

# PHYSICS 529

## INTRODUCTION TO HIGH ENERGY PHYSICS

KIRK McDONALD

308 JADWIN

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### RECOMMENDED READING:

- INTRO. TO HIGH ENERGY PHYSICS ; PERKINS , 3RD ED. (1987) ADDISON-WESLEY.

THE COURSE WILL ROUGHLY FOLLOW THE ORGANIZATION OF PERKINS, EXCEPT THAT MUCH OF CHAPTER 7 WILL BE COVERED BEFORE CHAP. 3

### WORKS THAT EXPAND UPON THE COURSE MATERIAL:

- WHOLE COURSE : GAUGE THEORIES & THE NEW PHYSICS, LEADER & PREDAZZI, CAMBRIDGE (1982).
- QED : RELATIVISTIC Q.M., BJORKEN & DRELL, MCGRAW-HILL (1964)  
: RELATIVISTIC QUANTUM THEORY, LANDAU & LIFSHITZ, VOL 4, PERGAMON (1971)
- QUARKS : INTRO. TO QUARKS & PARTONS ; CLOSE, ACADEMIC PRESS (1979)
- WEAK INT. : WEAK INT. OF LEPTONS & QUARKS ; COMINS & BUCKSBAUM, CAMBRIDGE (1983)
- GAUGE THEORIES : ELEMENTARY PRIMER FOR GAUGE THEORY ; MORIYASU, WORLD SCIENTIFIC (1983)  
: GAUGE THEORIES IN PARTICLE PHYSICS ; AITCHISON & HUY, HILGER (1982)  
GAUGE THEORIES OF S, U, & E-M INT ; QUIGG, BENJAMIN (1983)

### WORKS SIMILAR TO PERKINS:

- SUB-ATOMIC PHYSICS ; FRAUENFELDER & HENLEY, PRENTICE HALL (1974)
- ELEMENTARY PARTICLE PHYSICS ; CHEN & O'NEILL, ADDISON-WESLEY (1979)
- CONCEPTS OF PARTICLE PHYSICS ; GOTTFRIED & WEISSKOPF, OXFORD (1984)
- QUARKS & LEPTONS ; HALTEN & MARTIN, WILEY (1984)

### NOTABLE OLDER WORKS:

- UP TO 1950 : HIGH ENERGY PHYSICS, ROSSI, PRENTICE HALL (1952)
- UP TO MID '60S : INVARIANCE PRINCIPLES & ELEMENTARY PARTICLES, SAKURAI, PRINCETON (1964)  
INTRO. TO ELEMENTARY PARTICLES, WILLIAMS, ACADEMIC PRESS (1961, 1971)

### WORKS BY MASTERS PARTLY ACCESSIBLE AT 'ENTRY LEVEL'

- PHOTON-NADRON PHYSICS ; FEYNMAN, BENJAMIN (1972)
- PARTICLE PHYSICS ; T.D. LEE, HARWOOD (1981)

### THEORISTS TEXTBOOK:

- QUANTUM FIELD THEORY ; ITZYKSON & ZUBER, MCGRAW-HILL (1980)

### JOURNALISTS VIEW:

- THE PARTICLE CONNECTION ; SUTTON, SIMON & SCHUSTER (1984)

### SCI-FI BASED ON BUFF WORDS:

- BROKEN SYMMETRIES ; PREUSS, POCKET BOOKS (1983)

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COURSE OUTLINE

LECTURE 1. WHY HIGH ENERGY?

2. HISTORY; OVERVIEW OF THE STRONG INTERACTION
3. OVERVIEW OF WEAK & E-M INTERACTIONS
4. PHYSICS OF PARTICLE DETECTORS.
5. ELECTRO MAGNETIC STRUCTURE OF MATTER
6. ELASTIC SCATTERING OF ELECTRONS & HADRONS, INCLUDING SPIN
7.                    "
8. INELASTIC ELECTRON-HADRON SCATTERING
9. INVARIANCE PRINCIPLES AND CONSERVATION LAWS: T & P
10. CHARGE CONSERVATION; ISOSPIN; G-PARITY
11. 3 BODY DECAYS; PARTIAL WAVE ANALYSIS
12. PHENOMENOLOGY OF THE STRONG INTERACTION AT HIGH ENERGY
13. QUARK MODEL
14.                   "
15. HEAVY QUARK STATES
16. THE WEAK INTERACTION
17.                   "
18. NEUTRAL KAON DECAYS & CP VIOLATION
19. NEUTRINO INTERACTIONS
20. THE NEED FOR A BETTER THEORY
21. THE GLASHOW-SALAM-WEINBERG MODEL
22. TESTS OF THE GLASHOW-SALAM-WEINBERG MODEL
23. QUANTUM CHROMODYNAMICS
24. SPECULATIONS

WHY HIGH ENERGY?

OUR TOPIC IS THE NATURE OF THE UNIVERSE ON A VERY SMALL SCALE.

WHAT IS THE BEHAVIOR WE OBSERVE WHEN LOOKING THRU THE MOST POWERFUL MICROSCOPES?

OUR TOPIC WOULD SEEM TO BE THE OPPOSITE EXTREME FROM COSMOLOGY - THE NATURE OF THE UNIVERSE ON A VERY LARGE SCALE. IN NEITHER EXTREME IS THE UNIVERSE MUCH LIKE OUR EVERYDAY WORLD, CONTRARY TO SPECULATIONS BY SUCH PEOPLE AS PASCAL WHO INVOKE THE 'ANTHROPIC' PRINCIPLE.

WHY THEN IS THE TITLE OF THE COURSE 'HIGH ENERGY PHYSICS'?

IN THE PURSUIT OF THE SMALL WE FIND WE MUST THINK BIG. THIS WAS ALREADY UNDERSTOOD BY RUTHERFORD - THE FATHER OF SUB-ATOMIC PHYSICS.

WHEN WE TRY TO OBSERVE PHENOMENA ON A VERY SMALL SCALE, WHAT TECHNIQUE CAN WE USE? THERE ARE NO RULERS TO MEASURE DISTANCES OR SPRINGS TO MEASURE FORCES WHICH ARE SMALL ENOUGH. WE MUST HAVE SOME ASPECT OF OUR MEASURING APPARATUS - A PROBE - WHICH IS SMALLER THAN THE OBJECT TO BE STUDIED. ALSO WE MUST HAVE SOME MEANS OF BRINGING THE PROBE NEAR THE OBJECT.

