WEAK INTERACTIONS, CONTINUED

11. NON-LEPTONIC DECAYS OF K MESONS, THE $\Delta I = 1/2$ RULE, AND SPURIONS

IN THINKING ABOUT K DECAY WITH AN $\ell\nu$ PAIR IN THE FINAL STATE, THE $\Delta S = \Delta Q$ RULE AND THE CABIBBO ANGLE ARE SUFFICIENT TO CATEGORIZE THE OBSERVED PHENOMENA. HOWEVER, DECAYS WITH NO FINAL STATE LEPTONS ARE ALSO OBSERVED, AND ADDITIONAL INSIGHT IS REQUIRED TO UNDERSTAND THE EXPERIMENTAL DATA.

WE HAVE ALREADY INTRODUCED THIS TOPIC IN LECTURE 10, p 190 WHEN DISCUSSING ISOSPIN VIOLATIONS IN THE WEAK INTERACTION.

A SURPRISING EXPERIMENTAL FACT IS

$$\frac{\Gamma_{K^+\to\pi^+\pi^0}}{\Gamma_{K^0\to\pi\pi}} \sim 1.5\times10^{-3}$$

THIS IS INTERPRETED AS EVIDENCE FOR THE $\underline{\Delta I = 1/2 \text{ RULE}}$

IN THIS EXAMPLE WE HAVE 2π'S IN AN $S=0$ STATE SO THEY MUST HAVE $I = 0$ OR 2 TO SATISFY BOSE STATISTICS. THUS $K^+\to\pi^+\pi^0$ MUST BE AN $I = 1/2 \to I = 2$ TRANSITION WHILE $K^0\to\pi\pi$ CAN HAVE $I = 1/2 \to I = 0$. WE INFER THAT THE $\Delta I = 3/2$ TRANSITION IS SUPPRESSED, ALTHOUGH IT IS NOT ABSOLUTELY FORBIDDEN.

THE REASON FOR THE $\Delta I = 1/2$ RULE IS NOT WELL UNDERSTOOD AT PRESENT.

WE MIGHT DIGRESS BRIEFLY TO NOTE THAT THE SPIN ZERO COMBINATIONS OF $\pi^0\pi^0$ AND $\pi^+\pi^-$ ARE EIGENSTATES OF THE COMBINED OPERATION CP, WITH EIGENVALUE $CP = +1$. TO A GOOD APPROXIMATION CP IS CONSERVED IN THE WEAK INTERACTION. THIS HAS THE WELL-KNOWN CONSEQUENCE THAT SO FAR AS THE WEAK INTERACTION IS CONCERNED ONE SHOULD NOT TALK ABOUT THE K^0 AND $\bar K^0$ BUT RATHER

$$K_1^0 \equiv \frac{K^0+\bar K^0}{\sqrt2} \quad \text{AND} \quad K_2^0 \equiv \frac{K^0-\bar K^0}{\sqrt2}$$

THEN $CP|K_1^0\rangle = |K_1^0\rangle$ WHILE $CP|K_2^0\rangle = -|K_2^0\rangle$. FOR THIS WE SUPPOSE THAT $C|K^0\rangle = -|\bar K^0\rangle$ AS THE K^0 BELONGS TO THE SAME SU(3) OCTET AS THE π MESONS.

THUS $K_2^0 \not\to 2\pi$, BUT $K_2^0 \to 3\pi$ IS ALLOWED BY CP INVARIANCE.

THE RATE FOR $K_2^0\to3\pi$ WILL BE LOW DUE TO THE SMALLNESS OF THE 3 BODY PHASE SPACE. INDEED EXPERIMENTALLY ONE OBSERVES 2 CHARACTERISTIC LIFETIMES IN K^0 DECAY: $\tau_{K_S^0} \sim 8.9\times10^{-10}$ sec & $\tau_{K_L^0} \sim 5.2\times10^{-8}$ sec

WE DEFER MOST DETAILS OF THE INTRICACIES OF THE NEUTRAL K DECAYS UNTIL LECTURE 18. FOR NOW WE ONLY CONSIDER SOME SIMPLE FEATURES OF $K\to3\pi$ DECAY.

THE DALITZ PLOT ANALYSIS OF $K^+ \to \pi^+\pi^+\pi^0$ (p 201) INDICATES THAT ALL ORBITAL ANGULAR MOMENTA BETWEEN PION PAIRS VANISH. THEN A SPIN ZERO STATE OF 3 PIONS CAN ONLY HAVE $I = 1, 2,$ or 3 TO SATISFY BOSE STATISTICS (ONLY THE $I=0$ COMBO OF 3 π'S WAS A COMPLETELY ANTISYMMETRIC ISOSPIN WAVE FUNCTION.)

THE $\Delta I = 1/2$ RULE THEN SUGGESTS THAT ONLY THE $I=1$ 3π FINAL STATES OCCUR IN K DECAY. THIS YIELDS VARIOUS PREDICTIONS IF WE CAN WRITE DOWN THE $I=1$ ISOSPIN SYMMETRIC WAVE FUNCTIONS OF 3 π'S. THE COMPLICATION IS THAT THE $I=1$ 3π WAVE FUNCTION HAS A MIXED EXCHANGE SYMMETRY. WE MUST PICK OUT ONLY THE SYMMETRIC PART TO BE CONSISTENT WITH $J=0$.

