PH 529 LECTURE 16

THE WEAK INTERACTION

[REFERENCE: 'WEAK INTERACTION OF LEPTONS AND QUARKS' BY COMMINS & BUCKSBAUM, CAMBRIDGE U.P. 1983]

THE WEAK INTERACTION IS THE THIRD WHICH PLAYS AN IMPORTANT ROLE IN THE MICROWORLD, AFTER THE STRONG AND ELECTROMAGNETIC INTERACTIONS. AS MENTIONED IN LECTURE 2, IT WAS DISCOVERED IN THE FORM OF β DECAY IN 1896 BY BECQUEREL, AND FIRST IDENTIFIED AS BEING DISTINCT FROM OTHER FORMS OF RADIOACTIVITY BY RUTHERFORD IN 1899. UNTIL THE 1940'S THE WEAK INTERACTION APPEARED ONLY TO PLAY A ROLE IN THE DECAY OF NUCLEI. BUT AFTER THE DISCOVERY OF THE $\mu^\pm$ LEPTONS AND THE $\pi^\pm$ MESONS IT WAS REALIZED THAT THE WEAK INTERACTION GOVERNS THE DECAY OF THESE PARTICLES ALSO (BUT NOT THE π^0). INDEED IN THE LONG RUN THE WEAK INTERACTION FINDS ITS SIMPLEST EXPRESSION IN LEPTON DECAY, BEING SOMEWHAT MORE INTRICATE FOR QUARK-QUARK TRANSITIONS, AND RELATIVELY COMPLICATED FOR BARYON AND NUCLEAR DECAY.

AFTER SOME INTRODUCTORY REMARKS WE PUT INITIAL EMPHASIS ON THE WEAK INTERACTIONS OF QUARKS AND LEPTONS.

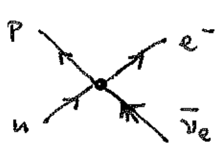
1. FERMI'S THEORY

SHORTLY AFTER THE DISCOVERY OF THE NEUTRON, FERMI [1934; FOR AN ENGLISH TRANSLATION SEE AM. J. PHYS, 36, 1150 (1968)] GAVE A VIEW OF β DECAY STRESSING AN ANALOGY WITH ELECTROMAGNETISM. WE ILLUSTRATE THIS WITH NEUTRON DECAY $\quad n \to p\, e^-\, \bar\nu_e$

THIS WAS CONSIDERED TO BE SOMETHING LIKE A TRANSITION $\quad n \to p\, \gamma^-$
IN WHICH THE 'CHARGED PHOTON' γ^- IS NOT OBSERVED, BUT COUPLES IMMEDIATELY TO $e^- \bar\nu_e$.

[IN WRITING $\bar\nu_e$ = ANTINEUTRINO WE ARE INVOKING A LATER CONCEPT OF LEPTON NUMBER CONSERVATION : e^-, ν_e HAVE $L_e = +1$, $e^+, \bar\nu_e$ HAVE $L_e = -1$]

THERE WAS NO EVIDENCE FOR LONG RANGE BEHAVIOR OF THE WEAK INTERACTION. FERMI ASSUMED THE TRANSITION $n \to p\, e^- \bar\nu$ TOOK PLACE AT A SINGLE POINT. THE MATRIX ELEMENT WAS WRITTEN TO LOOK MUCH LIKE PHOTON EXCHANGE, COMPRESSED TO A 'CONTACT' INTERACTION:



$$\mathcal{M} = \frac{G}{\sqrt{2}} \left(\bar u_p | \gamma_\alpha | u_n\right)\left(\bar u_{\bar\nu} | \gamma^\alpha | u_e\right)$$

THE $\sqrt{2}$ IS A HISTORICAL CONVENTION

γ_α IS A 4 VECTOR OF DIRAC MATRICES ; $|u_n) =$ 4-SPINOR WAVE FUNCTION...

THE SINGLE COUPLING CONSTANT $G/\sqrt{2}$ REPLACES THE COUPLING AND PHOTON PROPAGATOR OF E&M, e^2/q^2. THE DIMENSIONS OF G ARE THEN

$[G] = \frac{1}{E^2} = \frac{1}{M^2}$. BY OBSERVATIONS OF NUCLEAR β DECAY RATES FERMI & COLLEAGUES DETERMINED THAT

$$G \sim \frac{10^{-5}}{M^2_{PROTON}}$$

ALREADY IN 1935 YUKAWA NOTED THAT THE SHORT RANGE OF THE WEAK INTERACTION COULD BE EXPLAINED IF THE 'CHARGED PHOTON' REFERRED TO ABOVE WERE HEAVY. BUT HE THOUGHT THAT THE RANGE OF THE WEAK FORCE IS $\sim$ 1 FERMI $\Rightarrow M_{\gamma^-} \sim 200$ MeV $\sim M_\pi$.

I AM NOT SURE WHO WAS THE FIRST TO PUSH THE FERMI-YUKAWA IDEA ONE STEP FURTHER, TO NOTE THAT IF γ^- IS REALLY A HEAVY, CHARGED PHOTON THEN

$$\mathcal{M} \sim e^2 (\bar{u}_p | \gamma_\alpha | u_n) \frac{1}{q^2 - M^2_{\gamma^-}} (\bar{v}_{\bar{\nu}} | \gamma_\alpha | u_e)$$

SO IF $q^2 \ll M^2_{\gamma^-}$ $\quad \frac{G}{\sqrt{2}} \sim \frac{e^2}{M^2_{\gamma^-}}$ OR $M_{\gamma^-} \sim \sqrt{\frac{4\pi\sqrt{2}\alpha}{G}} \sim 100\, M_p$

[T.D. LEE CLAIMS IT WAS HIMSELF, P.R. 75, 905 (1949); IT DOES SEEM THAT HIS NAME OF W^- FOR γ^- HAS STUCK.] IN ANY CASE THIS IDEA WAS NOT TAKEN REALLY SERIOUSLY UNTIL THE MODEL OF GLASHOW, WEINBERG & SALAM EMERGED IN THE LATE 1960'S. WE HAVE ALREADY REMARKED IN LECTURE 3, p. 36, THAT WITHOUT THE γ^- (HEREAFTER CALLED W^-) THE FERMI THEORY BECOMES IMPLAUSIBLE AT VERY HIGH ENERGIES (HEISENBERG, 1936).

GAMOW & TELLER [P.R. 49, 895 (1936)] 'IMPROVED' ON THE FERMI THEORY, NOTING THAT IT COULD NOT ACCOUNT FOR THE ANGULAR MOMENTUM SELECTION RULES OBSERVED IN CERTAIN β DECAYS. THEY INTRODUCED 5 KINDS OF POSSIBLE MATRIX ELEMENTS. EACH IS AN OVERALL SCALAR, BEING THE PRODUCT OF TWO TERMS NOT NECESSARILY SCALARS THEMSELVES. (RECALL p. 89)

1. SCALAR - SCALAR	$(\bar{u}_p \| u_n)(\bar{v}_{\bar{\nu}} \| u_e)$	S
2. VECTOR - VECTOR	$(\bar{u}_p \| \gamma_\alpha \| u_n)(\bar{v}_{\bar{\nu}} \| \gamma^\alpha \| u_e)$	V
3. TENSOR - TENSOR	$(\bar{u}_p \| \sigma_{\alpha\beta} \| u_n)(\bar{v}_{\bar{\nu}} \| \sigma^{\alpha\beta} \| u_e)$	T
4. AXIAL VECTOR - AXIAL VECTOR	$(\bar{u}_p \| \gamma_5\gamma_\alpha \| u_n)(\bar{v}_{\bar{\nu}} \| \gamma_5\gamma^\alpha \| u_e)$	A
5. PSEUDOSCALAR - PSEUDOSCALAR	$(\bar{u}_p \| \gamma_5 \| u_n)(\bar{v}_{\bar{\nu}} \| \gamma_5 \| u_e)$	P

(NONE OF THESE ARE PARITY VIOLATING!)

IT WAS NOT EASY TO DISTINGUISH AMONG ALL THESE POSSIBILITIES FOR THE NEXT 20 YEARS! WE GET A ROUGH SENSE OF THE DIFFICULTY BY NOTING THAT IN NUCLEAR β DECAY $E_e \sim 1\,\mathrm{MeV} \Rightarrow P_p \sim P_e \sim 1\,\mathrm{MeV}$ THUS TERMS LIKE P_p/M_p ARE RATHER NEGLIGIBLE. SO WE MAY EVALUATE THE FACTORS $(\bar{u}_p|\Gamma|u_n)$ IN THE NON RELATIVISTIC LIMIT, AS ON p 90-91.

$S: \quad (\bar{u}_p|u_n) \to 2M$

$V: \quad (\bar{u}_p|\gamma_0|u_n) \to 2M$ ALSO $\quad (\bar{u}_p|\gamma_i|u_n) \to 0 + O(P_p/M)$

$T:$ NOTING $\sigma_{0j} = i\begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}$ THEN $(\bar{u}_p|\gamma_{0j}|u_n) \to 0$

WHILE $\sigma_{ij} = \epsilon_{ijk}\begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} \Rightarrow (\bar{u}_p|\sigma_{ij}|u_n) \to \pm 2M \langle p|\sigma_k|n\rangle$

2 SPINORS

$A: \quad \gamma_5\gamma_0 = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} \Rightarrow (\bar{u}_p|\gamma_5\gamma_0|u_n) \to 0$

BUT $\gamma_5\gamma_i = \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \Rightarrow (\bar{u}_p|\gamma_5\gamma_i|u_n) \to -2M\langle p|\sigma_i|n\rangle$

$P: \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \Rightarrow (\bar{u}_p|\gamma_5|u_n) \to 0$

IN THE NON-RELATIVISTIC LIMIT THE P COUPLING VANISHES, WHILE BOTH S AND V CANNOT AFFECT SPIN OR ANGULAR MOMENTUM

$$S, V \Rightarrow \Delta J = 0 \qquad \text{(FERMI TRANSITIONS)}$$

BUT A AND T DO AFFECT SPIN

$$A, T \Rightarrow \Delta J = 0, \pm 1 \quad \text{BUT } J=0 \not\to J=0 \quad \text{(GAMOW-TELLER)}$$

EXPERIMENT INDICATED BOTH FERMI AND GAMOW-TELLER TYPE TRANSITIONS OCCUR IN NATURE, WHICH REQUIRES AT LEAST TWO TYPES OF COUPLINGS. BUT TOTAL DECAY RATE EXPERIMENTS CANNOT DISTINGUISH S FROM V, OR A FROM T.

