Ph 406: Elementary Particle Physics
Problem Set 7

K.T. McDonald
kirkmcd@princeton.edu
Princeton University
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1. **Baryon Magnetic Moments in SU(2)**

   The requirement that the interaction Hamiltonian be an isoscalar leads to the understanding that the magnetic moment μ of spin-1/2 baryons has the form

   $$\mu = A + BI_3, \tag{1}$$

   for particles within an isospin multiplet, where the constants A and B differ from multiplet to multiplet.[1] Deduce the one nontrivial relation among magnetic moments of the basic spin-1/2 baryon octet that can be made from eq. (1). Is this prediction testable by the methods used to measure magnetic moments of short-lived baryons?

   If the form (1) also holds for the spin-3/2 baryon decuplet, what relations among their magnetic moments are implied?[2] Are these predictions testable?

2. **Baryon Magnetic Moments in SU(3)**

   If SU(3) were a good symmetry, then relations like (1) should also hold for so-called U-spin and V-spin, which provides relations among magnetic moments in of particles in different isospin multiplets. While SU(3) is not an exact symmetry for baryon "flavor," Gell-Mann[3] and Okubo[4] had good success in relating baryon masses by supposing the pattern of symmetry breaking was between, but not within, U-spin multiplets.

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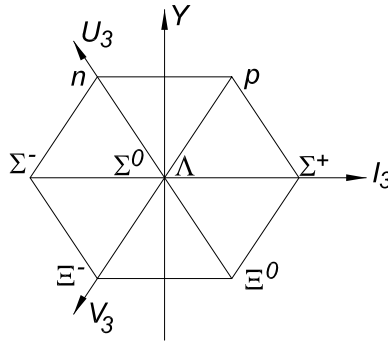
[1]R. Marshak, S. Okubo and G. Sudarshan, *Consequences of Charge Independence for the Magnetic Moments and Masses of Σ Hyperons*, Phys. Rev. **106**, 599 (1957),
http://kirkmcd.princeton.edu/examples/EP/marshak_pr_106_599_57.pdf.
   [2]The magnetic moment μ of a spin-3/2 particle can be in four states with respect to some z-axis, $\mu_z = \mu$, $\mu/3$, $-\mu/3$ and $-\mu$. *Note the appearance of a fractional quantum number.*
   [3]M. Gell-Mann, *The Eightfold Way: A Theory of Strong Interaction Symmetry*, CTSL-20 (1961),
http://kirkmcd.princeton.edu/examples/EP/gellmann_ctsl-20_61.pdf.
   [4]S. Okubo, *Note on Unitary Symmetry in Strong Interactions*, Prog. Theor. Phys. **27**, 949 (1962),
http://kirkmcd.princeton.edu/examples/EP/okubo_ptp_27_949_62.pdf.

Hence, it seems reasonable to suppose that eq. (1) holds only for I-spin, and that magnetic moments are the same for all members of a U-spin multiplet,[5] but differ for different U-spin multiplets. Show that the magnetic moments of the spin-1/2 baryon octet, and the so-called transition moment $\langle \Sigma^0 | \mu | \Lambda \rangle$, can then be expressed in terms of μ_p and μ_n.

A result from prob. 1 is needed here. Note that the $U_3 = 0$ states corresponding to the $I_3 = 0$ states Λ and Σ^0 can be obtained from the latter via rotation by $120°$ in SU(3) space (I_3-Y space).