TO DO THIS WE FIRST COMBINE 2 PIONS TO FORM SYMMETRIC ISOSPIN WAVE FUNCTIONS $\Rightarrow I = 0$ OR 2. THEN WE ADD A 3RD π IN SUCH A WAY AS TO GIVE $I=1$ FOR THE 3π SYSTEM. WITH THE AID OF THE C-G TABLES:

ADD THE 3RD π TO $I=2$ OF 2π: $|I=1,1\rangle = \sqrt{\frac{3}{5}}\,|I=2,2\rangle\,\pi^- - \sqrt{\frac{3}{10}}\,|I=2,1\rangle\,\pi^0 + \sqrt{\frac{1}{10}}\,|I=2,0\rangle\,\pi^+$

OR, ADD THE 3RD π TO $I=0$ OF 2π: $|I=1,1\rangle = |I=0,0\rangle\,\pi^+$

ANY LINEAR COMBINATION OF THESE TWO $|I=1,1\rangle$ WAVE FUNCTIONS IS STILL $|I=1,1\rangle$, BUT ONLY ONE SUCH COMBINATION IS EXCHANGE SYMMETRIC. THE GENERAL COMBO IS

$$\alpha\left\{\sqrt{\tfrac{3}{5}}\,\pi^+\pi^+\pi^- - \sqrt{\tfrac{3}{10}}\left(\frac{\pi^+\pi^0+\pi^0\pi^+}{\sqrt{2}}\right)\pi^0 + \frac{1}{\sqrt{10}}\left(\frac{\pi^+\pi^-}{\sqrt{6}} + \sqrt{\tfrac{2}{3}}\,\pi^0\pi^0 + \frac{\pi^-\pi^+}{\sqrt{6}}\right)\pi^+\right\}$$
$$+\ \beta\,\frac{\left(\pi^+\pi^- - \pi^0\pi^0 + \pi^-\pi^+\right)}{\sqrt{3}}\,\pi^+$$

WE DESIRE EQUAL AMOUNTS OF $\pi^+\pi^+\pi^-$, $\pi^+\pi^-\pi^+$ AND $\pi^-\pi^+\pi^+$ FOR SYMMETRY

SO $\sqrt{\frac{3}{5}}\,\alpha = \frac{\alpha}{\sqrt{60}} + \frac{\beta}{\sqrt{3}}$. ALSO $\alpha^2+\beta^2=1 \Rightarrow \alpha = 2/3 \quad \beta = \frac{\sqrt{5}}{3}$

LEADING TO $|3\pi, I=1,1\rangle = \frac{2}{\sqrt{5}}\,|\pi^+\pi^+\pi^-, \text{SYM}\rangle - \frac{1}{\sqrt{5}}\,|\pi^+\pi^0\pi^0, \text{SYM}\rangle$

WHERE $|abb, \text{SYM}\rangle \equiv \frac{1}{\sqrt{3}}\left(|abb\rangle + |bab\rangle + |bba\rangle\right)$

EXERCISE: SHOW $|3\pi, I=1,0\rangle = \sqrt{\frac{2}{5}}\,|\pi^+\pi^-\pi^0, \text{SYM}\rangle - \sqrt{\frac{3}{5}}\,|\pi^0\pi^0\pi^0\rangle$

NOW WE MAY READ OFF VARIOUS RESULTS, INVOKING THE $\Delta I = 1/2$ RULE

$$\frac{\Gamma_{K^+\to\pi^+\pi^+\pi^-}}{\Gamma_{K^+\to\pi^+\pi^0\pi^0}} = 4 \qquad \text{DATA: } 3.23 \pm 1$$

$$\frac{\Gamma_{K^0_L\to\pi^+\pi^-\pi^0}}{\Gamma_{K^0_L\to\pi^0\pi^0\pi^0}} = 2/3 \qquad \text{DATA: } .58 \pm .03$$

THERE ARE 3 BODY PHASE SPACE CORRECTIONS WHICH REDUCE THE EXPECTED RATE FOR FINAL STATES WITH MORE CHARGED PIONS (WHICH ARE HEAVIER). THE $\Delta I = 1/2$ RULE WORKS FAIRLY WELL IN THIS EXAMPLE.

THE RATES FOR $K^+ \to 3\pi$ AND $K^0_L \to 3\pi$ ARE NOT IMMEDIATELY RELATED BY THE ABOVE ANALYSIS, ALTHOUGH THEY ARE IN FACT RATHER SIMILAR.

A SOMEWHAT AD HOC PRESCRIPTION FOR THIS WAS GIVEN BY WENTZEL [P.R. 101, 1214 (1956)]. HE SUPPOSES THAT THE $\Delta I = 1/2$ RULE IS ENFORCED FOR STRANGENESS CHANGING DECAYS BY THE <u>SPURION</u> CONCEPT.