BASICALLY THE ONLY TECHNIQUE AVAILABLE IS TO THROW ONE SMALL OBJECT AT ANOTHER, AND TRY TO PICK UP THE PIECES PRODUCED IF THEY COLLIDE.

FEYNMAN: "TRYING TO LEARN ABOUT ELEMENTARY PARTICLE PHYSICS IS LIKE TRYING TO LEARN HOW TO WRITE A SYMPHONY BY THROWING A PIANO OFF A TEN-STORY BUILDING."

AROUND 1900 RADIOACTIVE SOURCES BECAME AVAILABLE, WHICH EJECTED PARTICLES WITH ENERGIES OF 1 MILLION ELECTRON VOLTS - QUITE LARGE AT THE TIME. THE "X-RAYS" AND "β-RAYS" EMITTED BY THESE SOURCES ARE CHARGED PARTICLES. THE "γ-RAYS" ARE PHOTONS.

SUPPOSE A PARTICLE OF CHARGE  $q$  HAS INITIAL KINETIC ENERGY  $E$ , AND IS INCIDENT ON A TARGET PARTICLE OF CHARGE  $Q$  INITIALLY AT REST.

AT CLOSEST APPROACH, SAY DISTANCE  $Y$ , THE ELECTROSTATIC POTENTIAL ENERGY IS

$$\frac{qQ}{Y}$$

THUS  $E = \frac{qQ}{Y} + \text{K.E. AT CLOSEST APPROACH}$

AT BEST,  $K.E_{\text{CLOSEST}} = 0$ , FOR A HEAD-ON COLLISION WITH A HEAVY TARGET.

$$\therefore \lambda = \frac{h\phi}{E} \sim \frac{1}{E}$$

TO PROBE SMALL DISTANCES WE NEED HIGH ENERGY

WHY NOT JUST 'LOOK' AT THE TARGET USING LIGHT, AND AVOID THE PROBLEM OF COULOMB REPUSSION? THIS IS FINE, BUT WE CANNOT LEARN MUCH ABOUT THE TARGET UNLESS THE WAVE LENGTH OF THE LIGHT IS SMALLER THAN THE TARGET.

WE NEED  $\lambda \lesssim r$

BUT  $E_{\text{PHOTON}} = h\nu = hc/\lambda$

THANKS TO EINSTEIN

SO WE CAN ONLY PROBE SIZES  $r \gtrsim \frac{hc}{E}$  WITH A PHOTON OF

ENERGY  $E$ . AGAIN  $r \sim 1/E$ !

IT IS AMUSING TO COMPARE MAGNITUDES RELATED TO OUR TWO METHODS.

MEMORABLE FACT:  $hc = 197 \text{ MeV} \cdot \text{FERMI}$  (OR,  $hc = 1234 \text{ MeV} \cdot \text{FERMI}$ )

$1 \text{ MeV} = 1 \text{ MILLION ELECTRON VOLTS} = 1.6 \times 10^{-13} \text{ JOULE}$

$1 \text{ FERMI} = 10^{-15} \text{ M} = 10^{-13} \text{ CM.}$

THUS USING LIGHT, WE CAN PROBE  $r \gtrsim \frac{1200}{E}$

WITH  $r$  MEASURED IN FERMI'S, AND  $E$  IN MeV.

RETURNING TO THE CHARGED PARTICLE PROBE, SUPPOSE WE PROBE ONE PROTON WITH ANOTHER, THEN  $q = Q = e = 1.6 \times 10^{-19} \text{ COULOMB}$

$$r \gtrsim \frac{e^2}{E} = \left( \frac{e^2}{hc} \right) \frac{hc}{E}$$

MEMORABLE FACT:  $\frac{e^2}{hc} = \frac{1}{137} \equiv \alpha = \text{FINE STRUCTURE CONSTANT}$

( $\alpha$  IS DIMENSIONLESS).

SO IT WOULD APPEAR THAT A PROTON OF 1 MeV IS 137 TIMES BETTER AS A PROBE THAN A PHOTON OF 1 MeV.

BUT THERE IS MORE TO QUANTUM MECHANICS THAN  $E = h\nu$ .

DE BROGLIE TELLS US THAT THERE IS A WAVELENGTH ASSOCIATED WITH THE PROTON:

$$\lambda = h/p \quad (p = \text{MOMENTUM})$$

WE THEN EXPECT WE CAN ONLY PROBE DISTANCES  $r > \lambda$

$$\text{OR } r > \frac{h}{p}$$

RECALL THE RELATIVISTIC RELATION  $E^2 = (pc)^2 + (mc^2)^2$ . SO FOR

A VERY HIGH ENERGY PROTON,  $E \approx pc$  (IF  $E \gg mc^2$ )

AND WE CAN PROBE  $r > \frac{hc}{E}$ , AS WITH A PHOTON.

BUT A PROTON OF 1 MeV KINETIC ENERGY,  $E$ , HAS  $E \ll mc^2$ , SO

$E = p^2/2m$  OR  $p = \sqrt{2mE}$ . HENCE WE CAN PROBE

$$r > \frac{h}{\sqrt{2m} \sqrt{E}} = \frac{hc}{\sqrt{2mc^2} \sqrt{E}}$$

MEMORABLE FACT: FOR A PROTON  $mc^2 \approx 938 \text{ MeV}$

$$\text{SO } r > \frac{1200}{45 \sqrt{E}} \text{ FERMI}$$

A 1 MeV PROTON IS STILL A BETTER PROBE THAN A 1 MeV PHOTON, BUT NOT BY AS MUCH AS RUTHERFORD ORIGINALLY THOUGHT.

THE QUANTUM MECHANICAL LIMITATION MAY ALSO BE THOUGHT OF IN TERMS OF HEISENBERG'S UNCERTAINTY RELATION

$$\Delta x \Delta p \gtrsim \hbar$$

AS A RESULT OF OUR PROBLEM WE CAN EXPECT A MAXIMUM CHANGE,  $\Delta p$ , IN THE PROBE MOMENTUM WHICH IS OF THE SAME ORDER AS  $p$  ITSELF. THEN

$$\Delta x \gtrsim \frac{\hbar}{p}$$

IS THE LIMIT OF POSITION INFORMATION WE CAN OBTAIN USING A PROBE OF MOMENTUM  $p$ . THIS IS THE SAME CONCLUSION AS OBTAINED VIA DE BROGLIE'S ARGUMENT.