2. PARITY VIOLATION AND THE V-A THEORY

THE SITUATION IMPROVED DRAMATICALLY IN 1956 WHEN LEE & YANG [P.R. 104, 254 (1956)] SUGGESTED THAT PARITY CONSERVATION MIGHT BE VIOLATED IN THE WEAK INTERACTION. THEY NOTED THAT THIS COULD BE DEMONSTRATED BY OBSERVING A CORRELATION BETWEEN DECAY PARTICLE DIRECTION AND INITIAL PARTICLE SPIN DIRECTION:

$$\langle \vec{\sigma} \cdot \vec{p} \rangle \neq 0$$

THIS WAS QUICKLY SHOWN TO BE THE CASE IN THE DECAY OF POLARIZED Co^{60} NUCLEI [WU ET AL, P.R. 105, 1413 (1957)]; AND IN THE DECAY CHAIN $\pi \to \mu \nu$, $\mu \to e \nu \bar{\nu}$ IN WHICH THE μ IS LEFT IN A POLARIZED STATE AFTER THE PARITY VIOLATING π DECAY [GARWIN ET AL, P.R. 105, 1415 (1957); FRIEDMAN & TELEGDI P.R. 105, 1681 (1957)]

IMPORTANT FURTHER INSIGHT INTO THE NATURE OF THE PARITY VIOLATION CAME FROM A SERIES OF EXPERIMENTS WHICH MEASURED THE NET HELICITIES OF ELECTRONS AND NEUTRINOS PRODUCED IN β DECAY OF UNPOLARIZED NUCLEI. SEE PERKINS SEC 6.5 FOR SOME DETAILS OF THESE INTERESTING EXPERIMENTS. CONSIDERABLE INGENUITY IS REQUIRED TO MEASURE THE ELECTRON SPIN DIRECTION ALONG ITS DIRECTION OF MOTION; THE EXPERIMENT TO MEASURE THE NEUTRINO HELICITY IS ASTOUNDING IN ITS SUBTLETY.

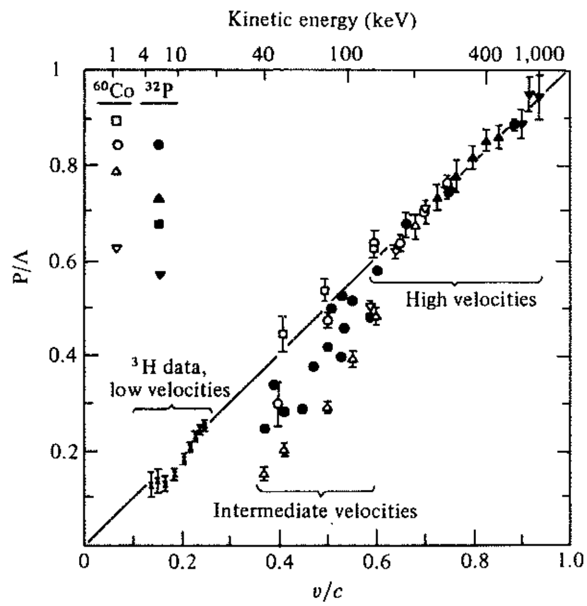
NET HELICITY IS DEFINED AS

$\dfrac{N_+ - N_-}{N_+ + N_-}$ WHERE N_+ = # OF ELECTRONS WITH SPIN UP ALONG DIRECTION OF MOTION, ETC

THE RESULTS ARE:

PARTICLE	NET HELICITY
ELECTRON	$-v/c$
POSITRON	$+v/c$
NEUTRINO	-1
ANTINEUTRINO	$+1$

Figure 5.1. Measured degree of longitudinal polarization for allowed e^- decay. Λ is a small correction for Coulomb and screening effects. The various data points represent measurements by a number of workers. (From Koks and Van Klinken 76. Reprinted with permission.)



THE RESULTS FOR THE NEUTRINO HELICITY ARE ESPECIALLY SUGGESTIVE. ON p116 WE NOTED THAT IN THE HIGH ENERGY LIMIT THE OPERATOR $\frac{1-\gamma_5}{2}$ ACTING ON THE 4-SPINOR OF A PARTICLE PRODUCES ONLY THE HELICITY -1 STATES, BUT IF IT ACTS ON AN ANTIPARTICLE, HELICITY $+1$ IS PRODUCED. IF THE NEUTRINO MASS IS SMALL, OR ZERO AS LONG BELIEVED, EVEN NUCLEAR β DECAY INVOLVES NEUTRINOS WHICH OBEY THE HIGH ENERGY LIMIT.

[PERKINS USES A DIFFERENT REPRESENTATION OF THE DIRAC MATRICES, WHICH UNFORTUNATELY CHANGES THE SIGN OF γ_5 IN THE HELICITY PROJECTION OPERATOR. PERHAPS THIS IS BECAUSE PERKINS IS FROM OXFORD, DIRAC CAMBRIDGE?]

THIS SUGGESTS THAT THE WEAK COUPLING $(\bar{v}_{\bar{\nu}} | \Gamma | u_e)$ MIGHT BE

$$(\bar{v}_{\bar{\nu}} | \gamma_\alpha (1-\gamma_5) | u_e) \quad \left[= (\bar{v}_{\bar{\nu}} | (1+\gamma_5)\gamma_\alpha | u_e) = \overline{(\bar{u}_e | \gamma_\alpha (1-\gamma_5) | v_{\bar{\nu}})} \right]$$

BOTH V AND A COUPLINGS ARE INCORPORATED, WHICH IS A POSSIBLE SOLUTION TO THE KNOWN NEED FOR BOTH FERMI AND GAMOW-TELLER COUPLINGS.

FURTHER, WE INTERPRET THE ABOVE COUPLING AS LEADING TO SPIN UP AND SPIN DOWN ELECTRONS WITH RELATIVE AMPLITUDES

$$|a_\pm) \approx \frac{1-\gamma_5}{2} |u_\pm)$$

WHERE $u_+ \sim \begin{pmatrix} 1 \\ 0 \\ P/E+M \\ 0 \end{pmatrix}$ AND $u_- \sim \begin{pmatrix} 0 \\ 1 \\ 0 \\ -P/E+M \end{pmatrix}$ AS ON P. 86

NOW $\frac{1-\gamma_5}{2} = \frac{1}{2}\begin{pmatrix} I & -I \\ -I & I \end{pmatrix}$ SO $|a_+) \sim \frac{1}{2}\begin{pmatrix} 1 - \frac{P}{E+M} \\ 0 \\ -1 + \frac{P}{E+M} \\ 0 \end{pmatrix}$; $|a_-) = \frac{1}{2}\begin{pmatrix} 0 \\ 1 + \frac{P}{E+M} \\ 0 \\ -1 - \frac{P}{E+M} \end{pmatrix}$

THEN $N_+ \sim (a_+|a_+) = \frac{1}{2}\left(1 - \frac{P}{E+M}\right)^2 = \frac{E-P}{E+M}$; $N_- = (a_-|a_-) = \frac{1}{2}\left(1 + \frac{P}{E+M}\right)^2 = \frac{E+P}{E+M}$

AND THE NET HELICITY IS $\frac{N_+ - N_-}{N_+ + N_-} = -\frac{P}{E} = -\frac{v}{c}$

IN THE HIGH ENERGY LIMIT, $N_+ \to 0$ AND ONLY NEGATIVE HELICITY, OR <u>LEFT-HANDED</u> ELECTRONS AND NEUTRINOS ARE PRODUCED.

THUS THE OBSERVED HELICITY MEASUREMENTS FOR BOTH ELECTRONS AND NEUTRINOS ARE CONSISTENT WITH THE V-A MATRIX ELEMENT. THIS WAS FIRST ADVOCATED BY FEYNMAN & GELL-MANN [P.R. <u>109</u>, 193 (1958)] AND BY SUDARSHAN & MARSHAK [P.R. <u>109</u>, 1860 (1958)].

IN PARTICULAR, A PURELY LEPTONIC DECAY SUCH AS $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ HAS MATRIX ELEMENT

$$\frac{G}{\sqrt{2}} (\bar{u}_\nu | \gamma_\alpha (1-\gamma_5) | u_\mu)(\bar{v}_{\bar{\nu}} | \gamma^\alpha (1-\gamma_5) | u_e)$$

HOWEVER IT WAS NOTED THAT IN NUCLEAR β DECAY A SLIGHTLY DIFFERENT FORM IS NEEDED. FOR EXAMPLE, IN $n \to p e^- \bar{\nu}$,

$$\mathcal{M} \sim \frac{G}{\sqrt{2}} (\bar{u}_p | \gamma_\alpha (1-\lambda\gamma_5) | u_n)(\bar{v}_{\bar{\nu}} | \gamma^\alpha (1-\gamma_5) | u_e)$$

WHERE EMPIRICALLY THE PARAMETER λ HAS VALUE 1.25.