IN A TRANSITION WITH $\Delta S = S_f - S_i = +1$ WE SUPPOSE THE INITIAL STATE HAS ABSORBED AN IMAGINARY 'SPURION' WITH QUANTUM NUMBERS

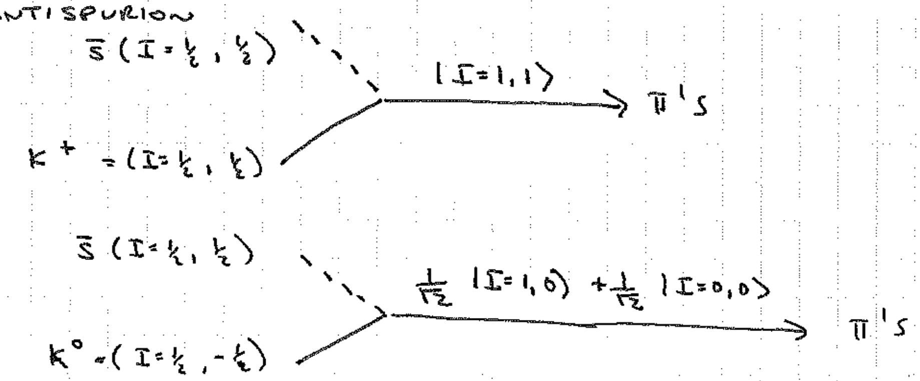
$$I = 1/2, \; I_3 = -1/2 \qquad S = +1$$

WHILE IF $\Delta S = -1$, AN 'ANTISPURION' WAS ABSORBED, WITH

$$I = 1/2 \quad I_3 = +1/2 \qquad S = -1$$

THE SPURION HAS NO MASS, CHARGE OR SPIN. IT IS JUST AN ACCOUNTING DEVICE. [UNLIKE THE QUARK, THE SPURION HAS NEVER GRADUATED BEYOND ITS BOOKKEEPING STATUS.] SPURIONS ARE SAID TO HAVE EITHER POSITIVE OR NEGATIVE PARITY, AS NEEDED TO MAKE PARITY APPEAR TO BE CONSERVED. HOWEVER THE SPURION CONCEPT MAKES NO PREDICTIONS ABOUT PARITY CONSERVATION. ITS ROLE IS TO IMPLEMENT THE $\Delta I = 1/2$ RULE.

FOR EXAMPLE, K^+ AND K^0 DECAY ARE $\Delta S = -1$ TRANSITIONS, SO WE USE THE ANTISPURION



ON P 190 WE ASKED YOU TO PREDICT RATIOS OF RATES FOR SOME SEMI-LEPTONIC K DECAYS. YOU MIGHT DO THIS BY INVENTING ANOTHER KIND OF SPURION, WHICH HAS $I_3 = -1/2$ WHEN IT EATS STRANGENESS

OTHER PEOPLE SUGGEST VIEWING $K \to \pi \ell \nu$ AS $K \to \pi K^*$, $K^* \to \ell\nu$

THEN USING THE DECOMPOSITION OF $I = 1$ STATES OF 3π, WE CAN WRITE

$$\langle \pi^+\pi^+\pi^- | \text{WEAK} | K^+\rangle = \frac{2}{\sqrt5}\, a$$

$$\langle \pi^+\pi^0\pi^0 | \text{WEAK} | K^+\rangle = -\frac{1}{\sqrt5}\, a$$

$$\langle \pi^+\pi^-\pi^0 | \text{WEAK} | K^0\rangle = \frac{1}{\sqrt2}\sqrt{\frac25}\, a = \frac{1}{\sqrt5}\, a$$

$$\langle \pi^0\pi^0\pi^0 | \text{WEAK} | K^0\rangle = -\frac{1}{\sqrt2}\sqrt{\frac35}\, a = -\sqrt{\frac{3}{10}}\, a$$

THEN WITH $K^0 = \frac{1}{\sqrt2}(K^0_S + K^0_L)$ (ASSUMES CP INVARIANCE)

WE CAN ALSO WRITE $\langle \pi^+\pi^-\pi^0 | \text{WEAK} | K^0_L\rangle = \sqrt{\frac25}\, a$

$$\langle \pi^0\pi^0\pi^0 | \text{WEAK} | K^0_L\rangle = -\sqrt{3/5}\, a$$

VARIOUS PREDICTIONS ARE NOW POSSIBLE

$$\frac{\Gamma_{K^0_L \to \pi^+\pi^-\pi^0}}{\Gamma_{K^+ \to \pi^+\pi^0\pi^0}} = 2 \qquad \text{DATA: } 1.71$$

$$\frac{\Gamma_{K^0_L \to 3\pi}}{\Gamma_{K^+ \to 3\pi}} = \frac{2+3}{4+1} = 1 \qquad \text{DATA: } 1.11 \qquad \text{ETC.}$$

IF WE SUPPOSE THE SMALL $\Delta I = 3/2$ TRANSITION CAN ALSO BE ASSOCIATED WITH A SPURION OF $I = 3/2$, $I_3 = -1/2$ ETC, THEN ANALYSIS OF $K \to 2\pi$ AND $K \to 3\pi$ LEADS ONE TO THE CONCLUSION THAT

$$\frac{a_{3/2}}{a_{1/2}} \sim .05$$ WHERE $a_{3/2}$ = AMPLITUDE FOR A $\Delta I = 3/2$ TRANSITION....

