THE QUARK MODEL, CONTINUED

WE HAVE SEEN HOW THE QUARK MODEL GOES BEYOND THE SYMMETRY SU(6) BY ASSIGNING MASSES AND MAGNETIC MOMENTS TO THE QUARKS. CRUDE UNDERSTANDING OF THE HADRON MASS SPECTRUM AND BARYON MAGNETIC MOMENTS WAS THEREBY OBTAINED. WE CONTINUE TO EXAMINE EVIDENCE THAT QUARKS HAVE A DYNAMICAL MEANING AS OPPOSED TO BEING MERELY FICTITIOUS BOOKKEEPING DEVICES.

6. ELECTROMAGNETIC TRANSITIONS.

WE SUPPOSE THE HADRONS ARE A KIND OF ATOM MADE FROM QUARKS. THEN WE MAY EXPLORE THE ATOMIC PHYSICS OF TRANSITIONS AMONG VARIOUS POSSIBLE STATES. WHILE THIS APPROACH IS SUGGESTIVE, WE WILL FIND ONLY APPROXIMATE AGREEMENT WITH EXPERIMENTAL FACT.

a. HADRON DECAY VIA SINGLE PHOTON EMISSION: $a \to b + \gamma$

IF WE CONSIDER MESONS AND BARYONS AS BOUND STATES OF QUARKS THEN THE LOWEST MASS SU(6) MULTIPLETS ARE ALL S WAVE STATES. ANY TRANSITIONS $a \to b + \gamma$ AMONG THESE STATES WILL THEN BE $L=0 \to L=0$, WITH $\Delta J = \pm 1$ OR 0, AND NO PARITY CHANGE. SUCH TRANSITIONS CANNOT BE DUE TO ELECTRIC DIPOLE RADIATION. EVEN ELECTRIC QUADRUPOLE RADIATION IS FORBIDDEN FOR $L=0 \to L=0$. THE SIMPLEST POSSIBILITY IS MAGNETIC DIPOLE RADIATION. THIS IS NOT SURPRISING FOR EXAMPLES LIKE $\omega \to \pi^0 \gamma$ OR $\Delta^+ \to p\gamma$ IN WHICH $\Delta S = 1$, WHICH REQUIRES A COUPLING TO SPIN IN THE DECAY PROCESS.

WE SKETCH A RATE CALCULATION.

$$\Gamma_{a \to b + \gamma} = \frac{k}{8\pi M_a^2} \frac{\sum_{\text{SPINS}} |\mathcal{M}|^2}{2S_a + 1} \qquad \text{(p. 193)}$$

THE MATRIX ELEMENT $\mathcal{M}$ DEPENDS ON THE MAGNETIC DIPOLE INTERACTION ENERGY

$$\langle f | \sum_{\text{QUARKS}} \bar{\mu} \cdot \bar{B} | i \rangle$$

$\langle f | \bar{\mu} | i \rangle$ IS OFTEN CALLED THE TRANSITION MAGNETIC MOMENT.

$\bar{B}$ IS THE MAGNETIC FIELD OF THE PHOTON: $\bar{B} = \bar{\nabla} \times \bar{A} = i\bar{k} \times \bar{\epsilon}\, e^{i\bar{k}\cdot\bar{r}}$

WHERE $\bar{\epsilon}$ = PHOTON POLARIZATION. FOR $k \gtrsim 200$ MeV, $e^{i\bar{k}\cdot\bar{r}} \sim 1$ OVER THE VOLUME OF THE HADRON (RADIUS ~ 1 FERMI).

THEN ON SUMMING OVER FINAL STATE SPINS

$$\Gamma = \frac{k^3}{8\pi M_a^2}\, 2 \cdot \frac{2S_b+1}{2S_a+1} \left| \langle f | \sum_{\text{QUARKS}} \bar{\mu} | i \rangle \right|^2$$

WHERE $\bar{\mu} = \mu_q \bar{\sigma} = \frac{e Q_q}{2 M_q} \bar{\sigma}$ WITH e = ELECTRON CHARGE.

To make the dimensions come out right, $|\mathcal{M}|^2$ apparently includes a piece $\sim 4 M_q^2 f$, where f = dimensionless fudge factor ~ 1, due to the overlap integral of the spatial parts of the initial and final states. If so,

$$\Gamma = \frac{f k^3}{\pi} \frac{2S_b+1}{2S_a+1} \left| \langle f | \sum_q \mu_q \vec{\sigma} | i \rangle \right|^2$$

It is sufficient to evaluate the matrix element for one particular set of i & f spins (if we suppose all initial spin states are equally likely, and we consider only Γ total which sums over final spins).

We consider an explicit example $\omega \to \pi^0 \gamma$ and evaluate the case $S_z = 0$.

$$\omega_0 = \tfrac{1}{2} (u\bar{u} + d\bar{d})(\uparrow\downarrow + \downarrow\uparrow)$$
$$\pi^0 = \tfrac{1}{2} (u\bar{u} - d\bar{d})(\uparrow\downarrow - \downarrow\uparrow)$$

Only σ_z gives non-zero matrix elements between the ω and π^0

For the u quark $\langle f | \mathcal{M} | i \rangle = \frac{\mu_u}{4}(1-(-1)) = \frac{\mu_u}{2}$

$\bar{u}$ . . . . . . $= \frac{\mu_{\bar{u}}}{4}(-1-1) = \frac{\mu_u}{2}$ using $\mu_{\bar{u}} = -\mu_u$

d $\quad -\frac{\mu_d}{4}(1-(-1)) = -\frac{\mu_d}{2}$

$\bar{d}$ $\quad -\frac{\mu_{\bar{d}}}{4}(-1-1) = -\frac{\mu_d}{4}$

so $|\mathcal{M}|^2 = |\mu_u - \mu_d|^2 = \left|\tfrac{3}{2}\mu_u\right|^2 = \left|\tfrac{3}{2}\cdot\frac{e\,\tfrac{2}{3}}{2M_u}\right|^2 = \frac{e^2}{4M_u}$

and $\Gamma_{\omega\to\pi^0\gamma} = f\frac{e^2}{4\pi}\frac{k^3}{3M_u^2} = \frac{\alpha f k^3}{3M_u^2} \sim 1.39\ \text{MeV}\cdot f$ if $M_u = 336$ MeV

Estimates of several meson transitions are summarized on p 251. With fudge factor $f \sim 1/2$ there is some agreement between 'calculation' and fact.

Estimates made along these lines for baryon transitions include

	Γ_{CALC} (MeV)	Γ_{EXPT} (MeV)
$\Sigma^0 \to \Lambda^0 + \gamma$	$8 \times 10^{-3} f$	$1.2 \pm .2 \times 10^{-2}$
$\Delta \to p + \gamma$	$0.4 f$	$0.6 \pm .1$

The decay $\Sigma^0 \to \Lambda^0\gamma$ is the dominant decay mode. Its width is relatively small due to the small photon energy.

The decay $\Delta \to p\gamma$ is known to be almost completely M1 rather than E2 from a partial wave analysis of the reaction $\gamma p \to \Delta \to p\pi^0$.

TABLE: MESON DECAYS $a \to b + \gamma$

	$\frac{2S_b+1}{2S_a+1} \lvert\langle f \lvert \vec{\mu} \rvert i \rangle\rvert^2$	Γ_{CALC} (MeV)	Γ_{EXPT} (MeV)
$\omega \to \pi^0 \gamma$	$\frac{1}{3}(\mu_u - \mu_d)^2$	1.39 f	.89 ± .05
$\rho \to \pi \gamma$	$\frac{1}{3}(\mu_u + \mu_d)^2$	.15 f	.07 ± .01
$\omega \to \eta \gamma$	$\frac{1}{6}(\mu_u + \mu_d)^2$	.01 f	.003 ± .002
$\rho \to \eta \gamma$	$\frac{1}{6}(\mu_u - \mu_d)^2$	.09 f	.05 ± .01
$\eta' \to \omega \gamma$	$\frac{1}{2}(\mu_u + \mu_d)^2$	.02 f	.008 ± .001
$\eta' \to \rho \gamma$	$\frac{1}{2}(\mu_u - \mu_d)^2$	.18 f	.08 ± .03
$\phi \to \eta \gamma$	$\frac{2}{3}\mu_s^2$	.11 f	.06 ± .01
$\phi \to \pi^0 \gamma$	0	0	.006 ± .002
$K^{*+} \to K^+ \gamma$	$(\mu_u + \mu_s)^2$	.15 f	.06 ± .03
$K^{*0} \to K^0 \gamma$	$(\mu_u + \mu_d)^2$	.22 f	.08 ± .04

EXTENSIVE CALCULATIONS OF A POSSIBLY MORE SOPHISTICATED NATURE CAN BE FOUND IN FEYNMAN ET AL, P.R. D3, 2701 (1971).

b. VECTOR MESON DECAYS TO e^+e^- OR $\mu^+\mu^-$

WE HAVE ALREADY CONSIDERED THESE DECAYS ON p106 & 210. NOW WE GIVE A QUARK MODEL VIEW, IN TERMS OF $q\bar{q}$ ANNIHILATION.

$$\text{RATE}\,(V \to e^+e^-) = \text{FLUX OF } q\bar{q} \cdot \sum_{\text{QUARKS}} \sigma_{q\bar{q} \to e^+e^-}$$

[FIRST CALCULATIONS OF THIS TYPE: NAMBU, PRL 8, 79 (1962)]

IF THE q AND $\bar{q}$ ARE TO ANNIHILATE INSIDE THE VECTOR MESON, THEN THEY MUST COLLIDE AT THE ORIGIN. HENCE FLUX = $2 V_q \lvert\psi(0)\rvert^2$

IN OUR 'ATOMIC' MODEL OF THE VECTOR MESON, $V_q < C$ IS NON RELATIVISTIC.