TO THIS DAY THE PARAMETER λ IS NOT WELL UNDERSTOOD, BUT REPRESENTS THE EXTRA COMPLEXITY OF THE WEAK INTERACTION OF BOUND STATES OF QUARKS.

We may rewrite the V-A coupling in various ways. Suppose we define a left handed particle as $|u_L) = (1-\gamma_5)|u)$; $(\bar{u}_L| = (\bar{u}|(1+\gamma_5)$

Then a vector coupling between 2 left handed particles is

$$(\bar{u}_{L_2}|\gamma_\alpha|u_{L_1}) = (\bar{u}_2|(1+\gamma_5)\gamma_\alpha(1-\gamma_5)|u_1) = (\bar{u}_2|\gamma_\alpha(1-\gamma_5)^2|u_1)$$
$$= 2(\bar{u}_2|\gamma_\alpha(1-\gamma_5)|u_1)$$

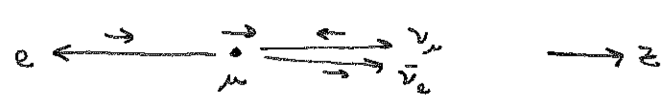
Thus after 34 years we are back to the original Fermi coupling, with the important, and parity violating, relation that only left-handed fermions (or right-handed antifermions) participate in the weak interaction. The absence of the right handed coupling is sometimes called <u>maximal</u> parity violation.

An immediate consequence concerns right handed neutrinos. Neutrinos only appear to participate in the weak interaction (and presumably the gravitational!). Then according to the rule that only left-handed neutrinos couple weakly, the right-handed neutrinos have no interactions at all! We might as well say that they don't exist. This is the essence of the <u>two component</u> theory of the neutrino. Note however that if the neutrino has non-zero mass, then the weak interaction couples very slightly to positive helicity neutrino, just as it does for positive helicity electrons when $v < c$.

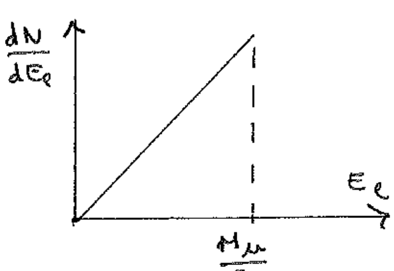
3. Muon Decay

Only in the late 1940's was it realized that muon decay is a weak interaction process. But since that time it has proven to be one of the best reactions for study, as it has no complications due to strong interactions.

A detailed calculation of muon decay is possible with the V-A theory. We consider $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ for μ^- with spin up along the z axis. As noted before, the decay configuration with the maximum energy electron has $\vec{P}_e$ along the $-z$ axis



Recall also that from 3 body phase space considerations (p. 196) we expect the decay electron energy spectrum to look something like



This was arrived at by integrating the 3-body phase space $dE_e\, d\Omega_e\, dE_\nu\, d\phi^*_\nu$ over the neutrino variables, which are not observable in practice. In the relativistic limit $\int dE_\nu \sim E_e$, as shown pictorially on p 196.

THE DECAY RATE CALCULATION IS THEN (p 192, 194)

$$d\Gamma = \frac{1}{2M_\mu} |\mathcal{M}|^2 \frac{1}{32\pi^3} E_e dE_e \frac{d\Omega_e}{4\pi}$$

IF $|\mathcal{M}|^2$ WERE INDEPENDENT OF THE 4 VECTORS OF THE $\bar{\nu}_e$ AND ν_μ

THIS CAN BE VERIFIED BY A CALCULATION OF $|\mathcal{M}|^2$ USING FEYNMAN'S TRACE TECHNIQUE. WE HAVE

$$\mathcal{M} = \frac{G}{\sqrt{2}} (\bar{u}_{\nu_\mu} | \gamma_\alpha (1-\gamma_5) | u_\mu)(\bar{v}_{\bar{\nu}} | \gamma^\alpha (1-\gamma_5) | u_e)$$

FOR WHAT IT'S WORTH, THE TABLE SUMMARIZES A FEW RULES FOR TRACES WITH γ_5 MATRICES.

Table 3.1. *Useful properties of γ matrices*

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

(a) $\mathrm{tr}\, \gamma_\mu\gamma_\nu = 4g_{\mu\nu}$

(b) $\mathrm{tr}\, \gamma_\alpha\gamma_\beta\gamma_\rho\gamma_\sigma = 4(g_{\alpha\beta}g_{\rho\sigma} - g_{\alpha\rho}g_{\beta\sigma} + g_{\alpha\sigma}g_{\beta\rho})$

(c) $\mathrm{tr}\, \gamma_\alpha\gamma_\beta\gamma_\rho\gamma_\sigma\gamma_5 = -4i\epsilon_{\alpha\beta\rho\sigma}$

(d) $\mathrm{tr}[\gamma_\alpha\gamma_\beta\gamma_\rho\gamma_\sigma(c_1 - c_2\gamma_5)]\, \mathrm{tr}[\gamma^\theta\gamma^\beta\gamma^\phi\gamma^\sigma(c_3 - c_4\gamma_5)] = 32(c_1c_3)(g_\alpha^\theta g_\rho^\phi + g_\alpha^\phi g_\rho^\theta) + 32c_2c_4(g_\alpha^\theta g_\rho^\phi - g_\alpha^\phi g_\rho^\theta)$ where c_1, c_2, c_3, and c_4 are constants.

(e) For $c_1 = c_2 = c_3 = c_4 = 1$, $\mathrm{tr}[\gamma_\alpha\gamma_\beta\gamma_\rho\gamma_\sigma(1 - \gamma_5)]\, \mathrm{tr}[\gamma^\theta\gamma^\beta\gamma^\phi\gamma^\sigma(1 - \gamma_5)] = 64g_\alpha^\theta g_\rho^\phi$

WE SIMPLY QUOTE THE RESULT FOR THE REST OF THE CALCULATION, USING THE CONVENTION

$$x \equiv \frac{E_e}{E_{e\,MAX}} = \frac{2E_e}{M_\mu} \qquad 0 \le x \le 1$$

Θ = ANGLE BETWEEN ELECTRON DIRECTION AND μ SPIN

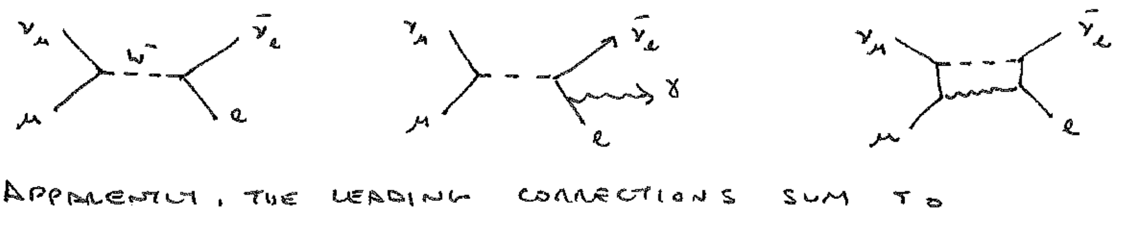
$$d\Gamma = \frac{G^2 M_\mu^5}{192\pi^3} [3 - 2x + (1-2x)\cos\Theta]\, 2x^2 dx \frac{d\Omega}{4\pi}$$

$$d\Gamma = \frac{G^2 M_\mu^5}{192\pi^3} (3-2x)\, 2x^2 dx$$

THE FUNCTION $x^2(3-2x)$ DIFFERS LITTLE FROM x FOR $1/3 < x < 1$

$$\Gamma = \frac{G^2 M_\mu^5}{192\pi^3}$$

BEFORE WE CAN USE THE LAST EXPRESSION TO EXTRACT G FROM THE DATA, WE MUST TAKE NOTE OF ELECTROMAGNETIC CORRECTIONS



APPARENTLY, THE LEADING CORRECTIONS SUM TO

$$\Gamma = \frac{G^2 M_\mu^5}{192\pi^3} \left(1 - \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right) f\left(\frac{M_e^2}{M_\mu^2}\right)$$

WHERE $f(y) = 1 - 8y + 8y^3 - y^4 + 12y^2 \ln(1/y)$

THEN $G = 1.16632 \pm .00004 \times 10^{-5}\ \mathrm{GeV}^{-2}$

$$= \frac{1.02679 \pm .00004}{M_p^2}$$

DETAILED EXAMINATION OF THE SPECTRUM OF THE DECAY ELECTRON MIGHT INDICATE DEPARTURES FROM THE V-A COUPLING. THIS IDEA HAS A LONG HISTORY DATING FROM AN ANALYSIS OF MICHEL [PROC. PHYS. SOC. LONDON A 63, 514 (1950)]. HE CONSIDERED THE POSSIBILITY OF VARIOUS AMOUNTS OF S, V, T, A AND P COUPLINGS, LEADING TO A VERY COMPLEX EXPRESSION FOR THE DECAY RATE! ONE RESULT OF THIS ANALYSIS CONCERNS THE ELECTRON ENERGY SPECTRUM:

$$\frac{dN_e}{dx} \sim x^2\left[2 - \frac{4}{3}\rho - \left(2 - \frac{16}{9}\rho\right)x\right]$$

WHERE $\rho = \dfrac{3C_V^2 + 6C_T^2 + 3C_A^2}{C_S^2 + 4C_V^2 + 6C_T^2 + 4C_A^2 + C_P^2}$ $\equiv$ MICHEL PARAMETER.