TO REPEAT: THE OBSERVATION OF SMALL DISTANCES REQUIRES HIGH ENERGY

OUR MAIN CONCLUSION LEADS US TO SEVERAL REMARKS.

1. COSMOLOGY. THE VERY HIGHEST ENERGIES OCCURRED IN THE BIG BANG. AS WE INCREASE OUR UNDERSTANDING OF HIGH ENERGY (= SMALL DISTANCE) PROCESSES, WE ALSO APPROACH AN UNDERSTANDING OF WHAT WENT ON DURING THE BIG BANG. THIS MAY TURN OUT TO BE THE MOST PROFOUND "APPLICATION" OF HIGH-ENERGY PHYSICS. THE ENTIRE UNIVERSE WAS QUITE SMALL AT EARLY TIMES, SO IT IS NOT SO SURPRISING THAT WE CAN RELATE OUR SMALL-DISTANCE STUDIES TO COSMOLOGY AFTER ALL.
2. NUCLEAR PHYSICS. THE CHARACTERISTIC SIZE OF THE NUCLEUS IS 1 FERMI, AND A TYPICAL ENERGY OF A NUCLEAR PROCESS IS 1 MeV. SO WITH

$$\lambda_{\text{PROBED}} \approx \frac{30 \text{ FERMI}}{\sqrt{E \text{ IN MeV}}} \quad \text{FOR P-P SCATTERING}$$

WE SEE THAT WE CANNOT LEARN MUCH ABOUT NUCLEAR FORCES UNLESS WE GO TO ENERGIES SIGNIFICANTLY HIGHER THAN 1 MeV. AS SUCH WE MAY ANTICIPATE THAT NUCLEAR PHYSICS WILL ALWAYS BE A SORT OF 'CHEMISTRY' - A COMPLICATED APPLICATION OF A PROCESS WHICH IS SIMPLE ONLY ON A SMALLER SCALE.

FERMI: "THE HIGH ENERGY LIMIT IS THE 'HYDROGEN ATOM' OF THE NUCLEAR INTERACTION."

THAT REMARK PROVED QUITE PROPHETIC AS OUR PRESENT UNDERSTANDING OF THE NUCLEAR FORCE IS THAT IT OBEYS 'ASYMPTOTIC FREEDOM' - IT GETS WEAKER (AND SIMPLER TO UNDERSTAND) AS THE INTERACTION ENERGY INCREASES.

3. EINSTEIN. HE TOLD US  $E=mc^2$ . AS WE RAISE THE ENERGY OF OUR PROBE IN THE QUEST FOR SIMPLICITY, SOMETHING COMPLICATED MAY HAPPEN. SOME OF THE ENERGY MAY BE CONVERTED INTO MATTER WHICH WASN'T THERE ORIGINALLY: NEW PARTICLES MAY BE CREATED!

WEISSKOPF: "BOILING THE VACUUM"

BEFORE UNDERSTANDING PROTON-PROTON SCATTERING WE WILL ENCOUNTER MANY NEW PARTICLES WITH MANY NEW PROPERTIES. KNITTING ALL THE COMPLEXITY WE WILL FIND INTO A UNIFIED TAPESTRY IS INDEED A GRAND ENTERPRISE OF THE HUMAN INTELLECT, AND WAS A SPECIAL GOAL OF EINSTEIN. IN RECENT YEARS GIANT STEPS HAVE BEEN MADE IN THE QUEST FOR UNIFICATION OF OUR VIEW OF THE VERY SMALL. IN THIS COURSE WE WILL TRY TO GIVE A SURVEY OF THE VAST RICHES OF PHENOMENA IN THE MICROWORLD, AND TO GIVE A SENSE OF THE THEORIES WHICH UNIFY OUR THINKING ABOUT THESE DIVERSE PHENOMENA.

4. MOMENTUM SPACE. ALTHOUGH OUR GOAL IS KNOWLEDGE OF THE NATURE OF TINY THINGS, WE MUST DEAL WITH LARGE ENERGIES AND MOMENTA. WE CANNOT DIRECTLY MEASURE DISTANCES OF  $10^{-13}$  CM WITH PERSON-SIZED EQUIPMENT, BUT WE CAN MEASURE, SAY, THAT A SINGLE PROTON HAS 10 GEV ENERGY WITH SUCH EQUIPMENT.

THIS FACT OF TECHNICAL LIFE HAS ITS ANALOGUE IN OUR THEORETICAL DESCRIPTION OF THE VERY SMALL. IT IS TYPICALLY EASIER TO DESCRIBE A MICRO-PROCESS IN MOMENTUM SPACE THAN IN COORDINATE SPACE. THE SIMPLE STATES ARE NOT THOSE OF PARTICLES WITH FIXED POSITIONS, BUT THOSE OF FIXED MOMENTUM AND ENERGY (PLANE WAVES).

RATHER THAN DEAL WITH SOME SPATIAL PROPERTY  $f(\vec{x})$ , WE WILL TYPICALLY DEAL WITH ITS FOURIER TRANSFORM IN MOMENTUM SPACE

$$F(\vec{p}) = \int e^{i\vec{p}\cdot\vec{x}} f(\vec{x}) d^3\vec{x} \quad \text{ETC.}$$

THUS THE NEED FOR HIGH ENERGY HAS THE CONSEQUENCE OF PUTTING OUR KNOWLEDGE OF THE SMALL AT ONE REMOVE (FOURIER TRANSFORM) FROM THE FORM WE MIGHT HAVE EXPECTED FROM CLASSICAL CONSIDERATIONS.

#### 5. HIGH-ENERGY TECHNOLOGY (PERKINS SEC 2.2)

THE PRODUCTION OF 'BEAMS' OF HIGH-ENERGY CHARGED PARTICLES INVOLVES MANIPULATION BY MACROSCOPIC ELECTRIC AND MAGNETIC FIELDS. LATER WE MAY GIVE EXAMPLES OF HOW NEUTRAL PARTICLE BEAMS CAN ALSO BE PRODUCED.