FROM p108 (WITH SOME EFFORT!) WE FIND

$$\sigma_{q\bar{q} \to e^+e^-} = \frac{4\pi}{3} \frac{\alpha^2 Q_q^2}{M_V^2} \frac{3 - V_q^2}{2 V_q} \qquad \text{ASSUMING } M_q \gg M_e$$

WE SHOULD ALSO NOTE THAT $\sigma_{q\bar{q}}$ INCLUDES A SPIN FACTOR $\frac{1}{2S_q+1}\frac{1}{2S_q+1} = \frac{1}{4}$

WHILE $\Gamma_{V \to e^+e^-}$ SHOULD INCLUDE $\frac{1}{2S_V+1} = \frac{1}{3}$

SO WE MUST CORRECT $\sigma_{q\bar{q}}$ BY A FACTOR OF $4/3$

WHEN SUMMING OVER THE POSSIBLE QUARK PAIRS, EACH $q\bar{q}$ COMBO CONTRIBUTES TO THE AMPLITUDE LINEARLY IN Q_q. SO WE DEFINE

$$Q \equiv \sum_{\text{QUARKS}} a_q Q_q$$

WHERE a_q = AMPLITUDE TO FIND THE $q\bar{q}$ PAIR INSIDE THE VECTOR MESON.

COMBINING ALL FACTORS IN THE LIMIT $v_q \ll c$

$$\Gamma_{V\to e^+e^-} = \frac{4}{3}\cdot\frac{4\pi}{3}\,\frac{\alpha^2 Q^2}{M_V^2}\cdot 3\,|\psi(0)|^2 = \frac{16\pi}{3}\,\frac{\alpha^2 Q^2}{M_V^2}\,|\psi(0)|^2$$

[VAN ROYEN & WEISSKOPF, NUOVO CIMENTO, 50, 617 & 51, 583 (1967)]

FOR EXAMPLE:

$$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \Rightarrow Q_\rho = \frac{1}{\sqrt{2}}\left(\frac{2}{3} - \left(-\frac{1}{3}\right)\right) = \frac{1}{\sqrt{2}}$$

$$\omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \Rightarrow Q_\omega = \frac{1}{\sqrt{2}}\left(\frac{2}{3} + \left(-\frac{1}{3}\right)\right) = \frac{1}{3\sqrt{2}}$$

$$\phi = s\bar{s} \Rightarrow Q_\phi = -1/3$$

TO THE EXTENT THAT $|\phi(0)|^2/M_V^2$ IS THE SAME FOR ρ, ω & ϕ

WE HAVE $\Gamma_{\rho\to e^+e^-} : \Gamma_{\omega\to e^+e^-} : \Gamma_{\phi\to e^+e^-} = \frac{1}{2} : \frac{1}{18} : \frac{1}{9} = \underline{9 : 1 : 2}$

DATA: 7.5 ± .7 .76 ± .2 1.2 ± .4 KeV

THE RATIOS ARE IN FAIR AGREEMENT WITH OUR MODEL.

IN ANALOGY WITH ATOMIC PHYSICS (p. 174) WE ESTIMATE THAT $|\psi(0)|^2 \sim \alpha_s^3 M_q^3$ α_s = STRONG COUPLING CONSTANT

THEN $\Gamma_{\rho\to e^+e^-} \sim \frac{8\pi}{3}\alpha^2\alpha_s^3\frac{M_q^3}{M_\rho^2} \sim 29\,\alpha_s^3$ KeV IF $M_q = 336$ MeV

IF $\alpha_s \sim 0.6$ WE ACHIEVE REASONABLE NUMERICAL AGREEMENT FOR THE ABSOLUTE DECAY RATES.

WE REMARK THAT THE 9:1:2 RATIO FOR ρ, ω & ϕ DECAY CAN BE DERIVED FROM SU(3) WITHOUT ANY MENTION OF QUARKS! ONE ASSUMES THAT THE COUPLING OF THE PHOTON TO SU(3) STATES IS LIKE THAT OF THE $|U=0,0\rangle$ STATE OF U-SPIN (pp 239-40) CHANGING THE NOTATION OF p. 240 SLIGHTLY

$$|\gamma\rangle \sim |U=0,0\rangle \sim \frac{\sqrt{3}}{2}\rho^0 + \frac{1}{2}\phi_8 = \frac{\sqrt{3}}{2}\rho^0 + \frac{1}{2}\left(\frac{1}{\sqrt{3}}\omega + \sqrt{\frac{2}{3}}\phi\right)$$

↖ p. 243

OR $|\gamma\rangle \sim 3\rho^0 + \omega + \sqrt{2}\,\phi$ SO FAR AS COUPLINGS ARE CONCERNED.

THIS IS A MORE FORMAL VERSION OF THE IDEA THAT THE PHOTON HAS PART ISOVECTOR AND PART ISOSCALAR NATURE, DISCUSSED ON p. 190. PEOPLE HAVE CARRIED THIS IDEA TO EXTREMES, SAYING THAT THE PHOTON TURNS INTO A ρ^0, ω, OR ϕ WHENEVER IT INTERACTS WITH HADRONS, ... (THE SO-CALLED VECTOR DOMINANCE MODEL).

c. $\pi^0, \eta, \eta' \to 2\gamma$

IN THE QUARK MODEL THE PSEUDOSCALAR MESONS ARE $q\bar{q}$ STATES WITH A $^1S_0$ WAVE FUNCTION IN SPIN AND SPACE. HENCE THE $M \to \gamma\gamma$ DECAYS ARE VERY SIMILAR TO THE $^1S_0 \to \gamma\gamma$ DECAY OF POSITRONIUM. WE THEN ESTIMATE

$$\Gamma_{M \to \gamma\gamma} \sim \alpha^2 \frac{|\psi(0)|^2}{M_q^2} Q^4$$

WHERE $Q^2 = \sum_{\substack{\text{QUARK}\\\text{PAIRS}}} a_q Q_q^2$, SINCE THE DECAY AMPLITUDE DEPENDS ON THE SQUARE OF THE QUARK CHARGE

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \Rightarrow Q^2_\pi = \frac{1}{\sqrt{2}}\left(\frac{4}{9} - \frac{1}{9}\right) = \frac{1}{3\sqrt{2}}$$

$$\eta \sim \frac{1}{\sqrt{6}}(2s\bar{s} - u\bar{u} - d\bar{d}) \Rightarrow Q^2_\eta = \frac{1}{\sqrt{6}}\left(2\left(\frac{1}{9}\right) - \frac{4}{9} - \frac{1}{9}\right) = -\frac{1}{3\sqrt{6}}$$

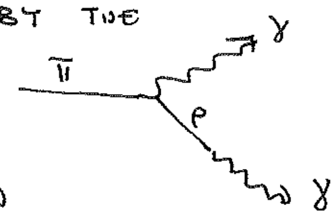
$$\eta' \sim \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \Rightarrow Q^2_{\eta'} = \frac{1}{\sqrt{3}}\left(4/9 + 1/9 + 1/9\right) = \frac{2}{3\sqrt{3}}$$

AND $\Gamma_{\pi^0 \to \gamma\gamma} : \Gamma_{\eta \to \gamma\gamma} : \Gamma_{\eta' \to \gamma\gamma} = \frac{1}{18} : \frac{1}{54} : \frac{4}{27} = 3 : 1 : 8$ IF $\frac{|\psi(0)|^2}{M_q^2}$ IS A CONSTANT

BUT $8 \pm .5$ eV $\quad 320 \pm 45$ eV $\quad 6 \pm 2$ KeV EXPERIMENTALLY

APPARENTLY $\frac{|\psi(0)|^2}{M_q^2}$ MUST DEPEND ON THE MESON INVOLVED.

AS BEFORE, $|\psi(0)|^2 \sim \alpha_s^3 M^3$ FOR SOME APPROPRIATE MASS M. SIMPLY $M = M_q$ WON'T DO. WE MAY BE GUIDED BY THE 'VECTOR DOMINANCE' VIEW OF THE DECAY PROCESS



THE π TURNS INTO A ρ^0 VIA M1 PHOTON EMISSION, AND THEN THE ρ^0 TURNS INTO A γ. WE SAW THAT THE RATE FOR M1 PHOTON EMISSION VARIES LIKE $k^3 \sim (M_\pi/2)^3$. SO WE INFER $M = M_{MESON}$

AND $$\Gamma \sim \alpha^2 \alpha_s^3 \frac{M^3}{M_q^2} Q^4$$

$$\Gamma_\pi : \Gamma_\eta : \Gamma_{\eta'} \sim 3M_\pi^3 : M_\eta^3 : 8M_{\eta'}^3 \sim 1 : 22 : 950$$

IN SOMEWHAT BETTER ACCORD WITH FACT.

NUMERICALLY WE ESTIMATE $\Gamma_{\pi\to\gamma\gamma} \sim \frac{\alpha^2 \alpha_s^3}{18} \frac{M_\pi^3}{M_u^2} \sim 66\,\alpha_s^3$ eV.

WITH $\alpha_s \sim 0.5$ WE ACHIEVE AGREEMENT WITH EXPERIMENT.

FOR THE SEQUEL WE REMARK THAT ADLER [P.R. 177, 2426 (1969)] MADE A QUARK MODEL CALCULATION OF $\pi^0 \to \gamma\gamma$ RELATING THIS TO CERTAIN MEASURABLE PARAMETERS OF THE WEAK DECAY $\pi^+ \to \mu^+ \nu$. THIS CALCULATION ACHIEVES EXCELLENT AGREEMENT WITH EXPERIMENT BY AVOIDING DIRECT USE OF AN ATOMIC MODEL WITH QUARK COUPLING α_s.