IN THE V-A THEORY $C_S = C_T = C_P = 0$, $C_A = C_V \Rightarrow \rho = 3/4$

IN GENERAL $0 \leq \rho \leq 1$

Figure 3.7. Results of experiments to determine ρ. Experimental points are plotted, together with a theoretical curve for $\rho = \frac{3}{4}$ corrected for radiative effects and ionization loss. (From Bardon et al. 65. Reprinted with permission.)

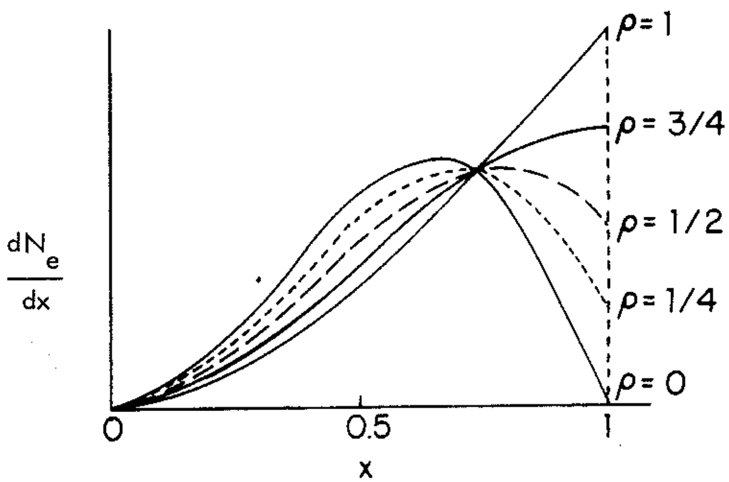
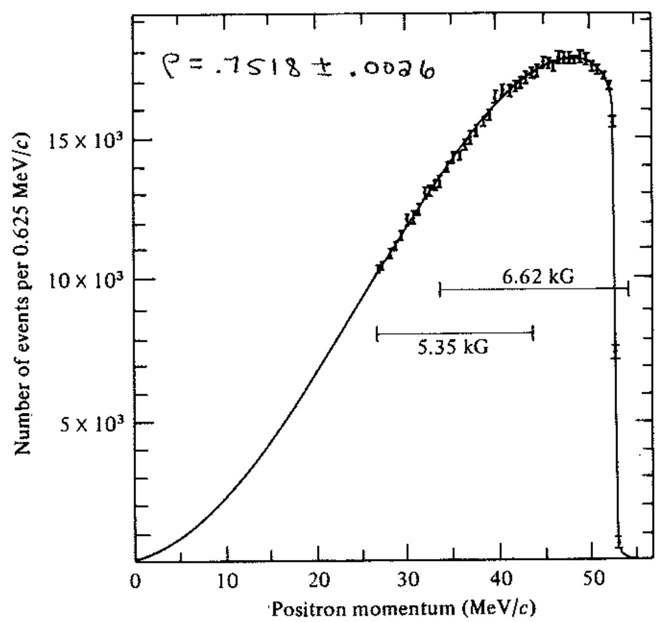
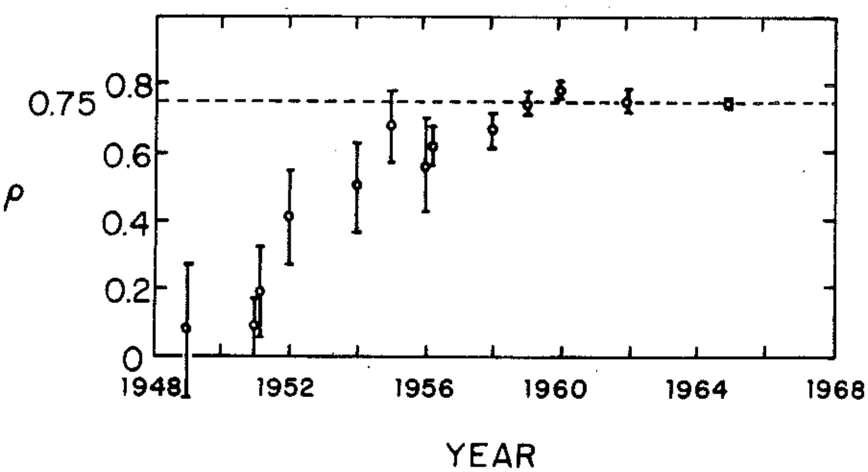


Fig. 21.1. Spectrum (21.9) for different ρ values.



NOTE THE SLOW RATE OF GROWTH OF THE MEASURED VALUE OF ρ WITH TIME. THIS GIVES A HINT OF THE DIFFICULTY OF THE EXPERIMENTS, AND CONSEQUENTLY IT WAS ALSO DIFFICULT TO ARRIVE AT THE V-A THEORY.



Experimental determination of the Michel parameter ρ versus time.

WHILE ONE CAN CONSIDER DEPARTURES FROM THE V-A THEORY OF ALMOST ANY FORM, IN RECENT TIMES A PARTICULAR VARIATION SEEMS ESPECIALLY INTERESTING. CONCEIVABLY IN THE VERY HIGH ENERGY LIMIT THE WEAK INTERACTION IS ACTUALLY PARITY CONSERVING. THIS COULD BE ACCOMODATED IF THERE EXISTS A PARTNER OF THE W BOSON WHICH COUPLES ONLY TO RIGHT-HANDED PARTICLES RATHER THAN LEFT-HANDED. IF WE CALL THIS NEW HEAVY BOSON THE W_R THEN IT WOULD LEAD TO A μ DECAY MATRIX ELEMENT

$$\mathcal{M} = e^2 \left(u_{\nu_\mu} | \gamma_\alpha (1+\gamma_5) | u_\mu \right) \frac{1}{q^2 - M_{W_R}^2} \left(\bar{v}_{\bar{\nu}_e} | \gamma^\alpha (1+\gamma_5) | u_e \right)$$

OR $\quad G_R \sim \dfrac{e^2}{M_{W_R}^2} \ll G \approx \dfrac{e^2}{M_W^2} \quad$ IF q^2 SMALL AND $M_{W_R} \gg M_W$. THUS WE WOULD HARDLY NOTICE THE V+A INTERACTION AT PRESENT ENERGIES. THE PROJECTION OPERATOR $\frac{1+\gamma_5}{2}$ PICKS OUT THE RIGHT HANDED PARTICLES. IN THE V+A CASE THE DECAY CONFIGURATION TO PRODUCE A MAXIMUM ENERGY ELECTRON IS

$$\begin{array}{c} \nu_\mu \\ \bar{\nu}_e \end{array} \longleftarrow \underset{\mu}{\bullet} \longrightarrow e$$

THIS CONFIGURATION IS FORBIDDEN IN THE V-A THEORY.

A RECENT EXPERIMENT [CARR ET AL. P.R.L. 51, 627 (1983)] EXAMINES THE CORRELATION BETWEEN μ SPIN AND ELECTRON DIRECTION FOR MAXIMUM ENERGY ELECTRONS.

THEY SET A LIMIT THAT

$$M_{W_R} > 380 \text{ GeV}$$

IE $\quad M_{W_R} > 4.5\, M_W$

NOW THAT WE KNOW $M_W \sim 80$ GeV

FOOTNOTE ON μ DECAY: A TRANSITION NOTABLE FOR ITS ABSENCE IS

$$\mu^+ \to e^+ \gamma$$

THIS COULD BE CONCEIVED OF AS A WEAK INTERACTION WITH AN ELECTROMAGNETIC CORRECTION. IF SO, WE MIGHT EXPECT A 1% BRANCHING RATIO. HOWEVER THE EXPERIMENTAL LIMIT IS $< 2 \times 10^{-10}$ [BOWMAN ET AL, P.R.L. 42, 556 (1979)]

THIS IS CONSIDERED STRONG EVIDENCE FOR LEPTON NUMBER CONSERVATION, AND THAT $\nu_\mu \neq \nu_e$.

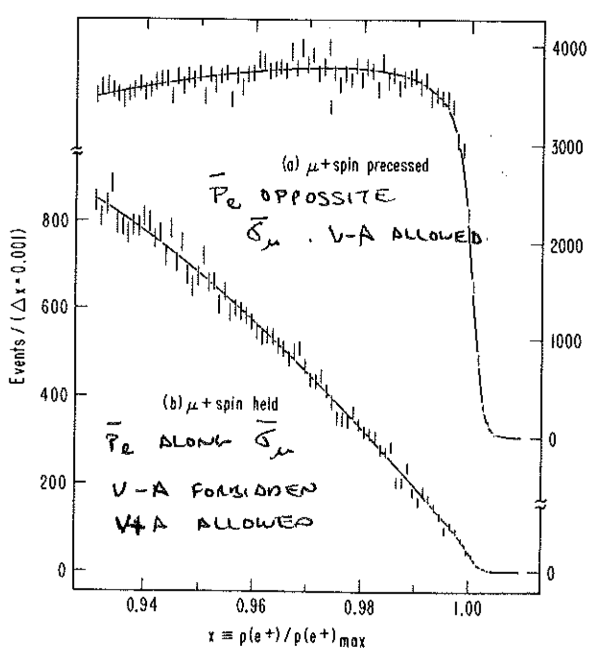


FIG. 3. Distributions (uncorrected for acceptance) in reduced positron momentum with the μ^+ spin precessed (curve a) and held (curve b). Errors are statistical. The edge in curve a corresponds to a resolution with a Gaussian part <0.2% rms. The fits are described in the text.