THE STRONGEST PRACTICAL ELECTRIC FIELD, OBTAINED INSIDE A RADIO-FREQUENCY (RF) CAVITY, IS ABOUT

$$10^7 \text{ VOLTS/METER} \quad (\text{M.K.S. UNITS})$$

THE STRONGEST PRACTICAL MAGNETIC FIELD (OBTAINED INSIDE A NON-SUPERCONDUCTING MAGNET) IS ABOUT

$$2 \text{ TESLA} \quad (= 20,000 \text{ GAUSS})$$

$$\text{THE LORENTZ FORCE IS} \quad \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (\text{M.K.S.})$$

IN HIGH ENERGY PHYSICS  $v \sim c$  ALMOST ALWAYS

$$\text{SO } F/q \text{ MAX} \sim 10^7 \text{ (M.K.S.) USING ELECTRIC FIELDS}$$

$$\text{AND } \sim 3 \times 10^8 \cdot 2 \sim 6 \times 10^8 \text{ USING MAGNETIC FIELDS}$$

# LECTURE 1 PH 529

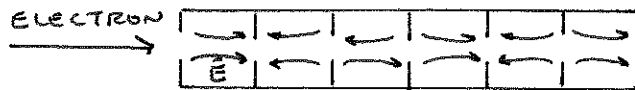
WHEN WE DESIRE TO STEER THE PATH OF A RELATIVISTIC CHARGED PARTICLE WE WILL PRIMARILY USE MAGNETIC FIELDS.

HOWEVER, A MAGNETIC FIELD CANNOT CHANGE THE ENERGY OF A PARTICLE:

$$\frac{dU}{dt} = \vec{F} \cdot \vec{v} = q \vec{E} \cdot \vec{v}$$

TO PRODUCE A HIGH-ENERGY PARTICLE WE MUST CONTRIVE TO USE THE RELATIVELY WEAKER ELECTRIC FIELD.

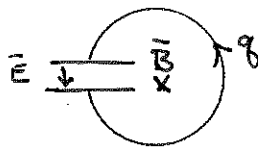
FOR EXAMPLE, TO GIVE AN ELECTRON 20 GEV OF ENERGY WE COULD PASS IT DOWN A COLUMN OF RADIO FREQUENCY CAVITIES, ALL WITH PROPER PHASING OF THE E-M WAVES INSIDE.



[WE ONLY GET ACCELERATION IF  $\vec{E} \cdot \vec{v} \neq 0$ , SO TEM WAVES ARE USELESS!]

WITH ENERGY GAIN OF 10 MeV/METER, WE NEED 2000 METERS > 1 MILE OF CAVITIES. HENCE THE BASIC PARAMETERS OF THE 2 MILE STANFORD LINEAR ACCELERATOR (SLAC).

IN THE EARLY DAYS OF HIGH ENERGY PHYSICS A FIELD STRENGTH OF  $10^7$  VOLTS/METER WAS STILL A DREAM. ONLY A FEW HUNDRED KV/METER COULD BE ACHIEVED IN THE 1920'S. THIS LED TO THE INNOVATION BY LAWRENCE (1930) OF THE CYCLOTRON, IN WHICH THE SAME ELECTRICAL FIELD COULD BE USED OVER AND OVER AGAIN. A MAGNETIC FIELD BENDS THE PARTICLES IN A CIRCLE, SO THEY COME BACK AND 'RECYCLE' THE ELECTRIC FIELD:

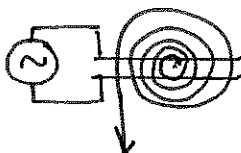


LAWRENCE'S INSIGHT: FOR MOTION IN A CIRCLE DUE TO  $\vec{B}$ ,  $\frac{mv^2}{r} = qvB$

SO  $\omega = \frac{v}{r} = \frac{qB}{m}$  WHICH IS INDEPENDENT OF THE ENERGY  $\frac{1}{2}mv^2$

HENCE THE ELECTRIC FIELD NEED NOT BE STATIC, BUT CAN BE INSIDE A R.F. CAVITY OF FREQUENCY  $\omega$ , WHICH IS MUCH EASIER TECHNICALLY AT HIGH FIELD STRENGTHS.

OF COURSE, THE RADIUS OF THE PARTICLE'S PATH INCREASE WITH ENERGY, SO THE PATH IS A SPIRAL.



TO REACH HIGH ENERGIES THE MAGNET MUST BE QUITE LARGE. THE 184" CYCLOTRON WEIGHS 4000 TONS.



# LECTURE 1 Ph 529

LAWRENCE'S INSIGHT APPLIES ONLY TO NON-RELATIVISTIC PARTICLES. AS  $v \rightarrow c$  WE CAN STILL INVOKE THE GENERAL FORM OF NEWTON'S LAW:

$$\vec{F} = \frac{d\vec{p}}{dt} = e\vec{v} \times \vec{B} = \frac{e}{m} \vec{p} \times \vec{B}$$

WHERE  $m = \gamma m_0 = m_0 \sqrt{1 - v^2/c^2}$

WE RECOGNIZE THIS AS UNIFORM PRECESSION ( $\leftrightarrow$  MOTION IN A CIRCLE) WITH FREQUENCY

$$\omega = \frac{eB}{\gamma m_0} = \frac{eBc^2}{E} \quad \text{WITH } E = \gamma m_0 c^2 = \sqrt{p^2 c^2 + m_0^2 c^4}$$

CYCLOTRONS HAVE NEVER BEEN MADE TO WORK FOR KINETIC ENERGIES BEYOND  $\sim 700$  MeV  $\leftrightarrow \gamma \sim 1.8$  FOR A PROTON.