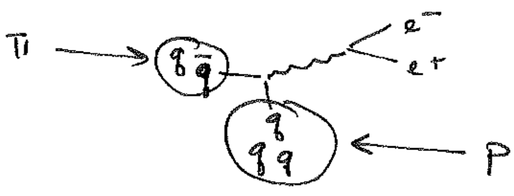
THE GENERAL CONCLUSION OF THIS SECTION IS THAT THE MODEL OF LOW MASS MESONS AND BARYONS AS QUARK ATOMS IS SUGGESTIVE, BUT DOES NOT HOLD UP WELL IN DETAIL. IN RETROSPECT WE MAY ARGUE THAT THE MASSES OF THE QUARKS ARE TOO SMALL COMPARED TO THE STRENGTH OF THE QUARK-QUARK FORCE FOR AN ATOMIC PICTURE OF AN ATOM TO BE ACCURATE. [IF THE CHARGE OF THE NUCLEUS IS GREATER THAN ~137e, ORDINARY ATOMS CAN'T REALLY EXIST....]

AS WE INDICATED IN LECTURE 8, DRAMATIC EVIDENCE FOR THE QUARK STRUCTURE OF THE PROTON CAME FROM BJORKEN'S ANALYSIS OF INELASTIC ELECTRON-PROTON SCATTERING (1969). A KEY STEP WAS THE ABANDONMENT OF THE ATOMIC PICTURE. INSTEAD WE FOUND EVIDENCE FOR POINTLIKE CONSTITUENTS OF THE PROTON WHICH CAN CARRY ANY FRACTION FROM 0 TO 1 OF THE PROTON'S LONGITUDINAL MOMENTUM. THIS IS NOT A STATIC VIEW, BUT A PICTURE OF THE PROTON 'ON THE FLY' WHICH ACHIEVES GREATEST VALIDITY IN THE HIGH ENERGY LIMIT.

7. THE DRELL-YAN PROCESS

A CONCEPTUAL VARIATION OF INELASTIC ELECTRON-PROTON SCATTERING OFFERS FURTHER EVIDENCE FOR THE DYNAMICAL EXISTENCE OF QUARKS INSIDE RAPIDLY MOVING MESONS AND BARYONS.

INSTEAD OF WAITING FOR A $q\bar{q}$ MESON STATE TO DECAY DUE TO ANNIHILATION OF THE QUARK AND ANTIQUARK, WE ARRANGE THAT THE ANTIQUARK ANNIHILATES WITH A QUARK FROM ANOTHER PARTICLE.



ONE POSSIBLE MODE OF ANNIHILATION IS THE ELECTROMAGNETIC PROCESS $\bar{q}q \to \gamma \to e^+e^-$, SKETCHED ABOVE.

Ph 529 Lecture 14

The reaction, $\pi P \to e^+e^-$ + anything, due to $q\bar{q}$ annihilation occurs in only about 1 in 10^6 πp interactions, but it is especially straightforward to calculate

$$d\sigma = \underbrace{f_\pi^{\bar{q}}(x_1)\,dx}_{\text{probability of finding the } \bar{q} \text{ in the } \pi \text{, carrying fraction } x_1 \text{ of the } \pi\text{'s momentum}} \; \underbrace{f_p^{q}(x_2)\,dx_2}_{\text{prob. of finding } q \text{ in } P \text{ with } x_2} \; \underbrace{\sigma_{\bar{q}q\to e^+e^-}}_{\frac{4\pi}{3}\frac{\alpha^2 Q_q^2}{M_{\bar{q}q}^2}}$$

A short kinematic calculation relates $M_{\bar{q}q} = M_{e^+e^-}$ to the quark momentum fractions x_1 & x_2

$$\vec{P}_{\bar{q}} = \frac{x_1\sqrt{s}}{2} \longrightarrow \longleftarrow \vec{P}_q = \frac{x_2\sqrt{s}}{2}$$

$$P_{e^+e^-} = P_{\bar{q}q} = P_{\bar{q}} - P_q = \frac{\sqrt{s}}{2}(x_1 - x_2) \qquad \left(\sqrt{s} = E_{CM}\right)$$

$$E_{e^+e^-} = E_{\bar{q}q} \sim P_{\bar{q}} + P_q = \frac{\sqrt{s}}{2}(x_1 + x_2)$$

$$M^2_{e^+e^-} = M^2_{\bar{q}q} = E^2_{\bar{q}q} - P^2_{\bar{q}q} = s\, x_1 x_2$$

$P_{e^+e^-}$ and $M^2_{e^+e^-}$ are readily measured by observing the final state electrons.

We can now write
$$\frac{d\sigma}{dx_1\,dx_2} = \frac{4\pi}{3}\alpha^2 \frac{s}{M^4_{e^+e^-}} \sum_{\text{quarks}} Q_q^2\, x_1 f_\pi^{\bar{q}}(x_1)\; x_2 f_p^{q}(x_2)$$

This exhibits a scaling behavior
$$\frac{M^2 d\sigma}{dx_1\,dx_2} = F\left(x_1, x_2, \frac{M^2}{s}\right)$$

which is observed experimentally.

A simple test of this analysis concerns the comparison

$$\frac{\sigma_{\pi^+ C\to e^+e^-}}{\sigma_{\pi^- C\to e^+e^-}}$$ in scattering of π's off a carbon target.

In terms of quarks, $\pi^+ = u\bar{d}$, $\pi^- = \bar{u}d$ and $C = 18(ud)$

Further, carbon is completely symmetric with respect to u and d quarks: $f_C^u(x) = f_C^d(x)$. You can use G-parity invariance to convince yourself that $f_{\pi^+}^{\bar{d}}(x) = f_{\pi^-}^{\bar{u}}(x)$

Then
$$\sigma_{\pi^+ C} \sim Q^2_{\bar{d}} = 1/9$$
$$\sigma_{\pi^- C} \sim Q^2_{\bar{u}} = 4/9$$
$$\Longrightarrow \frac{\sigma_{\pi^+ C}}{\sigma_{\pi^- C}} = \frac{1}{4}$$

EXPERIMENTALLY, THE CROSS SECTION RATIO APPROACHES 1/4 AS M^2/s GROWS LARGE.

WE HAVE NEGLECTED THE PRESENCE OF THE $\bar{q}q$ SEA, WHICH OCCURS IN ALL HADRONS. THE SEA QUARKS IN π^+ AND π^- HAVE IDENTICAL DISTRIBUTIONS, AND DILUTE THE RATIO, ESPECIALLY AT SMALL MASSES.

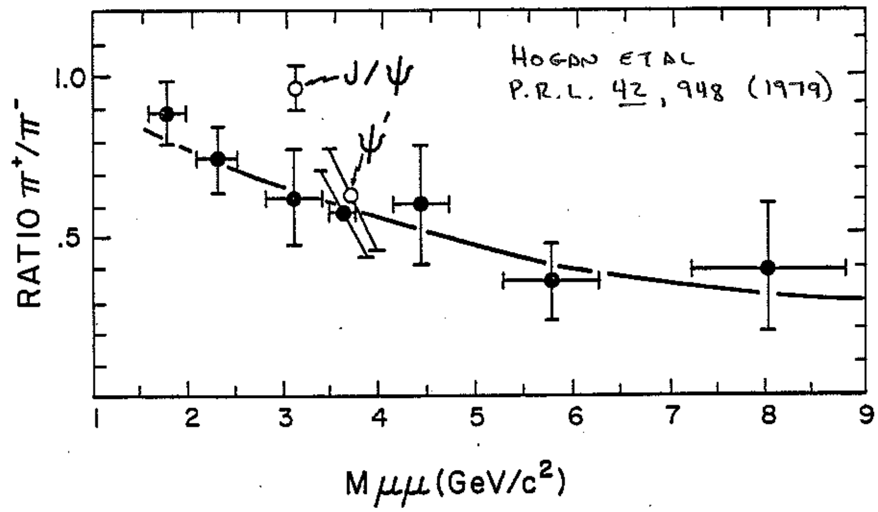


Figure 5-43. Production ratio of π^+ induced dimuons to π^- induced dimuons on carbon as a function of mass.

FURTHER ANALYSIS OF THESE REACTIONS ALLOWS ONE TO EXTRACT THE STRUCTURE FUNCTION $f_\pi^{\bar{q}}(x)$

SHOWN IN THE FIGURE. THE QUARKS IN THE PION TYPICALLY CARRY MORE MOMENTUM THAN QUARKS IN THE PROTON. THIS IS NOT TOO SURPRISING AS THERE ARE ONLY 2 QUARKS PER PION (NEGLECTING THE SEA).

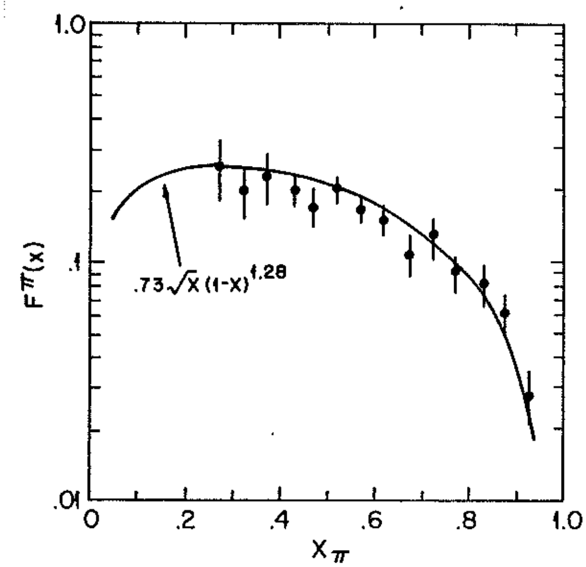


Figure 6-4. The pion structure function.