12. NON-LEPTONIC DECAYS OF HYPERONS

THE SIMPLEST EXAMPLE OF A HYPERON NON-LEPTONIC DECAY IS

$$\Lambda \to p\pi^- \quad \text{or} \quad n\pi^0$$

IF WE SUPPOSE THE $\Delta I = 1/2$ RULE HOLDS THEN WE HAVE, FROM C-G TABLES

$$|1/2, -1/2\rangle = -\sqrt{2/3}\, p\pi^- + \sqrt{1/3}\, n\pi^0$$

$$\Rightarrow \frac{\Gamma_{\Lambda \to p\pi^-}}{\Gamma_{\Lambda \to n\pi^0}} = 2 \qquad \text{DATA: } 1.79$$

THE DISCREPANCY CAN BE ACCOMODATED IF $a_{3/2}/a_{1/2} \sim .03$

THERE IS TECHNICAL INTEREST IN MORE DETAILED CONSIDERATION OF THE DECAY $\Lambda \to p\pi^-$. THIS IS BECAUSE OF THE EMPIRICAL FACT THAT IF Λ'S ARE PRODUCED IN THE REACTION $\pi^- p \to \Lambda K$, THEY EMERGE WITH NET POLARIZATION. THEN THE PARITY VIOLATION OF THE STRANGENESS CHANGING WEAK DECAY $\Lambda \to p\pi^-$ YIELDS A CORRELATION $\langle \vec{\sigma}_\Lambda \cdot \vec{P}_p \rangle \neq 0$.
WE CAN THUS DETERMINE THE DIRECTION OF $\vec{\sigma}_\Lambda$ VIA THE DECAY ANGULAR DISTRIBUTION. THIS IN TURN PERMITS MEASUREMENT OF THE MAGNETIC MOMENT OF THE Λ. (APPLY A MAGNETIC FIELD AND OBSERVE THE RATE OF SPIN PRECESSION.... LECTURE 13).

IF PARITY WERE CONSERVED THE $p\pi^-$ STATE WOULD HAVE ORBITAL ANGULAR MOMENTUM $l = 1$. BUT WITH PARITY VIOLATION $l = 0$ IS POSSIBLE ALSO.

WE WRITE a_S = AMPLITUDE TO DECAY TO AN S WAVE $p\pi$ STATE

a_P = " " " P-WAVE "

IN THE V-A THEORY, a_S CORRESPONDS TO THE VECTOR AMPLITUDE

a_P THE AXIAL VECTOR

WE NOW CONSIDER A Λ WITH SPIN UP ALONG THE Z AXIS. THE S WAVE $p\pi$ FINAL STATE WILL HAVE WAVE FUNCTION $Y_0^0 \, |p\uparrow\pi\rangle$, WHERE Y_0^0 DESCRIBES THE SPATIAL PART OF THE WAVE FUNCTION. SIMILARLY, THE $J_z = 1/2$ P WAVE STATE IS $-\sqrt{1/3}\, Y_1^0 |p\uparrow\pi\rangle + \sqrt{2/3}\, Y_1^1 |p\downarrow\pi\rangle$

THE MATRIX ELEMENT IS $\langle p\uparrow\pi | a_s Y_0^0 - \sqrt{\tfrac{1}{3}}\, a_p Y_1^0 | \Lambda\uparrow\rangle + \langle p\downarrow\pi | \sqrt{\tfrac{2}{3}}\, Y_1^1 a_p | \Lambda\uparrow\rangle$

WE SQUARE THE PIECES AND ADD TO GET THE RATE, AS THE FINAL STATE SPINS ARE DISTINGUISHABLE IN PRINCIPLE.

$$\Gamma \sim a_p^2 \cdot \tfrac{2}{3} \cdot \tfrac{3}{8\pi} \sin^2\theta + \tfrac{1}{4\pi}\left(a_s - \sqrt{\tfrac{1}{3}}\sqrt{3}\cos\theta \, a_p\right)^2$$

$$\sim \frac{a_p^2 + a_s^2}{4\pi}\left(1 - 2\,\frac{\mathrm{Re}\, a_s a_p^*}{a_p^2 + a_s^2}\cos\theta\right) \sim 1 - \alpha\cos\theta$$

OBSERVATION OF THE DECAY ANGULAR DISTRIBUTION DETERMINES

$$\alpha = .647 \pm .013$$

THE Λ IS REASONABLY COOPERATIVE IN LETTING ITS SPIN STATE BE MEASURED!

OTHER HYPERON DECAYS CAN BE DISCUSSED WITH THE AID OF THE SPURION CONCEPT.

WE COMPARE $\Sigma^+ \to n\pi^+$, $\Sigma^+ \to p\pi^0$ AND $\Sigma^- \to n\pi^-$ (WHY NOT Σ^0?)
THE $\Delta I = 1/2$ RULE ALLOWS BOTH $I = 1/2$ AND $I = 3/2$ FINAL STATES. EACH OF THESE TRANSITIONS CAN LEAD TO BOTH S AND P WAVE FINAL STATES. HOWEVER WE EMPHASIZE THE ISOSPIN BEHAVIOR.