SIDE REMARK; IF A PARTICLE CROSSES A LIMITED REGION OF MAGNETIC FIELD ITS MOMENTUM IS CHANGED BY

$$d\vec{p} = e v B dt = e v B \frac{dl}{v} = e B dl$$

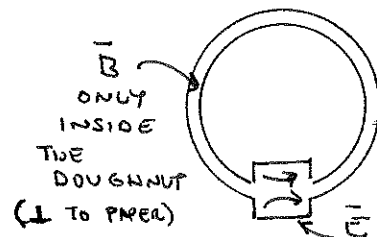
IN A DIRECTION TRANSVERSE TO ITS INITIAL MOTION. THIS LEADS TO THE JARGON OF DESCRIBING THE STRENGTH OF A MAGNET BY IS

$$\text{KICK} = \Delta p_{\perp} = e \int B dl$$

THE COMMON UNIT FOR KICK IS  $\frac{\text{MeV}}{c}$  (A MOMENTUM!)  $= \frac{2 \cdot 10^6}{3 \times 10^8} = \frac{e}{300}$  (MKS)

$$\text{THUS } \Delta p_{\perp} \left( \frac{\text{MeV}}{c} \right) = 300 \int B dl \text{ IN TESLA-METERS}$$

IN 1945 Mc MILLAN, AND ALSO VEKSLER, PROPOSED THE SYNCHROTRON, IN WHICH A SHORT LINEAR ACCELERATOR IS JOINED ONTO A DOUGHNUT OF MAGNETS. IN THIS SCHEME THE RADIUS OF THE PARTICLES ORBIT REMAINS CONSTANT, SO THAT ONLY A RELATIVELY SMALL VOLUME OF MAGNETIC FIELD IS NEEDED.



FROM ABOVE,  $\gamma = \frac{v}{\omega} = \frac{\gamma m_0 v}{eB} = \frac{p}{eB}$

TO KEEP  $\gamma$  CONSTANT  $B$  MUST RISE WITH MOMENTUM  $p$ .

SUPPOSE WE DESIRE  $p = 400 \text{ GeV}/c$  WITH 2 TESLA MAGNETS.

$$\text{THEN } \gamma = \frac{4 \times 10^{11}}{3 \times 10^8} \frac{e}{e \cdot 2} \sim 700 \text{ METERS, } C = 2\pi \gamma \sim 2 \text{ MILES}$$

WHICH ARE THE PARAMETERS OF FERMI LAB.

# LECTURE 1 PH 529

THE TASK OF CONFINING THE PARTICLES TO A 1" TUBE IN A MANNER WHICH IS STABLE AGAINST VARIOUS PERTURBATIONS IS ONE OF THE LARGEST SCALE APPLICATIONS OF CLASSICAL MECHANICS TO A PRESENT DAY SYSTEM. IN THE ALTERNATING GRADIENT SYNCHROTRON, PROPOSED BY COURANT ET AL (1952) THE BEAM IS KEPT STABLE SOMEWHAT IN THE MANNER OF A RAPIDLY VIBRATING INVERTED PENDULUM!

IN HIGH-ENERGY COLLISIONS IT IS NOT REALLY THE LABORATORY ENERGY OF THE PROBE = BEAM PARTICLE WHICH MATTERS, BUT RATHER THE CENTER OF MASS ENERGY OF THE BEAM + TARGET SYSTEM.

SUPPOSE THE BEAM PARTICLE IS HIGHLY RELATIVISTIC SO THAT  $E_{\text{BEAM}} \approx pc$ , AND ITS ENERGY-MOMENTUM 4-VECTOR IS

$$\begin{matrix} \text{TIME COMPONENT} & \xrightarrow{(pc, \vec{p}c)} & \text{SPACE COMPONENT} \end{matrix}$$

THE TARGET PARTICLE'S 4-VECTOR IS  $(mc^2, \vec{0})$

SO THE TOTAL ENERGY 4-VECTOR IS  $(pc + mc^2, \vec{p}c)$ .

THE SQUARE OF THIS IS JUST THE SQUARE OF THE CENTER OF MASS ENERGY:

$$E_{\text{CM}}^2 = 2(mc^2)(pc) + (mc^2)^2 \approx 2(mc^2) E_{\text{BEAM}}$$

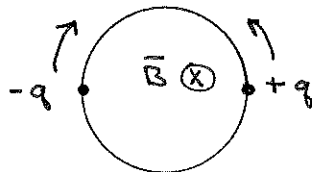
$$\text{OR } E_{\text{CM}} \sim \sqrt{2mc^2 E_{\text{BEAM}}}$$

HENCE THE 'USEFUL' ENERGY  $E_{\text{CM}}$  RISES ONLY AS THE SQUARE ROOT OF THE LABORATORY ENERGY  $E$ , IN THE HIGH-ENERGY LIMIT.

THIS WASTEFUL SITUATION COULD BE AVOIDED IF THE LABORATORY AND CENTER OF MASS FRAMES ARE THE SAME.

$$\xrightarrow{E} \quad \xleftarrow{E} \quad E_{\text{CM}} = 2E$$

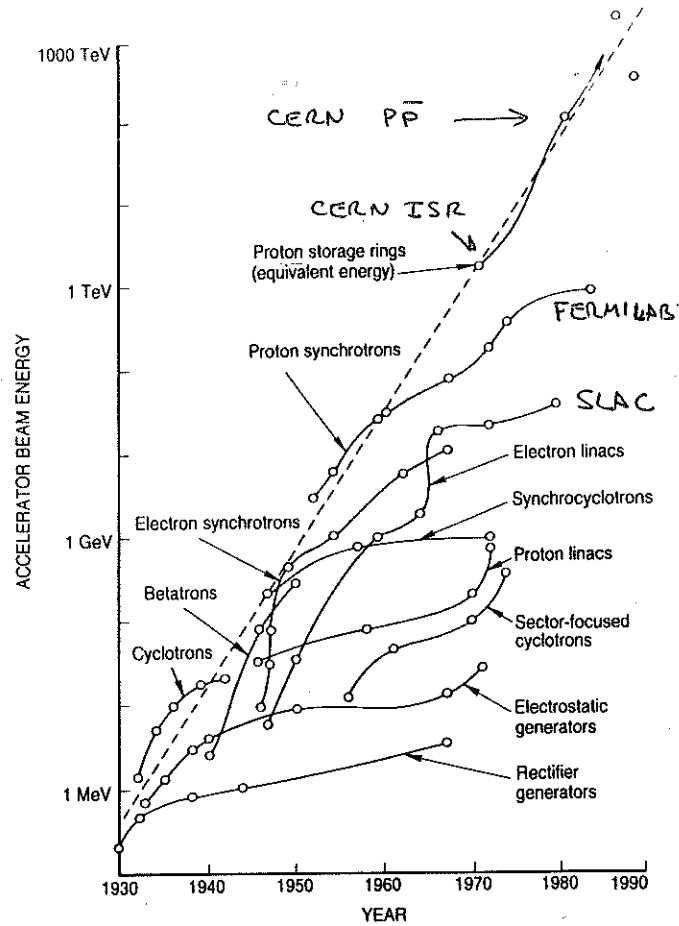
THIS MIGHT BE ARRANGED BY COLLIDING TWO LINEAR ACCELERATORS HEAD-ON, OR BY HAVING 2 CONCENTRIC CIRCULAR ACCELERATIONS (= STORAGE RINGS) (KERST, O'NEILL 1956). IF THE TWO COLLIDING PARTICLES ARE THE ANTI-PARTICLES OF EACH OTHER, THEN ONLY 1 RING IS NEEDED.