8. FREE QUARKS

CERTAINLY THE MOST STRIKING EVIDENCE IN FAVOR OF THE QUARK MODEL WOULD BE DIRECT EXPERIMENTAL DEMONSTRATION OF THE EXISTENCE OF FRACTIONALLY CHARGED QUARKS.

HOW WOULD WE KNOW A QUARK IF WE MET ONE? IF THE QUARK IS PRODUCED IN A HIGH ENERGY COLLISION WE WILL HAVE TO VIEW IT AS IT PASSES RAPIDLY BY. THE MAIN TECHNIQUE AVAILABLE TO INFER THE QUARK'S CHARGE IS TO PASS THE QUARK THRU A BLOCK OF MATTER AND OBSERVE THE dE/dx ENERGY LOSS TO ATOMS (LECTURE 4). THE QUARK INDUCED IONIZATION OF ATOMS WILL HAPPEN ONLY 1/9 OR 4/9 AS OFTEN, PER gm/cm^2 TRAVERSED,

AS FOR A PARTICLE OF CHARGE 1. NUMEROUS SEARCHES OF THIS TYPE HAVE FAILED TO PRODUCE QUARKS IN INTERACTIONS WITH $E_{cm} \lesssim 100$ GeV.

ANOTHER POSSIBILITY IS THAT QUARKS MIGHT BE LEFT OVER FROM THE EARLY UNIVERSE WHEN GREATER ENERGIES WERE AVAILABLE. ALL SEARCHES FOR PRIMORDIAL QUARKS HAVE ALSO BEEN NEGATIVE, EXCEPT FOR THE REPORT OF THE FAIRBANK GROUP [P.R.L. 46, 967 (1981)]. THEY CLAIM EVIDENCE FOR CHARGE $1/3\,e$ OBJECTS IN A 'MILLIKAN OIL DROP' EXPERIMENT. OTHER WORKERS FAIL TO REPRODUCE THIS RESULT.

COLORED QUARKS AND GLUONS

IN LECTURE 13 WE DISCUSSED THE IDEA OF 'COLOR'. SUPPOSEDLY, QUARKS HAVE AN ADDITIONAL QUANTUM NUMBER BESIDES FLAVOR, CHARGE, ETC, WHICH CAN TAKE ON 3 POSSIBLE VALUES, THE COLORS RED, GREEN, OR BLUE. IN PARTICULAR THIS EXTRA DEGREE OF FREEDOM ALLOWS THE QUARK WAVE FUNCTIONS OF BARYONS TO BE MADE ANTISYMMETRIC, CONSISTENT WITH FERMI-DIRAC STATISTICS.

AS WE DID PREVIOUSLY FOR THE QUARK CONCEPT, WE NOW EXPLORE THE POSSIBILITY THAT COLOR IS NOT JUST A BOOKKEEPING DEVICE, BUT AN IMPORTANT DYNAMICAL ASPECT OF THE STRONG INTERACTION. WE HAVE ALREADY INDICATED IN LECTURE 2 OUR PRESENT VIEW THAT COLOR IS THE CHARGE OF THE STRONG INTERACTION. ASSOCIATED WITH THIS COLOR CHARGE ARE MASSLESS, SPIN 1 QUANTA, THE GLUONS, WHICH MEDIATE THE STRONG INTERACTION, ANALAGOUSLY TO THE ROLE OF PHOTONS IN ELECTROMAGNETISM.

WE PRESENT 4 ARGUMENTS FOR COLOR OF AN ACCOUNTING NATURE, THEN WE SHOW HOW THE IDEA OF GLUONS HELPS UNDERSTAND SOME FEATURES OF THE MESON AND BARYON MASS SPECTRUM.

1. $\pi^0 \to \gamma\gamma$ DECAY

HISTORICALLY ONE OF THE FIRST CRITICAL APPLICATIONS OF THE IDEA OF COLOR, OTHER THAN FOR ANTISYMMETRIZING WAVE FUNCTIONS, WAS IN THE CALCULATION BY ADLER OF THE $\pi^0 \to \gamma\gamma$ DECAY RATE (p254). THE CALCULATION IS SOPHISTICATED, SO WE WILL ONLY INDICATE HOW IT IS MODIFIED BY QUARK COLOR.

AS ON P. 253, $\Gamma_{\pi^0 \to \gamma\gamma} \sim Q^4 \cdot$ OTHER FACTORS

WHERE $Q^2 = \sum_{\text{QUARKS}} a_q Q_q^2$ $\qquad a_q$ = AMPLITUDE TO FIND $q\bar{q}$

IF QUARKS COME IN THREE VARIETIES LABELLED BY COLOR, THE SUMMATION OVER COLORED QUARKS INCLUDES 3 TIMES AS MANY TERMS AS PREVIOUSLY CONSIDERED. ON THE OTHER HAND THE AMPLITUDE TO FIND A PARTICULAR COLORED QUARK IS $1/\sqrt{3}$ TIMES THE PREVIOUS CASE. THIS PRESERVES THE NORMALIZATION OF THE WAVE FUNCTION!

$$\pi^0 = \frac{1}{\sqrt{6}}\left(u_r\bar{u}_{\bar{r}} + u_g\bar{u}_{\bar{g}} + u_b\bar{u}_{\bar{b}} - d_r\bar{d}_{\bar{r}} - d_g\bar{d}_{\bar{g}} - d_b\bar{d}_{\bar{b}}\right)$$

IN THE STANDARD MODEL OF COLOR (GELL-MANN) QUARK CHARGE DEPENDS ON FLAVOR, BUT NOT ON COLOR $Q_{u_r} = Q_{u_g} = Q_{u_b} = 2/3$ ETC.

[SEE SECTION 7 BELOW FOR FURTHER COMMENTS ON THIS ASSIGNMENT]

THEN $Q^2_{\pi^0} = \frac{1}{\sqrt{6}}\left(3\cdot\frac{4}{9} - 3\cdot\frac{1}{9}\right) = \frac{1}{\sqrt{6}} = \frac{\sqrt{3}}{3\sqrt{2}}$

THIS IS COMPARED TO $Q^2_{\pi^0} = \frac{1}{3\sqrt{2}}$ FOUND ON P253, WITHOUT USING COLOR.

WE INFER $\Gamma_{\pi^0\to\gamma\gamma}\Big|_{\substack{\text{COLORED}\\\text{QUARKS}}} = 3\,\Gamma_{\pi^0\to\gamma\gamma}\Big|_{\substack{\text{PLAIN}\\\text{QUARKS}}}$

THE CLAIM IS THAT THE REST OF ADLER'S CALCULATION IS ACCURATE ENOUGH THAT THE FACTOR OF 3 DUE TO COLOR IS ESSENTIAL FOR OBTAINING AGREEMENT WITH EXPERIMENT.

[IN MOST DISCUSSIONS OF ADLER'S CALCULATION A FACTOR OF 9 IS REPORTED. BUT A FACTOR 1/3, ALSO DUE TO COLOR IS HIDDEN IN HIS DEFINITION OF f_π^2 ....]

2. $e^+e^- \to$ HADRONS

A MORE CLEAR CUT ARGUMENT IN FAVOR OF COLOR COMES FROM THE PRODUCTION OF HADRONS IN e^+-e^- COLLISIONS. THIS IS TYPICALLY COMPARED TO THE PRODUCTION OF $\mu^+\mu^-$ PAIRS.

$$R \equiv \frac{\sigma_{e^+e^-\to \text{HADRONS}}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$$

FOR PRODUCTION OF ANY POINTLIKE SPIN 1/2 PARTICLE & ANTIPARTICLE THE CROSS SECTION IS

$$\sigma_{e^+e^-\to a\bar{a}} = \frac{4\pi}{3}\frac{\alpha^2 Q^2}{s} \qquad \text{(P108)}$$

WHERE Q_a = CHARGE. WE ASSUME $M_a \ll \sqrt{s}$

THE HYPOTHESIS IS THAT ALL HADRONS ARE PRODUCED VIA THE REACTION $e^+e^- \to q\bar{q} \to$ HADRONS

ON p 229 WE GAVE A SCHEMATIC VIEW OF THE PROCESS BY WHICH THE QUARKS TURN INTO THE OBSERVED HADRONS.

THE QUARK MODEL ANALYSIS IS THEN

$$R = \sum_{\text{QUARKS}} Q_q^2$$

IF WE HAVE ONLY u, d AND s QUARK FLAVORS, THEN

$$R = 4/9 + 1/9 + 1/9 = 2/3$$

BUT IF QUARKS COME IN 3 COLORS, $R = 3(2/3) = 2$.