ALSO, $\mu \to 3e$ LESS THAN $\sim 10^{-10}$ OF THE TIME; BERTL ET AL, PHYS. LETT 140B, 299 (1984); BOLTON ET AL P.R.L. 53, 1415 (1984)

4. PRODUCTION AND DECAY OF THE τ LEPTON

SHORTLY AFTER THE DISCOVERY OF THE ψ PARTICLES AT SLAC EVIDENCE EMERGED FOR AN INTERESTING CLASS OF REACTIONS

$$e^+e^- \to \begin{cases} e^+e^- \\ e^+\mu^- \end{cases} + \text{NEUTRINOS}$$

THIS WAS INTERPRETED AS EVIDENCE FOR PRODUCTION OF PAIRS OF A NEW LEPTON, CALLED THE τ. [PERL ET AL. P.R.L. 35, 1489 (1975)]

IT IS SUPPOSED THAT THE τ IS ASSOCIATED WITH ITS OWN NEUTRINO, ν_τ SO THAT VARIOUS WEAK DECAYS OF THE τ ARE

$$\tau \to \begin{cases} \mu\ \bar{\nu}_\mu\ \nu_\tau \\ e\ \bar{\nu}_e\ \nu_\tau \\ \nu_\tau + \text{HADRONS} \end{cases}$$

PRODUCTION OF $\tau\bar{\tau}$ PAIRS THEN CAN LEAD TO e^+e^-, $e^+\mu^-$, $\mu^+\mu^-$ PAIRS WITH CONSIDERABLE ENERGY 'MISSING' IN THE FORM OF NEUTRINOS.

BY THE METHOD DISCUSSED BELOW THE MASS OF THE τ LEPTON IS DETERMINED TO BE

$$M_\tau = 1784 \pm 3 \text{ MeV}$$

THEN THE THRESHOLD FOR $\tau\bar{\tau}$ PRODUCTION IS 3568 MEV $\sim M_{\psi'} \sim 2M_D$

THESE COINCIDENCES ADDED TO THE DIFFICULTY IN DEMONSTRATING THE EXISTENCE OF THE τ, TO SAY THE LEAST.

A GOOD DETERMINATION OF THE PROPERTIES OF THE τ LEPTON WAS OBTAINED BY OBSERVING THE RATE OF $\tau\bar{\tau}$ PAIR PRODUCTION AS A FUNCTION OF e^+e^- CM ENERGY. ON p 281 WE NOTED THAT

$$\sigma_{q\bar{q} \to e^+e^-} = \frac{4\pi}{3} \frac{\alpha^2 Q_q^2}{E_{cm}^2} \frac{3-\beta^2}{2\beta} \qquad \left(\beta = \frac{v_q}{c}\right)$$

IF THE τ LEPTON HAS SPIN 1/2 AND CHARGE e, WE CAN RELATE $\tau\bar{\tau}$ PRODUCTION TO THE ABOVE EXPRESSION BY TIME REVERSAL INVARIANCE.

$$\sigma_{e^+e^- \to \tau\bar{\tau}} = \frac{p_\tau^2}{p_e^2} \cdot \frac{4\pi}{3} \frac{\alpha^2}{E_{cm}^2} \frac{3-\beta^2}{2\beta} \qquad (\text{p.}155)$$

NOW $p_\tau = E_\tau \beta = E_e \beta \sim p_e \beta$, SO

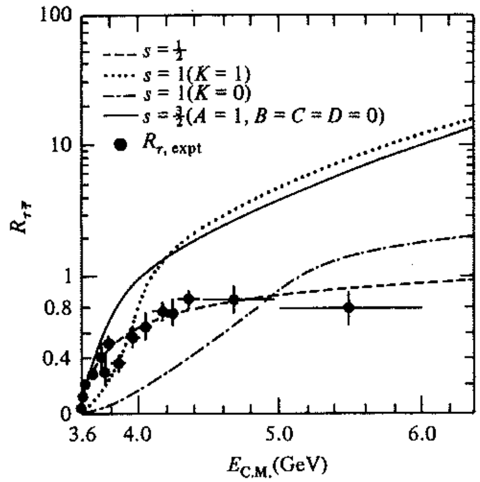
$$\sigma_{e^+e^- \to \tau\bar{\tau}} = \frac{4\pi}{3} \frac{\alpha^2}{E_{cm}^2} \frac{\beta(3-\beta^2)}{2}$$

THIS VANISHES WHEN $E_{cm} = 2M_\tau \Rightarrow \beta \to 0$, WHICH ALLOWS

THE DETERMINATION OF M_τ MENTIONED ABOVE.

THE SHAPE OF $\sigma(E_{cm})$ ALSO CONFIRMS THAT THE τ HAS SPIN $1/2$.

Figure 3.9. Comparison of $R_{\tau,\text{expt}}$ (Kirkby 79) with theoretical curves for pointlike particles with various spins. The constants A, B, C, and D are vertex parameters (Tsai 78). (From Perl 80. Reprinted with permission.)



IF THE WEAK INTERACTION OF THE τ LEPTON HAS THE SAME FORM AS THAT OF THE μ AND e, THEN WE MAY WRITE AT ONCE

$$\Gamma_{\tau \to \mu \bar{\nu}_\mu \nu_\tau} = \Gamma_{\tau \to e \bar{\nu}_e \nu_\tau} = \frac{G^2 M_\tau^5}{192\pi^3}$$

IF WE ADD THE INSIGHT OF COLORED QUARKS, THEN WE ALSO ESTIMATE

$$\Gamma_{\tau \to \nu_\tau + \text{HADRONS}} = 3 \cdot \frac{G^2 M_\tau^5}{192\pi^3} \qquad (\text{p. } 260)$$

THEN $$\Gamma_{\tau \to \text{ANYTHING}} = \frac{5\, G^2 M_\tau^5}{192\pi^3} \Rightarrow \tau_{\text{LIFETIME}} = 2.8 \times 10^{-13} \text{ SEC}$$

RECENT MEASUREMENT OF THE τ LIFETIME [JAROS ET AL P.R.L. 51, 955 (1983)] GIVES $\tau_\tau = 3.2 \pm 0.5 \times 10^{-13}$ SEC.

THIS IS GOOD CONFIRMATION OF THE IDEA OF 'UNIVERSALITY' OF THE WEAK INTERACTION.

5. $\pi^\pm \to e^\pm \nu$ DECAY

WE CONSIDERED THE DECAYS $\pi \to \mu\nu$ AND $\pi \to e\nu$ IN OUR INTRODUCTORY REMARKS IN LECTURE 3. IN THE V-A THEORY THESE DECAYS ARE SOMEWHAT SUPPRESSED IN THAT THE μ AND e MUST BE PRODUCED WITH POSITIVE HELICITY TO CONSERVE ANGULAR MOMENTUM, IF WE SUPPOSE THAT ANTI-NEUTRINOS ALWAYS HAVE POSITIVE HELICITY.

$$\bar{\nu}_e \xleftarrow{\;\leftarrow\;} \underset{\pi^-}{\bullet} \xrightarrow{\;\rightarrow\;} e^-$$

IN THE RELATIVISTIC LIMIT THE V-A COUPLING FOR SUCH A CONFIGURATION VANISHES. WE INDICATED THAT THIS SUPPRESSION LEADS TO A FACTOR M_e/M_π IN THE MATRIX ELEMENT, AS A MEASURE OF THE COUPLING TO THE 'WRONG' HELICITY AT LOW ENERGY. THEN $\dfrac{\Gamma_{\pi\to e\nu}}{\Gamma_{\pi\to\mu\nu}} = \left(\dfrac{M_e}{M_\mu}\right)^2$

WE NOW EXAMINE THIS SUPPRESSION IN GREATER DETAIL.

THE 2 BODY DECAY RATE FOR $\Gamma_{\pi\to\ell\bar\nu}$ IS CALCULATED AS

$$\Gamma = \frac{P_\ell}{8\pi M_\pi^2}|\mathcal{M}|^2 \qquad \text{(p 193)}$$

IN THE V-A THEORY WE MAY WRITE

$$\mathcal{M} = \frac{G}{\sqrt2}\left(\bar v_{\bar\nu}|\gamma^\alpha(1-\gamma_5)|u_\ell\right)\left(\text{VACUUM}|J_\alpha|\pi\right)$$

WE SUMMARIZE OUR IGNORANCE ABOUT WHAT HAPPENS TO THE π IN THE 'PION CURRENT' J_α WHICH SOMEHOW ELIMINATES THE PION UPON THE CREATION OF THE $\ell\bar\nu$ PAIR. THE CURRENT J_α IS UNDOUBTEDLY INFLUENCED BY DETAILS OF THE STRONG INTERACTION, AND IS NOT COMPLETELY CALCULABLE AT PRESENT. BUT TO MAKE THE OVERALL MATRIX ELEMENT A SCALAR, J_α HAS BOTH VECTOR AND AXIAL VECTOR PARTS.

IN THE PRESENT CASE ONLY THE AXIAL VECTOR PART OF J_α CAN BE NON-VANISHING. TO SEE THIS WE NOTE THAT THE ONLY 4 VECTOR WHICH CAN BE ASSOCIATED WITH A SPINLESS PION IS JUST ITS 4-MOMENTUM

$q_\alpha = (M_\pi, \vec 0)$ IN THE π REST FRAME.

THEN AS THE PION HAS NEGATIVE INTRINSIC PARITY, WE CONSIDER THAT J_α MUST BEHAVE AS AN AXIAL VECTOR NOT AN ORDINARY VECTOR I.E. $J_\alpha|\pi)$ MUST HAVE POSITIVE PARITY = PARITY OF THE VACUUM.