SOME OF THE GREATEST SUCCESSSES IN RECENT YEARS HAVE COME FROM  $e^+e^-$  AND  $p\bar{p}$  STORAGE RINGS.

ON THE FOLLOWING PAGE WE SHOW A 'LIVINGSTON' PLOT INDICATING HOW THE EFFECTIVE LABORATORY ENERGY OF PARTICLE ACCELERATORS HAS GROWN EXPONENTIALLY WITH TIME.

The growth of accelerator beam energies with time. For colliding-beam proton storage rings, finished, unfinished or recently abandoned, the points indicate the "equivalent" beam energy a fixed-target machine would require to achieve the same center-of-mass collision energy. Electron-positron storage rings cannot be included here in this way; the small "target" mass pushes these points way off the scale.



## UNITS

WE HAVE ALREADY EXPRESSED ENERGIES IN ELECTRON-VOLTS, REMINDING US OF HOW WE PRODUCE THAT ENERGY.

LENGTHS WILL OFTEN BE MEASURED IN FERMIS  $= 10^{-13}$  cm

AREAS OF CROSS-SECTIONS ARE OFTEN MEASURED IN BARNS

$$1 \text{ BARN} = 10^{-24} \text{ cm}^2 \quad (\neq 1 \text{ FERM}^2)$$

A BARN IS REALLY TOO BIG. WE WILL ALSO USE

$$\text{mb} = \text{MILIBARN} = 10^{-27} \text{ cm}^2$$

$\mu\text{b} = \text{MICROBARN}$

$\text{nb} = \text{NANOBARN}$

$\text{pb} = \text{PICOBARN}$

$$\text{fb} = \text{FEMTOBARN} = 10^{-15} \text{ BARN} = 10^{-39} \text{ cm}^2$$

WE WILL ALSO OFTEN USE THE THEORISTS' UNITS IN WHICH  $\hbar = c = 1$

IN DOING THIS WE CAN KEEP THE SIZE OF ONE UNIT (I.E. TIME) UNCHANGED, WHILE ALTERING THE OTHER TWO (I.E. LENGTHS AND MASSES).

BUT AS THIS IS A COURSE IN HIGH-ENERGY PHYSICS, WE MOST COMMONLY CHOOSE TO EMPHASIZE THE ENERGY SCALE, USING THE MeV (OR SOMETIMES GeV) AS THE BASIC UNIT.

$$\text{KEV} = 10^3 \text{ eV}; \text{MEV} = 10^6 \text{ eV}, \text{GeV} = 10^9 \text{ eV}; \text{TeV} = 10^{12} \text{ eV}$$

THE SCALES OF THEN A LENGTH AND TIME CAN BE RELATED TO THE ENERGY SCALE AS FOLLOWS:

$$1 = \hbar = 1.05 \times 10^{-34} \text{ JOULE SEC.} = \frac{1.05 \times 10^{-34}}{1.6 \times 10^{-13}} = \underline{6.6 \times 10^{-22} \text{ MeV SEC.}}$$

$$\text{SO } 1 \text{ SEC} = \frac{1}{6.6 \times 10^{-22} \text{ MeV}} \quad \text{TIME} \propto \frac{1}{\text{ENERGY}}$$

$$\text{ALSO } 1 = \hbar c = 6.6 \times 10^{-22} \text{ MeV SEC.} \cdot 3 \times 10^8 \text{ M/SEC.} \cdot \frac{1 \text{ FERMI}}{10^{-15} \text{ M}} = \underline{197 \text{ MeV-FERMI}}$$

$$\text{SO } 1 \text{ FERMI} = \frac{1}{197 \text{ MeV}} \quad \text{LENGTH} \propto \frac{1}{\text{ENERGY}}$$

OF COURSE MASS = ENERGY SINCE  $c^2 = 1$  IN  $E = mc^2$

WE ILLUSTRATE THE USE OF THESE RELATIONS BY COMMENTING ON TWO IMPORTANT CONCEPTS: MEAN LIFE AND CROSS SECTION

### MEAN LIFE

AN UNSTABLE PARTICLE DECAYS WITH MEAN LIFE  $\tau$ , ACCORDING TO

$$N = N_0 e^{-t/\tau} \quad N = \text{NUMBER OF PARTICLES REMAINING}$$

FOR EXAMPLE, THE  $J/\psi$  PARTICLE HAS MEAN LIFE  $\tau \sim 10^{-20}$  SEC

$$\text{IN ENERGY UNITS } \tau = \frac{10^{-20}}{6.6 \times 10^{-22} \text{ MeV}} = \frac{1}{.066 \text{ MeV}} = \frac{1}{66 \text{ KEV}}$$

WE WILL MORE OFTEN REFER TO THE WIDTH OF AN UNSTABLE PARTICLE

$$\underline{\Gamma} \equiv \frac{\hbar}{\tau} \rightarrow \frac{1}{\tau} \text{ IN OUR UNITS.} \quad \text{THUS } \Gamma_{J/\psi} = 66 \text{ KEV}$$

WHAT IS  $\Gamma$  THE WIDTH OF?