DATA COLLECTED FOR $\sqrt{s}$ BETWEEN 1.5 AND 3 GeV ARE 'REASONABLY' CONSISTENT WITH $R = 2$, AND CERTAINLY EXCLUDE $R = 2/3$

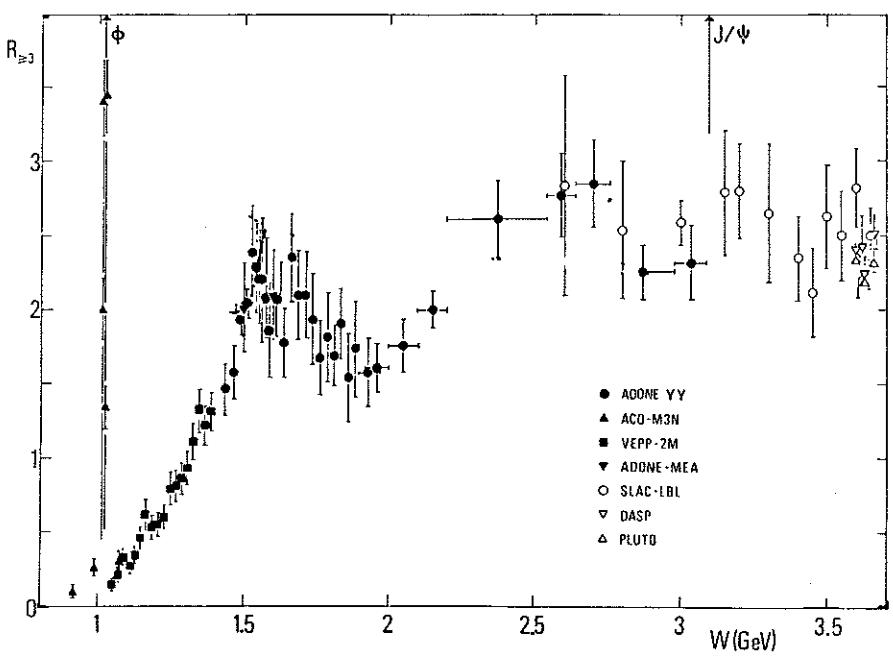


Fig. 5: The ratio $R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ [from Spinetti, 1979].

DATA COLLECTED WITH $\sqrt{s}$ BETWEEN 3 AND 10 GeV (p 260) SHOW CONSIDERABLE VARIATION WITH $\sqrt{s}$. BUT FOR $\sqrt{s} > 10$ R SETTLES DOWN TO A VALUE ABOUT 4 UNITS.

THE SUGGESTION IS THAT 2 ADDITIONAL QUARK FLAVORS ARE NOW BEING PRODUCED: CHARM, FOR $\sqrt{s} > 3.1$, AND BOTTOM, FOR $\sqrt{s} > 9.4$. WITH $Q_{\text{CHARM}} = 2/3$ AND $Q_{\text{BOTTOM}} = -1/3$, WE EXPECT R TO BE $3\frac{2}{3}$, INCLUDING THE FACTOR OF 3 FOR COLOR.

AN INTERESTING COMPLICATION IS DUE TO THE PRODUCTION OF $\tau^+\tau^-$ PAIRS, WHERE τ IS THE 'HEAVY LEPTON' PARTNER OF e AND μ.

THE τ DECAYS TO HADRONS ABOUT 60% OF THE TIME. HENCE ~ 80% OF THE TIME $e^+e^- \to \tau^+\tau^-$ LEADS TO HADRONS.

BUT $\sigma_{e^+e^- \to \tau^+\tau^-} = \sigma_{e^+e^- \to \mu^+\mu^-}$

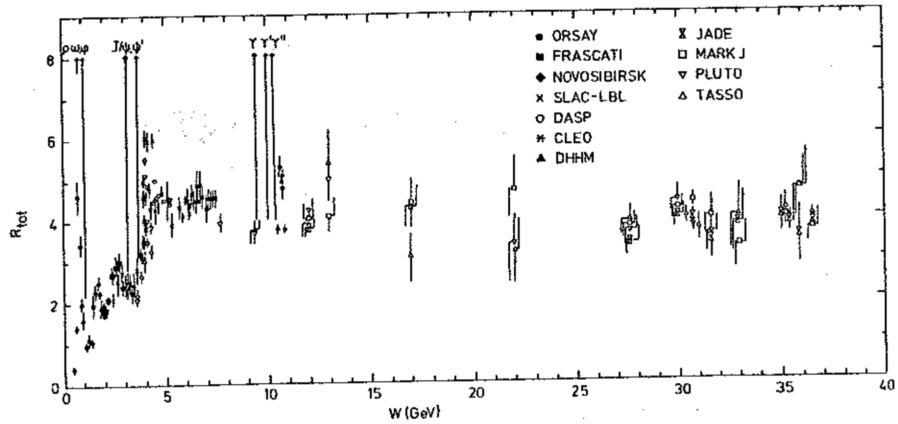


Figure 7a The ratio R of the total cross section for e^+e^- annihilation into hadrons to the μ pair cross section $\sigma_{\mu\mu} = 4\pi\alpha^2/3s$ (Quenzer 1977, Cordier et al 1979, Sidorov 1976, Perez-y-Jorba 1978, Schwitters 1976, Burmester et al 1977, Brandelik et al 1978a, Bacci et al 1979, Berger et al 1979a, Bartel et al 1979, Barber et al 1979a. Bock et al 1980, Brandelik et al 1980a, Cords 1980).

FOR $\sqrt{s} \gg M_\tau = 1784$ MeV. HENCE THERE IS AN APPARENT CONTRIBUTION OF ABOUT 0.8 TO R DUE TO τ PRODUCTION. THIS HAS BEEN REMOVED FROM THE ABOVE FIGURE.

3. $\tau \to$ HADRONS

ANOTHER ARGUMENT FOR COLOR CONCERNS THE FACT JUST STATED, THAT THE τ LEPTON DECAYS TO HADRONS ~60% OF THE TIME.

ALL τ DECAYS ARE WEAK DECAYS $\tau \to \nu_\tau$ + 'WEAK DOUBLET'

AS INDICATED BRIEFLY IN LECTURE 3, THE 'WEAK DOUBLETS' INCLUDE $\binom{e}{\nu_e}, \binom{\mu}{\nu_\mu}, \binom{\tau}{\nu_\tau}, \binom{u}{d}, \binom{c}{s}, \binom{t}{b} \dots ?$

ENERGY CONSERVATION ONLY ALLOWS THE DOUBLETS $\binom{e}{\nu_e} \binom{\mu}{\nu_\mu}$ & $\binom{u}{d}$ IN THE CASE OF τ DECAY. THE HADRONS ARE OBSERVED WHENEVER THE DOUBLET $\binom{u}{d}$ IS PRODUCED IN τ DECAY.

THE IDEA OF UNIVERSALITY OF THE WEAK INTERACTION IS THAT ALL DOUBLETS ARE EQUALLY LIKELY IN A WEAK DECAY (ENERGY PERMITTING).

IN THIS VIEW, $\tau \to$ HADRONS ~ 1/3 OF THE TIME.

BUT WITH COLORED QUARKS, WE HAVE 3 DOUBLETS $\binom{u}{d}_r \binom{u}{d}_g \binom{u}{d}_b$

AND $\tau \to$ HADRONS 3/5 OF THE TIME!

4. Color and the Quark Structure Functions

We inquire how color might modify the analysis of inelastic electron-proton scattering in terms of the quark structure functions $f(x)$. (Lecture 8)?

Recall that $f^q(x)\,dx$ is the probability of finding quark q carrying fraction x of the proton's momentum. If quarks come in 3 colors, we expect

$$f^{q_r}(x) = f^{q_g}(x) = f^{q_b}(x) = \frac{1}{3} f^{q}(x)$$

where f^q is the original structure function defined prior to the introduction of color. In e-p scattering, the photon couples to each quark according to the charge, so the cross section depends on Q_q^2

$$\sigma_{e\text{-}p} \sim \sum_{\text{quarks}} Q_q^2 f^q$$

After adding color, we sum over 3 times as many terms, each $1/3$ as large $\Rightarrow$ no change in cross-section!

The situation is slightly different for the Drell-Yan process $\pi p \to e^+e^- + X$ for which $\sigma \sim \sum_{\text{quarks}} Q_q^2 f_\pi^{\bar{q}}(x_\pi) f_p^{q}(x_p)$

On adding color, we have 3 times as many terms in the summation, each $1/9$ as large as before, as a red quark can only annihilate with an antired antiquark, etc.

$$\sigma_{\text{color}} = \frac{1}{3}\sigma_{\text{no color}} \qquad \text{(Drell-Yan)}$$

Experimentally, the cross sections are about $\frac{1}{2}$ to $\frac{2}{3}$ of the no color estimates. This result is interpreted as evidence for 'gluon radiative corrections', which increase the cross section above σ_{color} by 1.5 to 2. While this effect is considered a further success for the model of colored quarks and gluons, the argument is not simple anymore!

5. Color Singlet States

A somewhat ad hoc feature of the color idea as introduced in Lecture 13 is the statement that real hadrons can only be color singlet states. This claim establishes the applicability of Fermi-Dirac statistics for colored quark states, but hardly seems compelling in its own right. If we consider the gluons as the quanta which carry the strong force between colored quarks we can understand the favored nature of color singlet states.

GELL-MANN (1971) ADVANCED THE IDEA THAT COLOR HAS THE SIGNIFICANCE OF CHARGE FOR THE STRONG INTERACTION. THE 3 COLOR CHARGES ARE ASSOCIATED WITH 8 MASSLESS QUANTA, EACH CARRYING BOTH COLOR AND ANTICOLOR. THIS LAST IDEA IS A NON-TRIVIAL GENERALISATION FROM ELECTROMAGNETISM - WE COULD SAY THAT THE PHOTON CARRIES BOTH POSITIVE AND NEGATIVE CHARGE $\Rightarrow$ PHOTON IS NEUTRAL!