THE SPURION PICTURE IS

$S\ (I = \tfrac{1}{2}, -\tfrac{1}{2})$

$\Sigma^+\ (I = 1, 1)$

$$\longrightarrow \sqrt{\tfrac{1}{3}}\,|I = \tfrac{3}{2}, \tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\,|I = \tfrac{1}{2}, \tfrac{1}{2}\rangle \longrightarrow \begin{cases} p\pi^0 = \sqrt{\tfrac{2}{3}}\,|I = \tfrac{3}{2}, \tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}\,|I = \tfrac{1}{2}, \tfrac{1}{2}\rangle \\ n\pi^+ = \sqrt{\tfrac{1}{3}}\,|I = \tfrac{3}{2}, \tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\,|I = \tfrac{1}{2}, \tfrac{1}{2}\rangle \end{cases}$$

$S\ (I = \tfrac{1}{2}, -\tfrac{1}{2})$

$\Sigma^-\ (I = 1, -1)$

$$\xrightarrow{|I = \tfrac{3}{2}, -\tfrac{3}{2}\rangle} n\pi^-$$

THUS $\Sigma^+ \to p\pi^0$ HAS AMPLITUDE $\frac{\sqrt{2}}{3} A_{3/2} - \frac{\sqrt{2}}{3} A_{1/2}$

$\Sigma^+ \to n\pi^+$ $\qquad \tfrac{1}{3} A_{3/2} + \tfrac{2}{3} A_{1/2}$

$\Sigma^- \to n\pi^-$ $\qquad A_{3/2}$

THUS $\sqrt{2}\, A_{\Sigma^+ \to p\pi^0} = A_{\Sigma^- \to n\pi^-} - A_{\Sigma^+ \to n\pi^+}$

THE MERITS OF THIS EXPRESSION ARE NOT IMMEDIATELY OBVIOUS. BUT IF WE TURN TO EXPERIMENT A NUMBER OF FACTS EMERGE. THE THREE DECAY RATES ARE VERY NEARLY EQUAL, AND FURTHERMORE THE PHASES OF THE AMPLITUDES ARE DETERMINED TO BE ALL ~0 OR 180°. ALSO $\Sigma^+ \to n\pi^+$ IS FOUND TO BE ESSENTIALLY PURE P WAVE WITH PHASE ~ −180°. $\Sigma^- \to n\pi^-$ IS PURE S WAVE, AND $\Sigma^+ \to p\pi^0$ IS A 50-50 MIXTURE OF S AND P WAVE. THE LAST RESULT IMPLIES THAT THE S AND P WAVE STRENGTH IN $\Sigma^+ \to p\pi^0$ ARE EACH $1/\sqrt{2}$ TIMES SMALLER THAN THOSE IN THE OTHER TWO DECAYS, SO THAT ALL TOTAL RATES ARE EQUAL.

THEN INDEED $$\sqrt{2}\left(\frac{S}{\sqrt{2}} + \frac{P}{\sqrt{2}}\right) = S - (-P)$$

CONFIRMS $$\sqrt{2}\, A_{\Sigma^+ \to p\pi^0} = A_{\Sigma^- \to n\pi^-} - A_{\Sigma^+ \to n\pi^+}$$

THIS IS A SOMEWHAT OBSCURE BUT TECHNICALLY IMPRESSIVE VERIFICATION OF THE $\Delta I = 1/2$ RULE.

EXERCISE: SHOW THAT THE SPURION IDEA PREDICTS

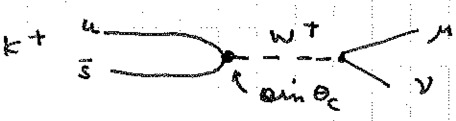
$$\frac{\Gamma_{\Xi^0 \to \Lambda^0\pi^0}}{\Gamma_{\Xi^- \to \Lambda^0\pi^-}} = \frac{1}{2} \qquad \text{DATA: } 0.55$$

$$\frac{\Gamma_{\Omega^- \to \Xi^0\pi^-}}{\Gamma_{\Omega^- \to \Xi^-\pi^0}} = 2 \qquad \text{DATA: } 2.94 \quad (?)$$

13. NEUTRAL CURRENTS AND THE GIM MECHANISM

IN ALL WEAK INTERACTIONS CONSIDERED THUS FAR TRANSITIONS ARE MADE BETWEEN PARTICLES WHOSE CHARGE DIFFERS BY ±1. THIS IS CLEAR IN THE INTERMEDIATE BOSON PICTURE, WHERE THE CHARGE OF THE W IS ±1

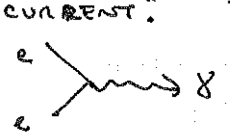
EXAMPLE: $K^+ \to \mu^+\nu$



i.e. $\begin{cases} u + \bar{s} \to W^+ \\ W^+ \to \mu^+ \nu \end{cases}$

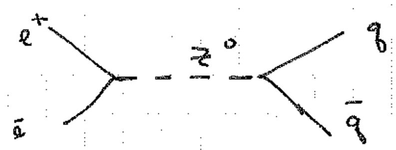
IN THE LITERATURE ONE FINDS THE PHRASE CHARGED CURRENT INTERACTION TO DESCRIBE THIS FEATURE.

[THIS NOMENCLATURE IS SOMEWHAT MISLEADING. A BETTER TERM WOULD BE "CHARGE-CHANGING CURRENT." THIS TERMINOLOGY IS EVEN WORSE FOR A VERTEX LIKE



. IT MIGHT BE SENSIBLE TO CALL THIS A "CHARGE CONSERVING CURRENT", BUT PEOPLE HAVE COME TO SAY NEUTRAL CURRENT INSTEAD.]