ALTOGETHER WE WRITE $(\text{VACUUM}|J_\alpha|\pi) = f_\pi\, q_\alpha$ — AXIAL VECTOR TRANSITION

WHERE f_π = FUDGE FACTOR WHICH INCLUDES THE EFFECT OF THE STRONG INTERACTION.

$$\text{THUS}\quad \mathcal{M} = \frac{G f_\pi}{\sqrt2}\left(\bar v_{\bar\nu}|\not q(1-\gamma_5)|u_\ell\right) \qquad (\not q = \gamma^\alpha q_\alpha)$$

BY ENERGY CONSERVATION $q_\alpha = p_\alpha + k_\alpha$

WHERE $p_\alpha = (E_\ell, \vec p_\ell)$

$k_\alpha = (E_{\bar\nu}, \vec p_{\bar\nu}) = (P_\ell, -\vec P_\ell)$

NOTE ALSO THAT

$M_\pi = E_\ell + E_\nu = E_\ell + P_\ell$

$$\mathcal{M} \sim (\bar{v}_{\bar\nu} | (\not{p} + \not{k})(1-\gamma_5) | u_\ell)$$
$$= \underbrace{(\bar{v}_{\bar\nu} | \not{k} (1-\gamma_5) | u_\ell)}_{0 \text{ AS } M_{\bar\nu} = 0} + (\bar{v}_{\bar\nu} | (1+\gamma_5) \underbrace{\not{p} | u_\ell)}_{M_\ell | u_\ell)}$$

$$\mathcal{M} = \frac{G f_\pi M_\ell}{\sqrt{2}} (\bar{v}_{\bar\nu} | 1+\gamma_5 | u_\ell)$$

ALREADY WE HAVE VERIFIED THE FACTOR OF M_ℓ IN THE MATRIX ELEMENT. EVALUATION OF $|\mathcal{M}|^2$ BY FEYNMAN'S TRACE METHOD YIELDS

$$|\mathcal{M}|^2 = 4 G^2 f_\pi^2 M_\ell^2 (P_\alpha K_\alpha) = 4 G^2 f_\pi^2 M_\ell^2 M_\pi P_\ell$$

3 MOMENTUM

SO $$\Gamma_{\pi \to \ell \bar\nu} = \frac{G^2 f_\pi^2}{2\pi} M_\pi M_\ell^2 \left(\frac{P_\ell}{M_\pi}\right)^2$$

FINALLY WE MAY FIND P_ℓ BY NOTING $K_\alpha = q_\alpha - P_\alpha$

SO $$K^2 = 0 = M_\pi^2 + M_\ell^2 - 2 \underbrace{q_\alpha P_\alpha}_{E_\ell M_\pi}$$

$$\Rightarrow E_\ell = \frac{M_\pi^2 + M_\ell^2}{2 M_\pi} \quad \text{AND} \quad P_\ell = M_\pi - E_\ell = \frac{M_\pi^2 - M_\ell^2}{2 M_\pi}$$

$$\Gamma_{\pi \to \ell \bar\nu} = \frac{G^2 f_\pi^2}{8\pi} M_\pi M_\ell^2 \left(1 - \frac{M_\ell^2}{M_\pi^2}\right)^2$$

AT LENGTH WE OBTAIN A MORE PRECISE COMPARISON

$$\frac{\Gamma_{\pi \to e \bar\nu}}{\Gamma_{\pi \to \mu \bar\nu}} = \left(\frac{M_e}{M_\mu}\right)^2 \left(\frac{1 - M_e^2/M_\pi^2}{1 - M_\mu^2/M_\pi^2}\right)^2 \sim 5.22 \left(\frac{M_e}{M_\mu}\right)^2$$

APPARENTLY IF ONE CONSIDERS THE EFFECT OF PHOTON RADIATIVE CORRECTIONS (MORE IMPORTANT IN $\pi \to e\nu$) THE ABOVE RATIO SHOULD BE REDUCED BY A FACTOR $1 - \frac{16.9\,\alpha}{\pi}$

THE PREDICTION IS THEN $$\frac{\Gamma_{\pi \to e\nu}}{\Gamma_{\pi \to \mu\nu}} \sim 1.24 \times 10^{-4}$$

DATA: RATIO $= 1.274 \pm .024 \times 10^{-4}$

WE MAY EXTRACT 2 OTHER INSIGHTS. $\tau_\pi = \frac{1}{\Gamma_{\pi \to \mu\nu}} = 2.6 \times 10^{-8}$ EXPERIMENTALLY

FROM THIS WE INFER $f_\pi \sim 128$ MeV $\sim .94\, M_\pi$.
ON DIMENSIONAL GROUNDS ALONE WE WOULD EXPECT $f_\pi \sim M_\pi$.

ALSO, WE MAY WRITE $M_\ell^2 = 2M_\pi(E_\ell - P_\ell) = 2M_\pi E_\ell\left(1-\frac{P_\ell}{E_\ell}\right) = (M_\pi^2+M_\ell^2)\left(1-\frac{v_\ell}{c}\right)$

USING FACTS GIVEN ON P 301.

THEN $\Gamma_{\pi\to\ell\bar\nu} \sim M_\ell^2 \sim 1-\frac{v_\ell}{c}$. SO WE MAY ALSO SAY

THAT THE AXIAL VECTOR DECAY OF THE PION LEADS TO A DECAY RATE PROPORTIONAL TO $1-\frac{v_\ell}{c}$. THIS IS YET ANOTHER WAY OF

REMARKING THAT THE PION DECAY RATE WOULD VANISH IN THE RELATIVISTIC LIMIT.

6. OTHER DECAYS OF $\pi^\pm$ MESONS

THE DECAY $\pi^+\to\pi^0 e^+\nu$ IS OBSERVED WITH A BRANCHING RATIO $1.05 \pm .04\times10^{-8}$ i.e. RATHER RARELY [McFARLANE ET AL P.R.L. 51, 249 (1983)]

THEN
$$\frac{\Gamma_{\pi^+\to\pi^0 e^+\nu}}{\Gamma_{\pi^+\to e^+\nu}} \sim \frac{1.02\times10^{-8}}{1.24\times10^{-4}} \sim 8\times10^{-5}$$

SUPPOSE WE MAKE A PHASE SPACE COMPARISON, AS ON P 177

$$\frac{\Gamma_{3\,BODY}}{\Gamma_{2\,BODY}} \sim \frac{Q^2}{300\,M_i\,P_f} \qquad \text{IF } |\mathcal{M}|^2 \text{ IS THE SAME IN BOTH CASES}$$

$$Q = M_{\pi^+} - M_{\pi^0} - M_e \sim 4\text{ MeV}$$
$$P_f = M_\pi/2 \qquad \Rightarrow \frac{\Gamma_3}{\Gamma_2} \sim 3\times10^{-6}$$

THUS WE INFER THAT $\mathcal{M}_{\pi^+\to\pi^0 e^+\nu} > \mathcal{M}_{\pi^+\to e^+\nu}$

THIS MAY NOT BE SO SURPRISING IN THAT THE SUPPRESSION OF $\pi\to e\nu$ DUE TO THE WRONG HELICITY OF THE ELECTRON NEED NOT HOLD IN $\pi^+\to\pi^0 e^+\nu$

IN THE V-A THEORY WE CAN WRITE
$$\mathcal{M}_{\pi^+\to\pi^0 e^+\nu} \sim \frac{G}{\sqrt2}\left(\bar u_\nu\,|\,\gamma^\alpha(1-\gamma_5)\,|\,v_{e^+}\right)\left(\pi^0\,|\,J_\alpha\,|\,\pi^+\right)$$

THIS TIME ONLY THE VECTOR PART OF J_α ACTS, AND WE SURMISE
$$J_\alpha = F_1(q^2)\,(k_{\pi^+}+k_{\pi^0})_\alpha + F_2(q^2)\,q_\alpha \qquad q_\alpha = (k_{\pi^+}-k_{\pi^0})_\alpha$$

FEYNMAN AND GELL-MANN ADVANCED THE VIEW THAT THE VECTOR PART OF THE CURRENT J_α IS ESSENTIALLY IDENTICAL TO THE ELECTROMAGNETIC CURRENT THAT WOULD BE RESPONSIBLE FOR THE DECAY $\pi^+\to\pi^0\gamma$ (IF PERMITTED). IN PARTICULAR, THE VECTOR CURRENT IS THEN THOUGHT TO BE CONSERVED: $q_\alpha J_\alpha = 0$

$\Rightarrow F_2 = 0$ IN OUR CASE. THIS IS THE SO-CALLED CONSERVED VECTOR CURRENT HYPOTHESIS (CVC).

THIS HYPOTHESIS TAKES FERMI'S ANALOGY BETWEEN THE WEAK AND ELECTROMAGNETIC INTERACTIONS ONE STEP FURTHER, LEADING THE WAY FOR THE SUBSEQUENT ELECTRO-WEAK UNIFICATION OF GLASHOW, WEINBERG & SALAM.

THE FACTOR $F_1(q^2)$ IS THEN TAKEN TO BE THE ELECTROMAGNETIC FORM FACTOR OF THE PION. NOW $q^2 \sim (M_{\pi^+} - M_{\pi^0})^2 \sim (4.6\,\mathrm{MeV})^2 \sim 0$

SO $F_1(q^2) \to 1$. THE RATE CALCULATION MAY NOW BE COMPLETED TO GIVE

$$\Gamma_{\pi^+ \to \pi^0 e \nu} = \frac{G^2 \Delta^5}{30\pi^3} + \text{E-M CORRECTIONS}$$

WHERE $\Delta = \dfrac{m_{\pi^+}^2 - M_{\pi^0}^2 + M_e^2}{2M_{\pi^+}}$ = ENERGY OF ELECTRON WHEN ν IS PRODUCED AT REST $= E_{e,\,MAX}$

$$\approx M_{\pi^+} - M_{\pi^0} \sim 4.6\,\mathrm{MeV}$$

THE CALCULATED BRANCHING RATIO IS THEN

$$\frac{\Gamma_{\pi^+ \to \pi^0 e \nu}}{\Gamma_{\pi^+ \to \mu^+ \nu}} = \frac{4}{15\pi^2} \frac{\Delta^5}{f_\pi^2 M_\pi M_\mu^2} \frac{1}{(1 - M_\mu^2/M_\pi^2)^2} \approx 1.07 \times 10^{-8}$$

IN RATHER GOOD AGREEMENT WITH EXPERIMENT. THIS WAS A STRONG CLUE THAT ELECTRO-WEAK UNIFICATION SHOULD BE POSSIBLE.