BUT GIVEN 3 KINDS OF COLOR CHARGE, WE CAN ARRANGE COLOR-ANTICOLOR COMBOS IN 2 DISTINCT WAYS, AS CATEGORIZED BY SU(3) MULTIPLETS

THE COLOR SINGLET: $\frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$

THE COLOR OCTET:

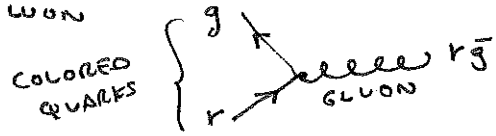
$g\bar{b}$ $\quad r\bar{b}$

$g\bar{r}$ $\quad \frac{1}{\sqrt{2}}(r\bar{r}-g\bar{g})$, $\frac{1}{\sqrt{6}}(2b\bar{b}-r\bar{r}-g\bar{g})$ $\quad r\bar{g}$

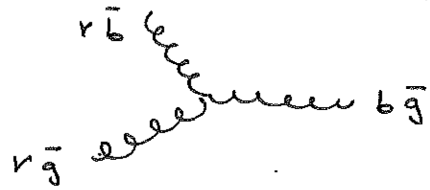
$b\bar{r}$ $\quad b\bar{g}$

IF WE SUPPOSE THE GLUON WERE A COLOR SINGLET, IT WOULD BE 'COLORLESS' SIMILAR TO THE PHOTON BEING CHARGELESS. HOWEVER GELL-MANN HAD THE INSIGHT THAT THE COMPLEXITIES OF THE STRONG INTERACTION ARE MUCH BETTER ACCOUNTED FOR IF THERE ARE ACTUALLY 8 GLUONS FORMING THE COLOR OCTET.

IN THIS VIEW QUARKS CAN CHANGE THEIR COLOR (BUT NOT THEIR FLAVOR) BY EMITTING OR ABSORBING A GLUON



THIS AMUSING POSSIBILITY IMMEDIATELY LEADS TO ANOTHER WHICH HAS NO ANALOGY IN ELECTROMAGNETISM: GLUONS CAN INTERACT WITH ONE ANOTHER:



RECALL THAT FURRY'S THEOREM (p.172) FORBIDS SUCH TRANSITIONS AMONG PHOTONS. WITH 3 COLORS THERE IS NO SIMPLE SYMMETRY SUCH AS 'COLOR CONJUGATION'!

WE WILL TRY TO THINK OF THE GLUONS AS THE QUANTA OF THE 'COLOR FIELD' WHICH IS SOMEWHAT ANALAGOUS TO THE ELECTROMAGNETIC FIELD. THIS NOTION MAY GIVE US A SEMI-CLASSICAL VIEW OF THE COLOR INTERACTION, BUT THE POSSIBILITY THAT 1 GLUON $\rightarrow$ 2 GLUONS IMPLIES A DEPARTURE FROM THE SUPERPOSITION PRINCIPLE FOR THE FIELD, WHICH IS A KEY TO OUR UNDERSTANDING OF CLASSICAL E & M. IT WILL BE DIFFICULT TO MAKE DETAILED SEMI-CLASSICAL ANALOGIES BETWEEN THE COLOR ELECTRIC AND MAGNETIC FIELDS AND THE CLASSICAL E & M FIELDS FOR THIS REASON. (BUT WE TRY ANYWAY ON OCCASION.)

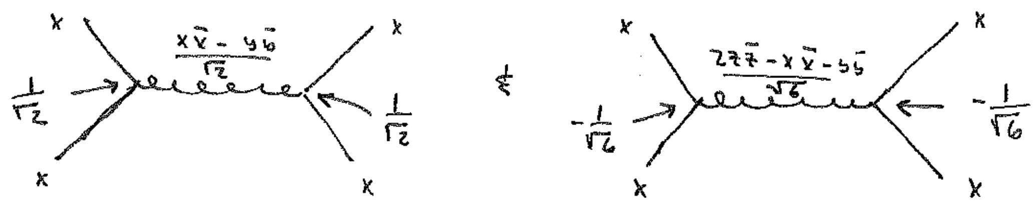
We now can give an argument how gluon exchange between quarks produces the strongest binding for color singlet quark configurations [Feynman, in the 5th Hawaii Conf. on Particle Physics (1973)].

We suppose that at least part of the quark-quark interaction can be understood from exchange diagrams.

This view cannot, however, explain the phenomenon of quark confinement: free quark pairs as well as bound states would be possible if only exchange diagrams are relevant.
In Lecture 12 we sketched an argument how the mysterious confining aspect of the strong interaction might be associated with a linear potential, $U \sim kr$. The gluon exchange diagrams we now consider are associated with a coulomb-like potential $U \sim -\frac{\alpha}{r}$ (in the limit that the gluons are massless). In Lecture 15 we will examine the effect of both kinds of potentials acting together....

The color force depends on quark color but <u>not</u> on quark flavor, by hypothesis. It is convenient then to label quarks only by their color in the following discussion. We let x, y, and z label any permutation of 3 distinct colors, r, g & b. Anti quarks will be labelled by anti color, $\bar{x}, \bar{y}$ or $\bar{z}$.

We compile a catalog of all types of gluon exchange between qq or $q\bar{q}$ pairs. We begin with a state of like colored quarks, xx (the flavors need not be the same!). In the scheme of 8 gluons there are 2 possible exchange diagrams.



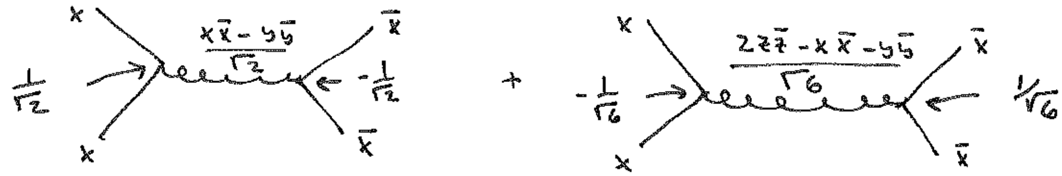
The exchanged gluons must lie at the center of the gluon octet. Whatever color x actually is, we are free to choose a representation of those gluon wavefunctions as shown.

Then we may read the relative coupling strengths at the vertices directly off the diagrams. The first diagram gives ampli $\sim \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$, while the second give $\left(-\frac{1}{\sqrt{6}}\right)^2 = \frac{1}{6}$. The two diagrams interfere,

so $$\langle xx | \text{color} | xx \rangle = 2/3$$

As for electromagnetism, we suppose that a positive exchange amplitude $\Rightarrow$ repulsion. (This is true for spin 1 exchange!, in the case of spin 0 or spin 2 exchange, like entities attract!)

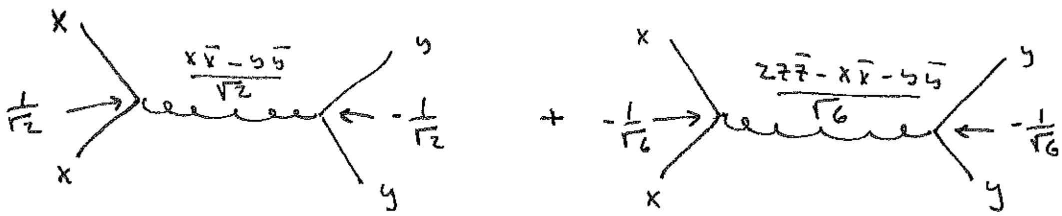
NEXT WE CONSIDER AN $x\bar{x}$ CONFIGURATION OF A $q\bar{q}$ PAIR. WE SUPPOSE THAT THE COUPLING OF A GLUON TO AN ANTIQUARK IS THE NEGATIVE OF THE COUPLING TO A QUARK



THUS $\langle x\bar{x} | \text{COLOR} | x\bar{x}\rangle = -2/3$ ATTRACTIVE !

SO FAR THIS IS MUCH LIKE ELECTRICITY, BUT THERE ARE 4 MORE CASES WITH NO E&M ANALOGUE.
NOW CONSIDER PAIRS INVOLVING TWO DISTINCT COLORS xy AND $x\bar{y}$

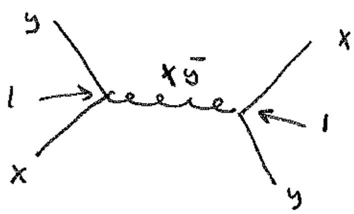
AGAIN 2 DIAGRAMS CONTRIBUTE TO EACH CASE



THUS $\langle xy | \text{COLOR} | xy\rangle = -\frac{1}{2} + 1/6 = -1/3$ ATTRACTIVE

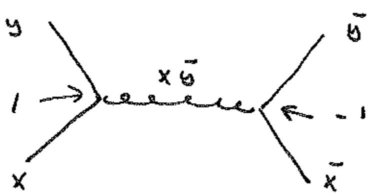
WHILE $\langle x\bar{y} | \text{COLOR} | x\bar{y}\rangle = 1/2 - 1/6 = +1/3$

BECAUSE QUARKS CAN CHANGE THEIR COLOR BY GLUON EMISSION OR ABSORPTION THERE IS ANOTHER DIAGRAM RELEVANT TO THE xy PAIR.

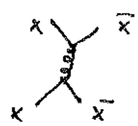


$\langle xy | \text{COLOR} | yx\rangle = +1$ COLOR INTERCHANGE

OF COURSE A QUARK CANNOT TURN INTO AN ANTIQUARK BY GLUON EMISSION! BUT THERE IS A FINAL DIAGRAM RELEVANT TO $x\bar{x}$ PAIRS



$\langle x\bar{x} | \text{COLOR} | y\bar{y}\rangle = -1$

[WHY DON'T WE CONSIDER DIAGRAMS LIKE 

↑ TIME ?

THE PROPAGATOR FACTOR EXPLAINS THIS. IN GLUON EXCHANGE, THE PROPAGATOR $\sim \frac{1}{q^2}$ WHERE $q^2 \to 0$ IS POSSIBLE, LEADING TO LARGE EFFECTS.