AN UNSTABLE PARTICLE IS NOT AN ENERGY EIGENSTATE, WHICH MUST LAST FOREVER IN PRINCIPLE. HEISENBERG TELLS US

$$\Delta E \Delta t \geq \hbar \quad \text{IN SOME SENSE}$$

$$\text{AT BEST} \quad \Delta E = \frac{\hbar}{\Delta t} = \frac{\hbar}{\tau} = \Gamma \quad \text{FOR AN UNSTABLE PARTICLE,}$$

THUS  $\Gamma$  IS THE SPREAD OF ENERGY, OR MASS, OF THE PARTICLE DUE TO ITS FINITE LIFE. (IF YOU EAT JUNK FOOD, YOU SHORTEN YOUR LIFE WHILE INCREASING YOUR MASS?)

THAT IS, IF WE MEASURE THE MASSES OF SEVERAL UNSTABLE PARTICLES OF THE SAME TYPE, WE WON'T ALWAYS GET THE SAME ANSWER, BUT VALUES SPREAD OVER A RANGE  $\Gamma$ .

SOME DETAILS: IF  $N = N_0 e^{-t/\tau} = N_0 e^{-\frac{\Gamma t}{\hbar}}$ , WE MIGHT EXPECT

THE WAVE FUNCTION OF THE PARTICLE TO BE

$$\psi(t) = \psi_0 e^{-i \frac{E_0 t}{\hbar}} e^{-\frac{\Gamma t}{2\hbar}} \quad \left[ \text{SOMETIMES WE SAY } E = E_0 - i \frac{\Gamma}{2} \right]$$

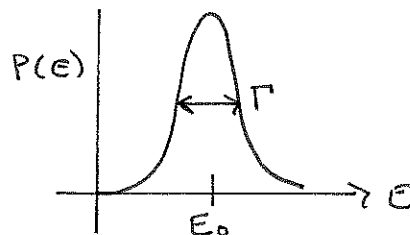
WHICH APPLIES ONLY FOR  $t > 0$ .  $t = 0$  = MOMENT OF CREATION OF THE UNSTABLE PARTICLE - WHICH WOULD NOT BE AROUND NOW IF IT EXISTED AT  $t = -\infty$ .

SUPPOSE WE WRITE THIS IN TERMS OF ENERGY EIGENSTATES

$$\psi(t) = \int dE f(E) e^{-i \frac{E t}{\hbar}} \quad (E \text{ REAL})$$

$$\text{THEN} \quad f(E) \sim \int_0^{\infty} dt \psi(t) e^{i \frac{E t}{\hbar}} = \int_0^{\infty} dt e^{i \frac{t}{\hbar} (E - E_0 + i \frac{\Gamma}{2})}$$

$$\begin{aligned} &\sim \frac{1}{E - E_0 + i \Gamma/2} \\ P(E) = |f(E)|^2 &\sim \frac{1}{(E - E_0)^2 + \Gamma^2/4} \end{aligned}$$



THE SHAPE OF THIS PROBABILITY DISTRIBUTION IS CALLED A BREIT-WIGNER RESONANCE CURVE.

WE MUST TRY TO ANSWER WIGNER'S QUESTION: A RESONANCE OF WHAT?

CROSS - SECTIONS

IT IS CUSTOMARY TO SUMMARIZE THE RESULTS OF A SCATTERING EXPERIMENT IN TERMS OF THE CROSS SECTION, WHICH HAS DIMENSIONS OF AREA. IN OUR UNITS, LENGTH  $\sim \frac{1}{\text{ENERGY}}$

SO CROSS SECTION  $\equiv \sigma \sim \frac{1}{\text{ENERGY}^2}$

FOR EXAMPLE A VERY BASIC PROCESS IS THE ANNIHILATION OF AN ELECTRON BY A POSITRON, RESULTING IN THE PRODUCTION OF A PARTICLE - ANTI PARTICLE PAIR OF  $\mu$  MESONS



AT VERY HIGH ENERGIES, SUCH THAT  $E_{cm} \gg m_e$  OR  $m_\mu$

TO ONLY RELEVANT ENERGY IN THIS PROCESS IS THE C.M FRAME ENERGY.

SO  $\sigma \sim \frac{1}{E_{cm}^2}$

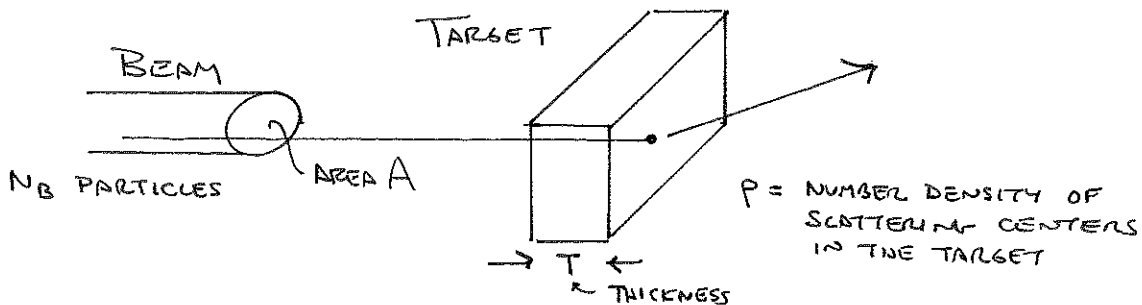
WE WILL LATER SHOW THAT  $\sigma = \frac{4\pi}{3} \frac{\alpha^2}{E_{cm}^2}$  TO VERY GOOD APPROX.

WHERE  $\alpha = \frac{1}{137} =$  FINE STRUCTURE CONSTANT.

SUPPOSE WE MEASURE  $E$  IN GeV AND WANT TO CONVERT  $\sigma$  BACK INTO  $\text{cm}^2$ . OUR BASIC FACT IS  $1 = 197 \text{ MeV Fermi} = .197 \text{ GeV f}$

SO  $\sigma \sim \frac{4}{(137)^2} \frac{(197 \text{ f})^2}{(E \text{ IN GeV})^2} = \frac{8846}{E^2} = \frac{8.8 \times 10^{-32} \text{ cm}^2}{E^2}$

WE REMIND YOU OF HOW CROSS SECTIONS ARE RELATED TO THE NUMBER OF SCATTERED PARTICLES IN AN EXPERIMENT.



$$N_{\text{SCATTERED}} = N_B \cdot \frac{\text{TOTAL CROSS SECTION OF ALL PARTICLES SWEEP BY THE BEAM}}{\text{AREA OF BEAM}}$$

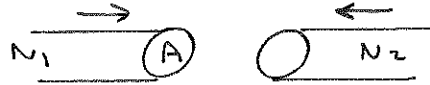
$$= \frac{N_B}{A} \cdot \sigma \cdot \rho T A = (N_B \rho T) \sigma$$

NOTE HOW THE BEAM AREA DIVIDES OUT (SO LONG AS ALL THE BEAM HITS THE TARGET). HENCE THIS RELATION HOLDS EVEN IF THE BEAM IS NOT UNIFORM OVER AREA A, AS IS TYPICAL.

