $M=+1$
+1/2	+1/2	1

m_1	m_2	$J=1$, $M=0$	$J=0$, $M=0$
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	$J=1$, $M=-1$
−1/2	−1/2	1

1×1/2

m_1	m_2	$J=3/2$, $M=+3/2$
+1	+1/2	1

m_1	m_2	$J=3/2$, $M=+1/2$	$J=1/2$, $M=+1/2$
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	$J=3/2$, $M=-1/2$	$J=1/2$, $M=-1/2$
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	$J=3/2$, $M=-3/2$
−1	−1/2	1

2×1/2

m_1	m_2	$J=5/2$, $M=+5/2$
+2	+1/2	1

m_1	m_2	$J=5/2$, $M=+3/2$	$J=3/2$, $M=+3/2$
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	$J=5/2$, $M=+1/2$	$J=3/2$, $M=+1/2$
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	$J=5/2$, $M=-1/2$	$J=3/2$, $M=-1/2$
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	$J=5/2$, $M=-3/2$	$J=3/2$, $M=-3/2$
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	$J=5/2$, $M=-5/2$
−2	−1/2	1

3/2×1/2

m_1	m_2	$J=2$, $M=+2$
+3/2	+1/2	1

m_1	m_2	$J=2$, $M=+1$	$J=1$, $M=+1$
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	$J=2$, $M=0$	$J=1$, $M=0$
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	$J=2$, $M=-1$	$J=1$, $M=-1$
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	$J=2$, $M=-2$
−3/2	−1/2	1

2×1

m_1	m_2	$J=3$, $M=+3$
+2	+1	1

m_1	m_2	$J=3$, $M=+2$	$J=2$, $M=+2$
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	$J=3$, $M=+1$	$J=2$, $M=+1$	$J=1$, $M=+1$
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	$J=3$, $M=0$	$J=2$, $M=0$	$J=1$, $M=0$
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	$J=3$, $M=-1$	$J=2$, $M=-1$	$J=1$, $M=-1$
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	$J=3$, $M=-2$	$J=2$, $M=-2$
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	$J=3$, $M=-3$
−2	−1	1

1×1

m_1	m_2	$J=2$, $M=+2$
+1	+1	1

m_1	m_2	$J=2$, $M=+1$	$J=1$, $M=+1$
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	$J=2$, $M=0$	$J=1$, $M=0$	$J=0$, $M=0$
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	$J=2$, $M=-1$	$J=1$, $M=-1$
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	$J=2$, $M=-2$
−1	−1	1

3/2×1

m_1	m_2	$J=5/2$, $M=+5/2$
+3/2	+1	1

m_1	m_2	$J=5/2$, $M=+3/2$	$J=3/2$, $M=+3/2$
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	$J=5/2$, $M=+1/2$	$J=3/2$, $M=+1/2$	$J=1/2$, $M=+1/2$
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	$J=5/2$, $M=-1/2$	$J=3/2$, $M=-1/2$	$J=1/2$, $M=-1/2$
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	$J=5/2$, $M=-3/2$	$J=3/2$, $M=-3/2$
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	$J=5/2$, $M=-5/2$
−3/2	−1	1

3/2×3/2

m_1	m_2	$J=3$, $M=+3$
+3/2	+3/2	1

m_1	m_2	$J=3$, $M=+2$	$J=2$, $M=+2$
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	$J=3$, $M=+1$	$J=2$, $M=+1$	$J=1$, $M=+1$
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	$J=3$, $M=0$	$J=2$, $M=0$	$J=1$, $M=0$	$J=0$, $M=0$
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	$J=3$, $M=-1$	$J=2$, $M=-1$	$J=1$, $M=-1$
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	$J=3$, $M=-2$	$J=2$, $M=-2$
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	$J=3$, $M=-3$
−3/2	−3/2	1

2×3/2

m_1	m_2	$J=7/2$, $M=+7/2$
+2	+3/2	1

m_1	m_2	$J=7/2$, $M=+5/2$	$J=5/2$, $M=+5/2$
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	$J=7/2$, $M=+3/2$	$J=5/2$, $M=+3/2$	$J=3/2$, $M=+3/2$
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	$J=7/2$, $M=+1/2$	$J=5/2$, $M=+1/2$	$J=3/2$, $M=+1/2$	$J=1/2$, $M=+1/2$
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	$J=7/2$, $M=-1/2$	$J=5/2$, $M=-1/2$	$J=3/2$, $M=-1/2$	$J=1/2$, $M=-1/2$
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	$J=7/2$, $M=-3/2$	$J=5/2$, $M=-3/2$	$J=3/2$, $M=-3/2$
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	$J=7/2$, $M=-5/2$	$J=5/2$, $M=-5/2$
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	$J=7/2$, $M=-7/2$
−2	−3/2	1

2×2

m_1	m_2	$J=4$, $M=+4$
+2	+2	1

m_1	m_2	$J=4$, $M=+3$	$J=3$, $M=+3$
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	$J=4$, $M=+2$	$J=3$, $M=+2$	$J=2$, $M=+2$
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	$J=4$, $M=+1$	$J=3$, $M=+1$	$J=2$, $M=+1$	$J=1$, $M=+1$
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	$J=4$, $M=0$	$J=3$, $M=0$	$J=2$, $M=0$	$J=1$, $M=0$	$J=0$, $M=0$
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	$J=4$, $M=-1$	$J=3$, $M=-1$	$J=2$, $M=-1$	$J=1$, $M=-1$
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	$J=4$, $M=-2$	$J=3$, $M=-2$	$J=2$, $M=-2$
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	$J=4$, $M=-3$	$J=3$, $M=-3$
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	$J=4$, $M=-4$
−2	−2	1

d functions

$$d^{\,1}_{0,0} = \cos\theta \qquad d^{\,1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{\,1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^{\,1}_{1,1} = \frac{1+\cos\theta}{2} \qquad d^{\,1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^{\,1}_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{\,3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{\,3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^{\,2}_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^{\,2}_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{\,2}_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^{\,2}_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^{\,2}_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2$$

$$d^{\,2}_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{\,2}_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{\,2}_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^{\,2}_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 36.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).