BUT FOR THE ANNIHILATION DIAGRAM, THE PROPAGATOR $\sim \frac{1}{M^2_{q\bar{q}}}$ $\Rightarrow$ TINY EFFECT COMPARED TO EXCHANGE.]

WE NOW HAVE ALL THE INGREDIENTS NEEDED TO CALCULATE THE RELATIVE BINDING OF QUARK STATES IN TERMS OF THE SYMMETRY OF THE COLOR PART OF THE QUARK WAVE FUNCTION.

FIRST THE $q\bar{q}$ MESON STATES. THE COLOR SINGLET WAVE FUNCTION IS, AS BEFORE: $\frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$

THE POSSIBILITIES FOR GLUON EXCHANGE INCLUDE 3 CASES OF TYPE $\langle x\bar{x}|\text{color}|x\bar{x}\rangle$ AND 6 OF $\langle x\bar{x}|\text{color}|y\bar{y}\rangle$

HENCE $\langle \text{singlet}|\text{color}|\text{singlet}\rangle = \left(\frac{1}{\sqrt{3}}\right)^2\left[3\left(-\frac{2}{3}\right) + 6(-1)\right] = -8/3$ UNITS

THIS IS A NET ATTRACTION, WHICH FAVORS BOUND STATES.

A TYPICAL COLOR OCTET STATE OF $q\bar{q}$ IS $|x\bar{y}\rangle$. THUS

$\langle \text{octet}|\text{color}|\text{octet}\rangle = \langle x\bar{y}|\text{color}|x\bar{y}\rangle = +1/3$

THIS IS A REPULSIVE EFFECT, AND IS SUGGESTIVE THAT BINDING WILL NOT OCCUR FOR COLOR OCTET CONFIGURATIONS!

EXERCISE: VERIFY THAT THE BINDING FACTOR IS ALSO $+1/3$ FOR THE $q\bar{q}$ COLOR OCTET STATES $\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$ AND $\frac{1}{\sqrt{6}}(2b\bar{b} - r\bar{r} - g\bar{g})$

THE QUESTION OF BINDING OF THE qqq BARYON STATES IS A BIT MORE COMPLICATED. THE QUARK-QUARK FORCES ARE ALWAYS BETWEEN PAIRS OF QUARKS, IF THEY ARISE FROM GLUON EXCHANGE. SO IT IS USEFUL TO CATEGORIZE THE COLOR SYMMETRY OF VARIOUS qq PAIRS INSIDE THE qqq STATES. THE POSSIBLE <u>COLOR</u> WAVE FUNCTIONS OF 3 QUARKS FALL INTO FOUR SU(3) MULTIPLETS, ACCORDING TO

$$3 \times 3 \times 3 = 1 + 8 + 8' + 10$$

THE COLOR SINGLET, 1, IS COLOR ANTISYMMETRIC UNDER THE INTERCHANGE ON ANY 2 QUARKS

THE COLOR DECUPLET, 10, IS COLOR SYMMETRIC

THE 2 COLOR OCTETS, 8 AND 8', HAVE 'MIXED SYMMETRY'. BY THIS WE MEAN THAT OF THE 3 PAIRS OF QUARKS, q_1q_2, q_2q_3 AND q_3q_1, ONE PAIR IS COLOR SYMMETRIC, ONE IS ANTISYMMETRIC AND THE 3RD IS NEITHER. (THIS FACT IS NOT SELF-EVIDENT.)

THIS SUGGESTS WE CALCULATE THE EFFECT OF GLUON EXCHANGE ON COLOR SYMMETRIC AND ANTISYMMETRIC qq STATES AS A WARM-UP EXERCISE.

COLOR SYMMETRIC: xx $\qquad \langle xx|\text{color}|xx\rangle = +2/3$

$\frac{xy + yx}{\sqrt{2}}$ $\qquad \frac{1}{2}\left(2\left(-\frac{1}{3}\right) + 2(1)\right) = +2/3$

ANTISYMMETRIC $\frac{xy - yx}{\sqrt{2}}$ $\qquad \frac{1}{2}\left(2\left(-\frac{1}{3}\right) - 2(1)\right) = -4/3$

THE COLOR ANTISYMMETRIC STATES OF qq HAVE A NET BINDING FORCE DUE TO GLUON EXCHANGE. THIS SHOWS THAT OUR EXPLANATION OF QUARK BINDING CANNOT BE COMPLETE. qq STATES HAVE NOT BEEN OBSERVED IN NATURE! AS INDICATED ON P.263, SOME CRITICAL FEATURE OF THE 'QUARK CONFINEMENT' MECHANISM STILL ELUDES OUR UNDERSTANDING. THE GLUON EXCHANGE DIAGRAMS REPRESENT CORRECTIONS TO THE LARGER CONFINEMENT EFFECT. THE LATTER IS APPARENTLY INSUFFICIENT TO BIND qq STATES EVEN WITH THE HELP OF THE ATTRACTIVE FORCE OF GLUON EXCHANGE FOR COLOR ANTISYMMETRIC PAIRS.

IN TERMS OF SU(3) MULTIPLETS OF qq, $3 \times 3 = \underset{\text{ASYM.}}{3^*} + \underset{\text{SYM.}}{6}$

SO GLUON EXCHANGE FAVORS qq PAIRS IN THE 3^* MULTIPLET OF COLOR...

TURNING NOW TO THE qqq STATES, WE HAVE 3 CASES:

COLOR SINGLET: ALL 3 qq PAIRS ARE ANTISYMMETRIC IN COLOR $\Rightarrow 3\left(-\frac{4}{3}\right) = -4$ UNITS OF BINDING

COLOR OCTET: 1 ASYM, 1 SYM & 1 REGULAR xy PAIR $\Rightarrow -\frac{4}{3} + \frac{2}{3} + \left(-\frac{1}{3}\right) = -1$ UNIT

COLOR DECUPLET: 3 SYMMETRIC PAIRS $\Rightarrow 3\left(\frac{2}{3}\right) = +2$ UNITS

THE COLOR SINGLET STATES ARE CLEARLY THE MOST STRONGLY BOUND. THE COLOR OCTET STATES WILL TEND TO 'DECAY' INTO AN ASYMMETRIC qq PAIR AND AN EXTRA QUARK. AS INDICATED ABOVE, THE qq PAIR MUST BE UNSTABLE ALSO. WE INFER THAT A COLOR OCTET WILL 'DECAY' DOWN TO A COLOR SINGLET RAPIDLY, BEFORE THE STATE EMERGES INTO THE 'REAL WORLD' FOR OBSERVATION. COLOR DECUPLET STATES ARE 'CLEARLY' UNSTABLE.

6. MASS RELATION BETWEEN DIFFERENT SU(3) FLAVOR MULTIPLETS

IN SECTION 4 OF LECTURE 13 WE CONSIDERED A QUARK MODEL CALCULATION OF MESON AND BARYON MASSES:

$$M = \sum_i M_{\text{QUARK}} + U(\text{SPIN, SU(3) MULTIPLET})$$

(WE CHANGE THE SIGN SO THAT $U < 0 \Rightarrow$ STRONGER BINDING)
WE PURSUE THIS CALCULATION FURTHER IN LIGHT OF THE NEW INSIGHTS PROVIDED BY THE IDEA OF GLUON EXCHANGE.

EMPIRICALLY THE BINDING U MUST DEPEND ON THE QUARK SPIN CONFIGURATION. THE $S=0$ MESONS HAVE LOWER MASS THAN THE $S=1$ MESONS, AND THE $S=\frac{1}{2}$ BARYON OCTET HAS LOWER MASS THAN THE $S=\frac{3}{2}$ DECUPLET. THE LARGE SIZE OF THESE MASS SPLITTINGS SUGGESTS THAT THE CAUSE IS A STRONG INTERACTION EFFECT.

IN SECTION 5, LECTURE 13 WE CONSIDERED A SPIN DEPENDENT EFFECT, THE MAGNETIC HYPERFINE INTERACTION:

$$U_{E\&M} = -\frac{8\pi}{3} \sum_{\text{PAIRS}} |\psi(0)|^2 \bar{\mu}_i \cdot \bar{\mu}_j \qquad \text{WITH } \bar{\mu} = \frac{e\bar{\sigma}}{2m}$$

SUPPOSE THAT THE SPIN OF A QUARK ALSO PRODUCES A COLOR MAGNETIC DIPOLE FIELD, DESCRIBED BY A COLOR MAGNETIC MOMENT

$$\bar{\mu}_s = \frac{f\bar{\sigma}}{2m}$$ WHERE f = COLOR COUPLING FACTOR

$f^2 \sim \alpha_{STRONG}$ · NUMERICAL FACTOR NEAR 1

THAT IS, WE TAKE THE ANALOGY BETWEEN COLOR CHARGES AND FIELD WITH ELECTRIC CHARGES AND FIELDS QUITE SERIOUSLY!

THEN $$U_{COLOR} \sim -\frac{2\pi}{3} \sum_{PAIRS} \frac{|\psi(0)|^2 f_i f_j}{m_i m_j} \bar{\sigma}_i \cdot \bar{\sigma}_j$$

WE SIMPLY ASSUME $f_i f_j = \begin{cases} -\alpha_s & \text{FOR A } q\bar{q} \text{ PAIR} \\ +\alpha_s & \text{" } qq \text{ "} \end{cases}$

IN ANALOGY WITH E&M.