THE QUANTITY  $N_B \rho T$  IS SOMETIMES CALLED THE LUMINOSITY  $\mathcal{L}$ .

AND HAS UNITS  $1/(\text{cm}^2 \text{sec})$ , WHERE HERE  $N_B = \#$  OF BEAM PARTICLES / SEC.

IN A COLLIDING BEAM EXPERIMENT, WE HAVE TWO BEAMS,  $N_1$  &  $N_2$ , EACH OF AREA  $A$ , WHICH CROSS  $f$  TIMES / SEC



BY THE DEFINITION OF THE CROSS SECTION

$$N_{\text{SCATT}} = f \frac{N_1 N_2}{A} \sigma \quad (= \# \text{ OF SCATTERS / SEC})$$

$\mathcal{L} = \text{LUMINOSITY OF COLLIDING BEAMS}$

### REACTION RATES

WE SHALL OCCASIONALLY SKETCH CALCULATIONS OF LIFETIMES AND CROSS SECTIONS. BASICALLY THESE CALCULATIONS ARE APPLICATIONS OF FERMI'S GOLDEN RULE

$$\text{RATE} = |M|^2 \rho$$

$M =$  REACTION MATRIX ELEMENT

$\rho =$  SUITABLY NORMALIZED DENSITY OF FINAL STATES

$$\text{FOR UNSTABLE PARTICLES, DECAY RATE} = \frac{1}{\tau} = \Gamma$$

IN THE CASE OF SCATTERING A SINGLE BEAM PARTICLE OFF A SINGLE TARGET PARTICLE,

$$\text{RATE} = \sigma \cdot v_{\text{RELATIVE}} \quad (= \# \text{ OF SCATTERS / SEC / UNIT VOLUME})$$

ASSUMING A NORMALISATION OF 1 BEAM PARTICLE AND 1 TARGET PARTICLE PER UNIT VOLUME. THEN  $v_{\text{RELATIVE}}$  MEASURES THE VOLUME SWEEPED OVER IN 1 SECOND

### CONVENTIONS

WE WILL TRY TO FOLLOW THE CONVENTIONS SUMMARIZED ON THE FOLLOWING PAGE. SOME OF THESE DIFFER FROM THOSE OF PERKINS.

Notational conventions

In this representation one has for the transpose  $T$  of the  $\gamma$  matrices:

$$\left. \begin{aligned} \gamma^{jT} &= \gamma^j & \text{for } j = 0, 2, 5, \\ \gamma^{jT} &= -\gamma^j & \text{for } j = 1, 3. \end{aligned} \right\} \quad (9)$$

For the Hermitian conjugates<sup>†</sup> one has

$$\left. \begin{aligned} \gamma^{0\dagger} &= \gamma^0, & \gamma^{5\dagger} &= \gamma^5, \\ \gamma^{j\dagger} &= -\gamma^j & \text{for } j = 1, 2, 3. \end{aligned} \right\} \quad (10)$$

The combination

$$\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu] \quad (11)$$

is often used.

The scalar product of the  $\gamma$  matrices and any four-vector  $A$  is defined as

$$A \equiv \gamma^\mu A_\mu = \gamma^0 A^0 - \gamma^1 A^1 - \gamma^2 A^2 - \gamma^3 A^3. \quad (12)$$

The particle spinors  $u$  and the anti-particle spinors  $v$ , which satisfy the Dirac equations

$$\left. \begin{aligned} (\not{p} - m)u(p) &= 0, \\ (\not{p} + m)v(p) &= 0, \end{aligned} \right\} \quad (13)$$

respectively, are related by

$$\left. \begin{aligned} v &= i\gamma^2 u^*, \\ v &= -iu^T \gamma^0 \gamma^2, \end{aligned} \right\} \quad (14)$$

where  $\bar{v} \equiv v^\dagger \gamma^0$  (similarly  $\bar{u} \equiv u^\dagger \gamma^0$ ).

Note that our spinor normalization differs from Bjorken and Drell. We utilize

$$u^\dagger u = 2E, \quad v^\dagger v = 2E, \quad (15)$$

the point being that (15) can be used equally well for massive fermions and for neutrinos. For a massive fermion or anti-fermion (15) implies

$$\bar{u}u = 2m, \quad \bar{v}v = -2m.$$

With this normalization the cross-section formula (B.1) of Appendix B in Bjorken & Drell (1964) holds for both mesons and fermions, massive or massless. For further details and properties of the  $\gamma$  matrices see Appendix A of Bjorken & Drell (1964).

Our notation generally follows that of Bjorken & Drell: 'Relativistic Quantum Mechanics' (McGraw Hill, 1964). We use natural units:  $\hbar = c = 1$ .

The metric tensor is

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (1)$$

Space-time points are denoted by the contravariant four-vector  $x^\mu$  ( $\mu = 0, 1, 2, 3$ )

$$x^\mu = (t, \mathbf{x}) = (t, x, y, z), \quad (2)$$

and the four-momentum vector for a particle of mass  $m$  is

$$p^\mu = (E, \mathbf{p}) = (E, p_x, p_y, p_z), \quad (3)$$

where

$$E = \sqrt{\mathbf{p}^2 + m^2}. \quad (4)$$

Using (1), the scalar product of two four-vectors,  $A, B$ , is defined as

$$A \cdot B = A_\mu B^\mu = g_{\mu\nu} A^\mu B^\nu = A^0 B^0 - \mathbf{A} \cdot \mathbf{B}. \quad (5)$$

The  $\gamma$  matrices for spin half particles satisfy

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \quad (6)$$

and we use a representation in which

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^j = \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix}, \quad j = 1, 2, 3, \quad (7)$$

where  $\sigma_j$  are the usual Pauli matrices. We define

$$\gamma^5 = \gamma_5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \quad (8)$$