THESE IDEAS WERE EXTENDED TO THE AXIAL CURRENT BY GOLDBERGER AND TREIMAN [P.R. 111 354 (1958)]. THEY ADVOCATED THE NOTION OF THE PARTIALLY CONSERVED AXIAL CURRENT (PCAC), WHICH STATES THAT THE AXIAL CURRENT IN HADRON TRANSITIONS WOULD BE CONSERVED IF ONLY M_π WERE ZERO. FOR EXAMPLE, FROM p 300,

$$J_\alpha(\pi^+ \to \ell\nu) = q_\alpha f_\pi \;,\; \text{so } q_\alpha J_\alpha = q_\alpha^2 f_\pi = m_\pi^2 f_\pi \to 0 \text{ if } m_\pi \to 0.$$

THEY APPLIED THIS TO NEUTRON DECAY, WHICH INCLUDES A NON-VANISHING AXIAL CURRENT TERM IN $(\bar{u}_p | J_\alpha | u_n) = (\bar{u}_p | \gamma_\alpha (1 - \lambda\gamma_5) | u_n)$

THEY ESTIMATE $\lambda = \dfrac{g f_\pi}{\sqrt{2} M_p} \sim 1.25$

WHERE $\dfrac{g^2}{4\pi}$ ≡ PION-NUCLEON COUPLING CONSTANT, A PRECURSOR TO α_s

THE PION-NUCLEON COUPLING CONSTANT IS OBSERVED TO BE ~15 IN LOW ENERGY EXPERIMENTS...

8. $\tau \to \pi \nu_\tau$

THIS DECAY IS CONCEPTUALLY RELATED TO $\pi \to \tau \nu_\tau$, WHICH IS OF COURSE FORBIDDEN BY ENERGY CONSERVATION. BUT WE CAN READILY ADAPT OUR CALCULATION OF $\pi \to \ell \nu_\ell$, SIMPLY BY EXCHANGING M_π AND m_ℓ EVERYWHERE, EXCEPT FOR THE FACTOR m_ℓ^2 WHICH ARISES OUT OF THE $(\bar{v}_\nu | \gamma (1-\gamma_5) | u_\ell)$.

THERE IS ALSO AN ADDITIONAL FACTOR $\frac{1}{2S_\tau+1} = 1/2$ AS THE INITIAL PARTICLE NOW HAS SPIN 1/2. THEN

$$\Gamma_{\tau\to\pi\nu_\tau} = \frac{G^2 f_\pi^2 M_\tau^3}{16\pi}\left(1 - \frac{M_\pi^2}{M_\tau^2}\right)^2$$

AND $$\frac{\Gamma_{\tau\to\pi\nu_\tau}}{\Gamma_{\tau\to ALL}} \approx \frac{12\pi^2}{5}\left(\frac{M_\pi}{M_\tau}\right)^2 \sim \frac{1}{10} \qquad \text{USING } f_\pi \sim M_\pi$$

EXPERIMENTALLY THIS BRANCHING RATIO IS 10% WITHIN ERRORS.

9. $K^\pm \to \ell^\pm \nu$ AND $K^\pm \to \pi^0 \ell^\pm \nu$

OUR CONSIDERATIONS ARE READILY EXTENDED TO THE WEAK DECAYS OF THE CHARGED K MESON, WHICH ARE NOTABLE FOR NON-CONSERVATION OF STRANGENESS.

AT ONCE WE ESTIMATE $$\Gamma_{K\to\ell\nu} = \frac{G^2 f_K^2}{8\pi} M_K M_\ell^2 \left(1 - \frac{M_\ell^2}{M_K^2}\right)^2$$

AGAIN $$\frac{\Gamma_{K\to e\nu}}{\Gamma_{K\to\mu\nu}} = \left(\frac{M_e}{M_\mu}\right)^2 \left(\frac{1 - M_e^2/M_K^2}{1 - M_\mu^2/M_K^2}\right) \sim 2\times10^{-5}$$ INCLUDING THE E&M CORRECTION

DATA: RATIO $= 2.43 \pm .11 \times 10^{-5}$

WE MAY ALSO FORM THE RATIO

$$\frac{\Gamma_{K\to\mu\nu}}{\Gamma_{\pi\to\mu\nu}} = \frac{f_K^2 M_K (1 - M_\mu^2/M_K^2)^2}{f_\pi^2 M_\pi (1 - M_\mu^2/M_\pi^2)^2} \sim 16.9 \left(\frac{f_K}{f_\pi}\right)^2$$

FROM THE DATA THIS RATIO IS $\sim 1.34 \Rightarrow \frac{f_K}{f_\pi} \sim .28$

RECALLING THAT f_π AND f_K REPRESENT STRONG INTERACTION EFFECTS, WE WOULD EXPECT $f_K \sim f_\pi$ TO FIRST ORDER.

BUT THEN THE K DECAY INVOLVES STRANGE QUARKS ....

LET US ALSO COMPARE $K^+ \to \pi^0 e^+ \nu$ WITH $\pi^+ \to \pi^0 e^+ \nu$

ACCORDING TO OUR CLAIMS OF p. 303 WE EXPECT

$$\frac{\Gamma_{K^+ \to \pi^0 e^+ \nu}}{\Gamma_{\pi^+ \to \pi^0 e^+ \nu}} \sim \left(\frac{\Delta_K}{\Delta_\pi}\right)^5 \qquad \text{WHERE} \quad \Delta_K = \frac{M_{K^+}^2 - M_{\pi^0}^2 + m_e^2}{2 M_{K^+}} \approx \frac{M_{K^+}}{2}$$

$$\text{AND} \quad \Delta_\pi \sim M_{\pi^+} - M_{\pi^0}$$

SO WE ESTIMATE THE RATIO AS $\sim 4 \times 10^8$

EXPERIMENTALLY THE RATIO IS QUITE CLOSE TO 10^7

THE K DECAY SEEMS SUPPRESSED BY A FACTOR OF 40 ....

10. HYPERON DECAYS, THE $\Delta S = \Delta Q$ RULE, AND THE CABIBBO ANGLE

IF WE CONSIDER THE WEAK DECAYS OF THE HYPERONS WE ARE ALSO PRESENTED WITH PUZZLES, BOTH AS TO MAGNITUDES OF DECAY RATES, AND IN THE FORM OF THE V−A INTERACTION.

WE HAVE ALREADY REMARKED HOW DETAILED ANALYSIS (NOT EXPLAINED IN THE NOTES) OF NUCLEAR β DECAY, SUCH AS $n \to p e^- \bar\nu$ LEADS ONE TO INFER THAT THE $n-p$ WEAK CURRENT HAS THE FORM

$$(\bar u_p | J_\alpha | u_n) \quad \text{WITH} \quad J_\alpha = \gamma_\alpha (1 - \lambda \gamma_5) \quad \text{AND} \quad \lambda = \frac{C_A}{C_V} \simeq 1.25$$

CONSIDER THE CORRESPONDING DECAY OF THE Λ HYPERON: $\Lambda \to p e^- \bar\nu$. ANALYSIS OF THIS DECAY INDICATES A SIMILAR FORM FOR THE WEAK CURRENT BETWEEN p AND Λ, BUT WITH $\lambda = -.73$

ANOTHER PECULIARITY IS NOTED BY COMPARING DECAY RATES.

$$\frac{\Gamma_{\Lambda \to p e \nu}}{\Gamma_{n \to p e \nu}} \sim 3 \times 10^9 \quad \text{EXPERIMENTALLY}$$

BUT WE ESTIMATE $\dfrac{\Gamma_{\Lambda \to p e \nu}}{\Gamma_{n \to p e \nu}} \sim \left(\dfrac{\Delta_\Lambda}{\Delta_n}\right)^5 \sim \left(\dfrac{M_\Lambda - M_p}{M_n - M_p}\right)^5 \sim 5 \times 10^{10}$

THE STRANGENESS CHANGING Λ DECAY APPEARS TO BE SUPPRESSED BY ROUGHLY A FACTOR OF 15.

ON THE OTHER HAND $\dfrac{\Gamma_{\Sigma^+ \to \Lambda e^+ \nu}}{\Gamma_{n \to p e^- \nu}} \sim 2.3 \times 10^8$ EXPERIMENTALLY

WHILE WE ESTIMATE FOR THIS $\left(\dfrac{M_\Sigma - M_\Lambda}{M_n - M_p}\right)^5 \sim 6 \times 10^8$

NOTE THAT $\Sigma^+ \to \Lambda e^+ \nu$ DOES NOT CHANGE STRANGENESS.

OUR VIEW OF THE WEAK INTERACTION SEEMS TO BE INADEQUATE FOR STRANGENESS CHANGING DECAYS.