The present data are, in terms of the nuclear/nucleon magneton $\mu_N = e\hbar/2m_p c$,

$$\langle p | \mu | p \rangle = \mu_p = 2.79 \mu_N, \tag{2}$$
$$\mu_n = -1.91 \mu_N, \tag{3}$$
$$\mu_\Lambda = -0.613 \pm 0.004 \mu_N, \tag{4}$$
$$\mu_{\Sigma^+} = 2.46 \pm 0.01 \mu_N, \tag{5}$$
$$\langle \Sigma^0 | \mu | \Lambda \rangle = -1.61 \pm 0.08 \mu_N, \tag{6}$$
$$\mu_{\Sigma^-} = -1.16 \pm 0.03 \mu_N, \tag{7}$$
$$\mu_{\Xi^0} = -1.25 \pm 0.02 \mu_N, \tag{8}$$
$$\mu_{\Xi^-} = -0.651 \pm 0.003 \mu_N, \tag{9}$$

3. **Baryon Magnetic Moments in the SU(6) Constituent-Quark Model**

Deduce the quark flavor + spin wavefunctions for the spin-up states of the spin-1/2 baryon octet, and use these to predict their magnetic moments.

In this model we suppose the quarks have Dirac magnetic moments,

$$\mu_q = \frac{Q_q \hbar}{2m_q c}, \tag{10}$$

where m_q is the (constituent-model) mass of the quark.[6] Use the data given in prob. 2 for μ_p, μ_n and μ_Λ to determine m_u, m_d and m_s, and give model predictions for the remaining baryon-octet magnetic moments (including the transition moment $\langle \Sigma^0 | \mu | \Lambda \rangle$).

The quarks are fermions, so the total wavefunction should be antisymmetric under quark exchange. There is no orbital angular momentum in the quark wavefunctions for the baryon octet, so the spatial part of the wavefunctions are symmetric under quark exchange. In the SU(6) symmetry of quark flavor + spin it seems most natural to consider the baryon octet and decuplet as comprising the symmetric multiplet **56** = (**8**,**2**) + (**10**,**4**), which seemed initially to contradict that quarks are fermions. Only in the 1970's did it become clear that quarks carry color charge that obeys an SU(3)

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[5]The Dirac magnetic moment a spin-1/2 particle is $Q\hbar/2Mc$, so if SU(3) is a good symmetry all baryon octet masses are the same, and the magnetic moments should be the same within U-spin multiplets for which the members have the same charge. And, the magnetic moments of the $U = 1$ and $U = 0$ multiplets differ only by their sign. This factoid turns out to be equivalent to the requirement that the sum of the baryon octet magnetic moments is zero if SU(3) is a good symmetry.

[6]If SU(3) were a good symmetry for quark flavor then all quarks would have the same mass m_q.

color symmetry in which hadrons are color singlets,[7] which are antisymmetric under exchange.[8] Hence, you are to construct $SU(18) = SU(3)_{\text{flavor}} \times SU(2)_{\text{spin}} \times SU(3)_{\text{color}}$ wavefunctions in which the $SU(3)_{\text{flavor}} \times SU(2)_{\text{spin}}$ part is exchange symmetric, and the $SU(3)_{\text{color}}$ part is antisymmetric (which latter part need not be displayed in your solution).

4. Show that the relative color-force amplitudes for the $q\overline{q}$ colored-quark states $\psi_{\text{color}} = (r\overline{r} - g\overline{g})/\sqrt{2}$ and $(2b\overline{b} - r\overline{r} - g\overline{g})/\sqrt{6}$ due to the exchange of a single color-octet gluon are $1/3$ (as are the related amplitudes $\langle r\overline{g}|\text{color}|r\overline{g}\rangle$, $etc.$, discussed on pp. 262-265, Lecture 14 of the Notes). The implication is that color-octet $q\overline{q}$ states are not bound.

[7]H. Fritzsch, M. Gell-Mann and H. Leutwyler, *Advantages of the Color Octet Gluon Picture*, Phys. Lett. **47B**, 365 (1973), http://kirkmcd.princeton.edu/examples/EP/fritzsch_pl_b47_365_73.pdf.
[8]The only antisymmetric qqq multiplet in color $SU(3)$, $\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{8} + \mathbf{8^\star} + \mathbf{10}$, is the $\mathbf{1}$. The r, g, b color singlet wavefunction is $\psi_{\text{color}} = (rgb - grb + gbr - bgr + brg - rbg)/\sqrt{(6)}$.