WE CATALOG SOME RESULTS ABOUT SPIN EXPECTATION VALUES. NOTE THAT $\langle\bar{\sigma}\rangle = 2\langle\bar{S}\rangle$ WHERE $\bar{S}$ = SPIN $1/2$ OPERATOR

THEN $\langle\bar{\sigma}_1\cdot\bar{\sigma}_2\rangle = 4\langle\bar{S}_1\cdot\bar{S}_2\rangle = 2\left[(\bar{S}_1+\bar{S}_2)^2 - \bar{S}_1^2 - \bar{S}_2^2\right]$

WE ALSO DEFINE $\bar{S} = \bar{S}_1 + \bar{S}_2$ = TOTAL SPIN, AND RECALL $\langle\bar{S}^2\rangle = S(S+1)$

SO $$\langle\bar{\sigma}_1\cdot\bar{\sigma}_2\rangle = 2\left[S(S+1) - 3/2\right]$$

$$= \begin{cases} +1 & \text{IF } S=1 \\ -3 & \text{IF } S=0 \end{cases}$$ FOR $q\bar{q}$ STATES

[THIS RESULT WAS USED ON P. 245]

FOR THE qqq STATES WE NEED TO SUM OVER THE 3 POSSIBLE PAIRS

$$\langle\bar{\sigma}_1\cdot\bar{\sigma}_2 + \bar{\sigma}_2\cdot\bar{\sigma}_3 + \bar{\sigma}_3\cdot\bar{\sigma}_1\rangle = 4\langle\bar{S}_1\cdot\bar{S}_2 + \bar{S}_2\cdot\bar{S}_3 + \bar{S}_3\cdot\bar{S}_1\rangle$$

$$= 2\langle(\bar{S}_1+\bar{S}_2+\bar{S}_3)^2 - \bar{S}_1^2 - \bar{S}_2^2 - \bar{S}_3^2\rangle$$

$$= 2\left[S(S+1) - 9/4\right]$$

$$= \begin{cases} +3 & \text{IF } S = 3/2 \\ -3 & \text{IF } S = 1/2 \end{cases}$$

SO FOR $q\bar{q}$, $U(S=1) > U(S=0)$

BUT FOR qqq $U(S=3/2) < U(S=1/2)$

NOTING THE SIGNS OF $f_i f_j$

FURTHERMORE $|U(S=3/2) - U(S=1/2)| = 3/2\,|U(S=1) - U(S=0)|$

THESE LAST TWO RESULTS ARE CONTRARY TO EXPERIMENTAL FACT.

THE ANALYSIS OF THE COLOR FORCE IN SECTION 4 ABOVE ALLOWS US TO IMPROVE THINGS CONSIDERABLY. THE COLOR DIPOLE-DIPOLE INTERACTION IS PRESUMABLY DUE TO GLUON EXCHANGE, SO BETTER ESTIMATES OF THE COUPLING FACTOR $f_i f_j$ CAN BE HAD FROM OUR RECENT EFFORTS.

FOR THE $q\bar{q}$ COLOR SINGLET, $f_i f_j \to -\frac{8}{3}\alpha_s$

THE DRAMATIC RESULT OF THE GLUON EXCHANGE ANALYSIS IS THAT THE STRONG FORCE IS ATTRACTIVE BETWEEN QUARK PAIRS INSIDE A qqq COLOR SINGLET STATE. HENCE WE IDENTIFY

$$f_i f_j \to -\frac{4}{3}\alpha_s \qquad \text{FOR } qq \text{ PAIRS INSIDE BARYONS}$$

NOTE THAT THE $q\bar{q}$ ATTRACTION IS TWICE THAT OF THE qq CASE. THESE RESULTS IMPLY THAT THE MESON MASS DIFFERENCES, DUE TO SPIN EFFECTS, WILL BE LARGER THAN THE BARYON DIFFERENCES, AND THAT IN BOTH CASES HIGH SPIN CORRESPONDS TO HIGH MASS.

THUS THE ADDITION OF COLOR TO THE QUARK MODEL GREATLY IMPROVES OUR UNDERSTANDING OF THE HADRON MASS SPLITTINGS, WHICH WAS RATHER SKETCHY IN LECTURE 13.

THE MASS ANALYSIS IS READILY CARRIED FURTHER BY NOTING THE QUARK MASSES IN DETAIL.

IN THE SUMMARY TABLES

$$a \equiv \frac{8\pi}{9}\alpha_s |\psi(0)|^2_{\text{BARYON}}$$

$$b \equiv \frac{8\pi}{9}\alpha_s |\psi(0)|^2_{\text{MESON}}$$

DIMENSIONALLY, a & b VARY AS $1/r^3$, SO THE FITS TO a & b ARE CONSISTENT WITH OUR PREVIOUS RESULTS THAT MESONS ARE SLIGHTLY SMALLER THAN BARYONS.

Table 13. Meson masses with hyperfine splittings incorporated.

Meson	Coeff. of m_u or m_d	Coeff. of m_s	$\Delta E\|_{Hfs}$	Prediction (MeV/c^2)
$\pi(138)$	2	0	$-6b/m_u^2$	140
K(496)	1	1	$-6b/m_u m_s$	485
$\eta(549)$	2/3	4/3	$-2b/m_u^2 - 4b/m_s^2$	559
$\rho(776)$, $\omega(783)$	2	0	$2b/m_u^2$	780
K*(892)	1	1	$2b/m_u m_s$	896
$\phi(1020)$	0	2	$2b/m_s^2$	1032

m_u = 310 MeV/c^2; m_s = 483 MeV/c^2; b/m_u^2 = 80 MeV/c^2.

Table 12. Baryon masses with hyperfine splittings incorporated.

Baryon	Coeff. of m_u or m_d	Coeff. of m_s	$\Delta E\|_{Hfs}$	Prediction (MeV/c^2)
N(939)	3	0	$-3a/m_u^2$	939
$\Lambda(1116)$	2	1	$-3a/m_u^2$	1114
$\Sigma(1193)$	2	1	$a/m_u^2 - 4a/m_u m_s$	1179
$\Xi(1318)$	1	2	$a/m_s^2 - 4a/m_u m_s$	1327
$\Delta(1232)$	3	0	$3a/m_u^2$	1239
$\Sigma^*(1384)$	2	1	$a/m_u^2 + 2a/m_u m_s$	1381
$\Xi^*(1533)$	1	2	$a/m_s^2 + 2a/m_u m_s$	1529
$\Omega(1672)$	0	3	$3a/m_s^2$	1682

m_u = 363 MeV/c^2; m_s = 538 MeV/c^2; a/m_u^2 = 50 MeV/c^2

7. THE HAN-NAMBU COLOR SCHEME

SOME OF THE EARLY WORK ON COLOR WAS DONE BY HAN & NAMBU [P.R. 139B, 1006 (1965)] WHO GAVE AN INTERESTING VARIATION. THEY SUPPOSED THAT QUARKS HAVE INTEGER CHARGES, BUT THE CHARGE DEPENDS ON THE COLOR! THIS COULD BE A SIMPLE EXPLANATION AS TO WHY FRACTIONALLY CHARGED QUARKS HAVEN'T BEEN OBSERVED.

FLAVOR

COLOR	u	d	s
r	1	0	0
g	1	0	0
b	0	-1	-1 ← CHARGE

AVERAGING OVER COLOR $\langle Q_u \rangle = 2/3 \qquad \langle Q_d \rangle = \langle Q_s \rangle = -1/3$

THUS WE WOULD FIND FRACTIONAL CHARGES TO THE EXTENT THAT COLOR CAN BE IGNORED.

IT IS QUITE HARD TO DEVISE EXPERIMENTAL TESTS TO DISTINGUISH THE HAN-NAMBU SCHEME FROM THE GELL-MANN SCHEME OF FRACTIONAL CHARGE INDEPENDENT OF COLOR.

FOR EXAMPLE, $R = \dfrac{\sigma_{e^+e^- \to \text{HADRONS}}}{\sigma_{e^+e^- \to \mu^+\mu^-}}$ IS THE SAME IN BOTH SCHEMES

LIKEWISE THE EFFECT OF COLOR ON $\Gamma_{\pi^0 \to \gamma\gamma}$ IS A FACTOR OF 3 IN BOTH CASES.

$$Q^2_{\pi^0} = \begin{cases} \frac{1}{\sqrt{6}}\left[3\left(\frac{4}{9}\right) - 3\left(\frac{1}{9}\right)\right] = \frac{1}{\sqrt{6}} & \text{GELL-MANN} \\ \frac{1}{\sqrt{6}}\left[2(1) - 1(1)\right] = \frac{1}{\sqrt{6}} & \text{HAN-NAMBU} \end{cases} \qquad (\text{cf. p253})$$

TRY THIS FOR $\eta \to \gamma\gamma$

A DISTINCTION ARISES IN THE DECAY $\eta' \to \gamma\gamma$

$$Q^2_{\eta'} = \begin{cases} \frac{1}{3}\left[3\left(\frac{4}{9}\right) + 3\left(\frac{1}{9}\right) + 3\left(\frac{1}{9}\right)\right] = 2/3 & \text{GELL-MANN} \\ \frac{1}{3}\left[\ 2 \quad + 1 \quad + 1\ \right] = 4/3 & \text{HAN-NAMBU} \end{cases}$$

THUS THE HAN-NAMBU MODEL PREDICTS $\Gamma_{\eta' \to \gamma\gamma}$ 4 TIMES LARGER THAN IN THE STANDARD COLOR MODEL. IT IS BELIEVED THAT THE CALCULATION OF THE REMAINING FACTORS IN $\Gamma_{\eta' \to \gamma\gamma}$ IS KNOWN WELL ENOUGH THAT EXPERIMENTAL DATA EXCLUDE THE HAN-NAMBU SCHEME.

[CHANOWITZ, P.R.L. 44, 59 (1980)].