As a step towards the unification of electromagnetism and the weak interaction various people considered the possibility of a <u>weak</u> <u>neutral</u> <u>current</u>. In the intermediate boson picture this would correspond to diagrams like



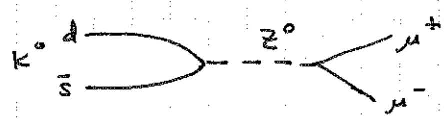
etc

The particle Z^0 has the quantum numbers of the photon : spin 1, neutral.... but is very heavy, so that its interaction is weak.

When this idea was introduced around 1960, experimental evidence in its favor was meager. There is almost no low-energy interaction in which the Z^0 could participate as the dominate quantum exchange — because it is so similar to the photon. The best candidate appeared to be

$$K^0 \to \mu^+\mu^- \qquad \text{(Consider also } K^+ \to \pi^+ e^+ e^- \ldots\text{)}$$

A possible diagram:



Photon exchange is forbidden by strangeness conservation of E&M.

A simple rate estimate would be $\dfrac{\Gamma_{K^0 \to \mu^+\mu^-}}{\Gamma_{K^+ \to \mu^+\nu}} \sim 1$

However the experimental result is $\dfrac{\Gamma_{K^0_L \to \mu^+\mu^-}}{\Gamma_{K^0_L \to \text{all}}} \sim 9 \times 10^{-9}$

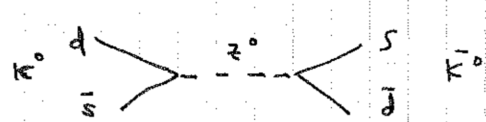
and $\Gamma_{K^0_L \to \text{all}} \sim \Gamma_{K^+ \to \mu^+\nu}$

If the Z^0 exists, the above diagram must be heavily suppressed (and this is our best hope at low energies!)

Another argument against neutral currents and the Z^0 was the result

$$M_{K^0} - M_{\bar{K}^0} < 10^{-14} M_{K^0} \qquad \text{(Lecture 18)}$$

If the Z^0 exists, a diagram



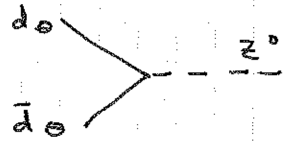
allows a first order weak transition between the K^0 and $\bar{K}^0$. If so, there must be a $K^0 - \bar{K}^0$ mass difference given by

$$\Delta M \sim \text{transition amplitude} \sim G M_K^3 \sim 10^{-5} M_K$$

Again a dramatic suppression is implied by experiment!

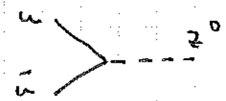
AN INTERESTING ARGUMENT TO SAVE THE Z^0 WAS GIVEN BY GLASHOW, ILIOPOULOS AND MAIANI [P. R. D2, 542 (1970)].

THEY FIRST SUPPOSED THAT ANY POSSIBLE STRANGENESS CHANGING NEUTRAL CURRENT WOULD ARISE FROM A COUPLING LIKE



, INVOKING THE CABIBBO MODEL

THE OTHER QUARK COUPLING



CAUSES NO PROBLEM.

THE $Z^0 \to q\bar{q}$ TRANSITIONS ARE THEN

$$Z^0 \to u\bar{u} + d_\theta \bar{d}_\theta = u\bar{u} + (d\cos\theta_c + s\sin\theta_c)(\bar{d}\cos\theta_c + \bar{s}\sin\theta_c)$$

$$= u\bar{u} + d\bar{d}\cos^2\theta_c + s\bar{s}\sin^2\theta_c + \underbrace{(d\bar{s} + \bar{s}d)}_{\text{TROUBLE}}\cos\theta_c\sin\theta_c$$

THE BRILLIANT INSIGHT OF GIM IS TO COMBINE THE HYPOTHESIS OF THE CHARMED QUARK WITH THE CABIBBO MODEL. THEY SUGGEST THAT THE CHARMED QUARK FORMS PART OF A NEW WEAK DOUBLET WITH ITS PARTNER NOT THE STRANGE QUARK, BUT RATHER

$$s_\theta = s\cos\theta_c - d\sin\theta_c$$

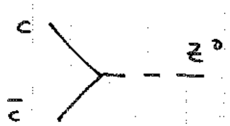
IN THIS VIEW THE $Q = -1/3$ QUARKS WHICH INTERACT WEAKLY ARE OBTAINED BY A ROTATION FROM THE STRONGLY INTERACTING QUARKS

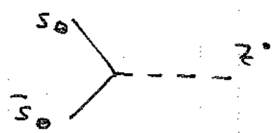
$$\begin{pmatrix} d_\theta \\ s_\theta \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

THE WEAK CHARGED CURRENT THEN CAUSES TRANSITIONS AMONG MEMBERS OF THE DOUBLETS

$$\begin{pmatrix} u \\ d_\theta \end{pmatrix} \quad \begin{pmatrix} c \\ s_\theta \end{pmatrix} \qquad \begin{matrix} Q = 2/3 \\ Q = -1/3 \end{matrix}$$