TOWARDS RESOLVING THIS DIFFICULTY WE FIRST NOTE A RULE, GIVEN BY FEYNMAN & GELL-MANN (1958), THE SO-CALLED $\underline{\Delta S = \Delta Q}$ <u>RULE</u>

IN STRANGENESS CHANGING LEPTONIC OR SEMILEPTONIC DECAYS WE HAVE $\Delta S = \Delta Q_{\text{HADRONS}}$

$K^+ \to e^+ \nu \qquad S_f - S_i = -1 = Q_f - Q_i$ (FOR THE <u>HADRONS</u>)

$\Lambda \to p e \nu \qquad S_f - S_i = +1 = Q_p - Q_\Lambda$

ETC

NOTE THAT $\Lambda \to p\pi$ AS WELL AS $n\pi^0$. THESE HAVE $\Delta Q = 0$, BUT $\Delta S = 1$. HOWEVER THESE ARE NON-LEPTONIC DECAYS, FOR WHICH WE DON'T APPLY THE RULE.

A MORE NON-TRIVIAL APPLICATION OF THE $\Delta S = \Delta Q$ RULE IS THAT

$$K^0 \to \pi^- e^+ \nu \quad \text{BUT} \quad K^0 \not\to \pi^+ e^- \bar{\nu}$$

[ON P191 WE ARGUED THIS FACT ON THE BASIS OF THE $\Delta I = \frac{1}{2}$ RULE.]

CABIBBO [P.R.L. <u>10</u>, 531 (1963)] GAVE AN EXPLANATION OF MANY OF THE PUZZLING FEATURES OF STRANGENESS CHANGING WEAK DECAYS. THIS IS MOST SIMPLY EXPRESSED IN THE QUARK MODEL — WHICH WASN'T INVENTED UNTIL 1964.

THE BASIC HYPOTHESIS IS THAT THE WEAK INTERACTION OF u, d & s QUARKS ALWAYS INVOLVES TRANSITIONS BETWEEN THE u QUARK AND A CERTAIN LINEAR COMBINATION OF THE d & s QUARKS

$$u \xleftrightarrow{\text{WEAK INT.}} \cos\theta_C \, d + \sin\theta_C \, s$$

$\theta_C \equiv$ CABIBBO ANGLE

THIS TRANSITION VIOLATES FLAVOR CONSERVATION AS WELL AS ISOSPIN INVARIANCE.

SOMETIMES PEOPLE SAY THAT THE WEAK INTERACTION 'SEES' A DIFFERENT DOWN QUARK THAN THE STRONG AND ELECTROMAGNETIC INTERACTIONS. THEY WRITE $d_\theta \equiv \cos\theta_C \, d + \sin\theta_C \, s$

THEN $u \leftrightarrow d_\theta$. IN THIS VIEW, ALL WEAK INTERACTIONS ARE BETWEEN MEMBERS OF A SERIES OF DOUBLETS

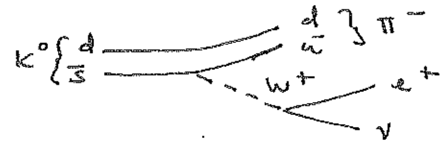
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix} \quad + \text{ MORE LATER}$$

IN ALL CASES, ONLY LEFT-HANDED PARTICLES INTERACT WEAKLY!

THIS VIEW EXPLAINS THE $\Delta S = \Delta Q$ RULE AT ONCE.

$u \longleftrightarrow \sin\theta_c\, s$ OR $\bar{u} \longleftrightarrow \sin\theta_c\, \bar{s}$ CLEARLY OBEY $\Delta S = \Delta Q$. INSTEAD OF MEMORIZING THE RULE, WE CAN ALWAY DRAW QUARK DIAGRAMS OF THE DECAY TO REMIND US

$K^0 \to \pi e \nu$



THE PATTERNS OF RELATIVE DECAY RATES ARE NOW LARGELY EXPLAINED ALSO.

IN $\pi^+ \to \mu^+ \nu$ WE HAVE $\pi^+ = u\bar{d} \to \cos\theta_c\, d\bar{d} + \mu^+ \nu$
$\hookrightarrow$ NOTHING

WHILE IN $K^+ \to \mu^+ \nu$ $K^+ = u\bar{s} \to \sin\theta_c\, s\bar{s} + \mu^+ \nu$
$\hookrightarrow$ NOTHING

SO WE REVISE OUR ESTIMATE : $$\frac{\Gamma_{K^+ \to \mu^+ \nu}}{\Gamma_{\pi^+ \to \mu^+ \nu}} \approx \tan^2\theta_c \frac{f_K^2}{f_\pi^2} \frac{M_K}{M_\pi} \left(\frac{1 - M_\mu^2 / M_K^2}{1 - M_\mu^2 / M_\pi^2} \right)^2$$

THIS ALLOWS $f_K \sim f_\pi$ IF $\tan\theta_c \sim .28$

OR, IN $n \to p e^- \nu$ WE HAVE $udd \to \cos\theta_c\, uud + e^- \nu$

$\Lambda \to p e^- \nu$ $uds \to \sin\theta_c\, uud + e^- \nu$

SO $$\frac{\Gamma_{\Lambda \to p e \nu}}{\Gamma_{n \to p e \nu}} \sim \tan^2\theta_c \left(\frac{M_\Lambda - M_p}{M_n - M_p} \right)^5$$

FITTING MANY SUCH EXAMPLES, ONE CONCLUDES THAT

$\theta_c \sim 13°$ $\sin\theta_c \sim .228$ $\cos\theta_c \sim .974$

CABIBBO WENT FURTHER. HE EXPLAINED THE VARIATIONS IN $\lambda = \frac{C_A}{C_V}$ FOR VARIOUS HYPERON TRANSITIONS BY SUPPOSING THE WEAK CURRENT OPERATOR $J_\alpha = \gamma_\alpha (1 - \lambda \gamma_5)$ BEHAVES LIKE A MEMBER OF AN SU(3) OCTET, RATHER THAN A SINGLET.

THEN (HYPERON$_2$ | J_α | HYPERON$_1$) $\sim$ (OCTET STATE | OCTET OPERATOR | OCTET STATE)

THIS VIEW LEADS TO RELATIONS AMONG THE λ PARAMETER FOR VARIOUS TRANSITIONS BASED ON RULES OF SU(3) ALGEBRA.

ALL POSSIBLE HYPERON TRANSITIONS MAY BE SUMMARIZED IN TERMS OF ONLY 2 PARAMETERS, CALLED D AND F IN THE TABLE BELOW. WE SIMPLY SUMMARIZE THE RESULTS WITHOUT DERIVING THEM.

Table 6.3. *Baryon semileptonic decays*

Transition			Cabibbo's hypothesis			
ΔS		Branching ratio		C_V	C_A	Experiment
0	$n \to pe^-\bar{\nu}$	100%	$\cos\theta_1$	1	$D+F$	$C_V = 1$, $C_A = 1.25$, $C_A/C_V = 1.25$,
	$\Sigma^- \to \Sigma^0 l^- \bar{\nu}$	—	$\cos\theta_1$	$\sqrt{2}$	$\sqrt{2}F$	—
	$\Sigma^- \to \Lambda^0 l^- \bar{\nu}$	$e^- 0.60(06) \times 10^{-4}$	$\cos\theta_1$	0	$\sqrt{\frac{2}{3}}D$	$C_V/C_A = 0.24(23)$
	$\Sigma^+ \to \Sigma^0 l^+ \nu$	$e^+ 2.02(47) \times 10^{-5}$	$\cos\theta_1$	0	$\sqrt{\frac{2}{3}}D$	—
	$\Xi^- \to \Sigma^0 l^- \bar{\nu}$	—	—	-1	$D-F$	—
1	$\Sigma^- \to n l^- \bar{\nu}$	$e^- 1.08(04) \times 10^{-3}$ $\mu^- 0.45(04) \times 10^{-3}$	$\sin\theta_1$	-1	$D-F$	e^-: $C_A/C_V = 0.385(070)$
	$\Lambda^0 \to p l^- \bar{\nu}$	$e^- 8.13(29) \times 10^{-4}$ $\mu^- 1.57(35) \times 10^{-4}$	$\sin\theta_1$	$\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{1}{6}}(D+3F)$	$C_V = 1.229(035)$, $C_A = -0.903(046)$, $C_A/C_V = -0.734(031)$
	$\Xi^- \to \Lambda^0 l^- \bar{\nu}$	$(0.69 \pm 0.18) \times 10^{-3}$	$\sin\theta_1$	$\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{1}{6}}(D-3F)$	—
	$\Xi^0 \to \Sigma^+ l^- \bar{\nu}$	1.1×10^{-3}	$\sin\theta_1$	1	$D+F$	—
	$\Xi^- \to \Sigma^0 l^- \bar{\nu}$	—	$\sin\theta_1$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}(D+F)$	—
	$\Omega^- \to \Lambda e^- \bar{\nu}_e$	$\sim 10^{-2}$	$\sin\theta_1$	—	—	—

IN A QUARK MODEL ANALYSIS ONE FINDS AN ADDITIONAL RELATION: $F = \frac{2}{3}D$, LEAVING ONE MYSTERIOUS PARAMETER.

EMPIRICALLY $\lambda = 1.25$ FOR NEUTRON DECAY $= D + F$ IN THE MODEL

SO WE INFER THAT $D = 3/4$, $F = 1/2$

AN EXTENSIVE AMOUNT OF DATA ON HYPERON DECAYS WAS ORGANIZED BY THE CABIBBO MODEL. THIS REPRESENTED ANOTHER EARLY SUCCESS FOR THE SU(3) SCHEME.

HOWEVER IT IS STILL UNCLEAR WHY $\theta_{\text{CABIBBO}} \neq 0$.