MORE RELEVANT TO THE PROBLEM AT HAND IS THAT THERE WOULD ALSO BE 2 MORE NEUTRAL CURRENT COUPLINGS TO QUARKS





THE NEW COUPLING IS THEN $Z^0 \leftrightarrow c\bar{c} + s\bar{s}\cos^2\theta_c + d\bar{d}\sin^2\theta_c - (s\bar{d} + d\bar{s})\cos\theta_c\sin\theta_c$

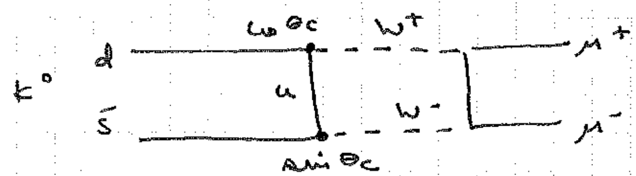
THE TOTAL COUPLING IS NOW $Z^0 \leftrightarrow u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c}$ (+ LEPTON TERMS)

AT THE EXPENSE OF INTRODUCING A NEW QUARK, THE UNWANTED

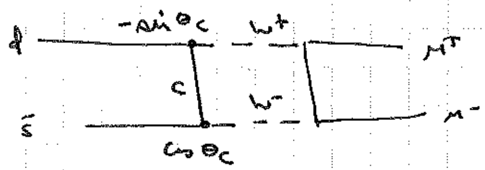
STRANGENESS CHANGING NEUTRAL CURRENTS HAVE BEEN ELIMINATED. HISTORY HAS SHOWN THAT THIS WAS A GOOD TRADE!

OF COURSE, IT IS NOW ALMOST IMPOSSIBLE TO FIND EVIDENCE FOR NEUTRAL CURRENTS IN A LOW-ENERGY INTERACTION, BUT THEY ARE NOT EXCLUDED IN PRINCIPLE. IN FACT NEUTRAL CURRENTS WERE FIRST IDENTIFIED IN CERTAIN NEUTRINO SCATTERING EXPERIMENTS IN 1974 (LECTURE 19).

THE GIM MECHANISM ALSO CLEARED UP SOME OTHER SUBTLETIES. IN $K^0 \to \mu^+\mu^-$, EVEN AFTER SUPPRESSING THE FIRST ORDER WEAK NEUTRAL CURRENT THERE IS A SECOND ORDER POSSIBILITY, INVOLVING A PAIR OF W^+ AND W^- BOSONS



THE RATE FOR THIS PROCESS MIGHT BE $\sim G^2 \sin^2\theta_C \sim 10^{-11}$ TIMES THAT FOR $K^+ \to \mu^+\nu$. BUT WITH THE CHARMED QUARK, ANOTHER DIAGRAM IS POSSIBLE



THE COMBINED AMPLITUDES ARE THEN $\sim G \sin\theta_C\cos\theta_C \left(\frac{1}{q^2 - M_u^2} - \frac{1}{q^2 - M_c^2}\right)$

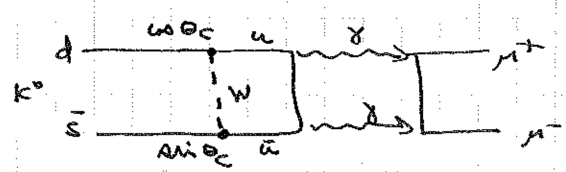
THIS VANISHES TO THE EXTENT THAT $M_u \sim M_c$

FROM THE RESULT $\frac{\Gamma K_L^0 \to \mu^+\mu^-}{\Gamma K_L^0 \to \text{ALL}} \sim 10^{-8}$ (ACTUALLY ONLY A LIMIT $< 10^{-7}$ WAS KNOWN IN 1970)

GIM ESTIMATED THAT $M_C \lesssim 3$ GeV, OTHERWISE SUFFICIENT CANCELLATION MIGHT NOT OCCUR.

IT MAY BE WORTH REMARKING THAT THERE IS ANOTHER WAY FOR THE DECAY $K_L^0 \to \mu^+\mu^-$ TO TAKE PLACE, NAMELY $K_L^0 \to \gamma\gamma \to \mu^+\mu^-$

THIS IS STILL A WEAK INTERACTION, AS E&M DOESN'T CHANGE STRANGENESS!

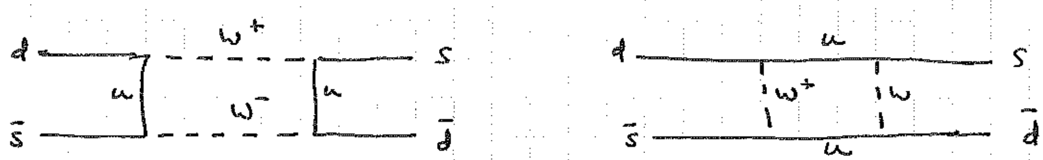


EXERCISE: SHOW THAT IF CP INVARIANCE HOLDS, $K_S^0 \not\to \gamma\gamma \to \mu^+\mu^-$. COULD WE HAVE $K^0 \to \gamma \to \mu^+\mu^-$?

FROM THE ABOVE DIAGRAM WE ESTIMATE $\frac{\Gamma_{K_L^0 \to \mu^+\mu^-}}{\Gamma_{K_L^0 \to \text{ALL}}} \sim \alpha^4 \sim 10^{-8}$

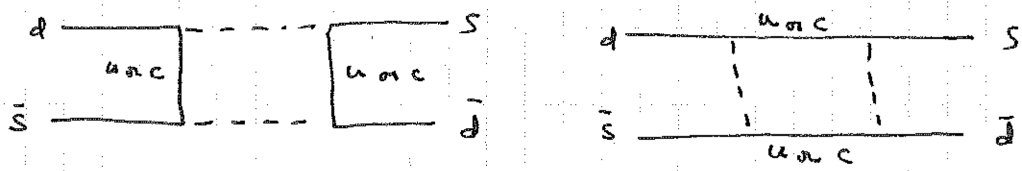
IN REASONABLE AGREEMENT WITH EXPERIMENT.

LIKEWISE, EVEN A SECOND-ORDER CHARGED-CURRENT INTERACTION LEADS TO SIGNIFICANT $K^0 - \bar{K}^0$ MASS SPLITTING.



$$\Delta M \sim \text{AMPL} \sim \sin^2\theta_c \, G^2 M^5 \sim 10^{-11} M_K$$

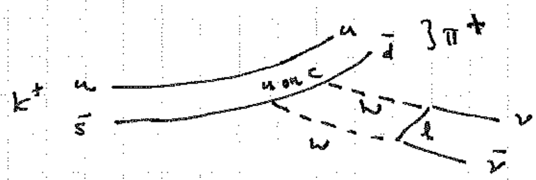
BUT WITH THE CHARM QUARK THE DIAGRAMS ARE NOW



THE DIAGRAMS WITH u AND c EXCHANGE, OR c AND u CANCEL THOSE WITH u AND u OR c AND c, IN THE LIMIT $M_u = M_c$.

FOOTNOTE: THE $K_S^0 - K_L^0$ MASS DIFFERENCE PROVIDES OUR ONLY PRESENT LABORATORY FOR STUDYING SECOND ORDER WEAK INTERACTION EFFECTS. BUT THE SEARCH IS ON FOR OTHER METHODS. PEOPLE HERE AT PRINCETON ARE BUILDING AN EXPERIMENT TO SEARCH FOR THE REACTION

$K^+ \to \pi^+ \nu\bar{\nu}$ WHICH MIGHT HAVE DIAGRAM



WE MAY ESTIMATE THAT $\Gamma_{K^+ \to \pi^+\nu\nu} < \Gamma_{K_L^0 \to \mu^+\mu^-}$

AS THE LATTER CAN ONLY PARTLY BE DUE TO 2ND ORDER WEAK EFFECTS.

$$\text{i.e.} \quad \frac{\Gamma_{K^+ \to \pi^+\nu\bar{\nu}}}{\Gamma_{K^+ \to \text{ALL}}} < 10^{-8}$$

ON THE OTHER HAND $M_{K_S^0} - M_{K_L^0} \sim 10^{-14} M_K \sim$ 2ND ORDER WEAK AMPL.

THIS SUGGESTS $\Gamma \sim 10^{-28} M_K{}^5 \sim 10^{-15}$ MeV OR $\dfrac{\Gamma_{K^+ \to \pi^+\nu\bar{\nu}}}{\Gamma_{K^+ \to \text{ALL}}} \gtrsim 10^{-14}$

TAKING THIS ARGUMENT AS PROVIDING A LOWER LIMIT. THE EXPERIMENT WILL BE VERY HARD!

14. DECAYS OF CHARMED MESONS

THE GIM EXTENSION OF THE CABIBBO MODEL ALLOWS US TO MAKE VARIOUS ESTIMATES OF DECAY RATES OF THE CHARMED MESONS D^+, D^0 AND F^+ ($= c\bar{d}$, $c\bar{u}$ AND $c\bar{s}$ RESPECTIVELY)

a. LEPTONIC DECAYS.

$[M_{D^+} = 1870 \quad M_{D^0} = 1865 \quad M_F \sim 1975 \text{ MeV}]$

THE D^+ AND F^+ MESONS CAN DECAY TO $\ell\nu$. NOTE THAT ℓ CAN BE e, μ, OR τ AS $M_{D,F} > M_\tau$

THE TRANSITION $c \to d$ IS SUPPRESSED BY $\sin\theta_C$ IN AMPLITUDE ACCORDING TO THE GIM MODEL, WHILE $c \to s \sim \cos\theta_C$. THEN COMPARING WITH p 301 WE HAVE AT ONCE

$$\Gamma_{D^+\to\ell\nu} \sim \sin^2\theta_C \, \frac{G^2 f_{D^+}^2}{8\pi} M_{D^+} M_\ell^2 \left(1 - \frac{M_\ell^2}{M_D^2}\right)^2$$

$$\Gamma_{F^+\to\ell\nu} \sim \cos^2\theta_C \, \frac{G^2 f_F^2}{8\pi} M_F \, M_\ell^2 \left(1 - \frac{M_\ell^2}{M_F^2}\right)^2$$

THE HADRONIC FUDGE FACTORS ARE ESTIMATED TO BE $f_D \sim f_F \sim M_\pi$

IT IS AMUSING TO NOTE THAT DECAY TO $\tau\nu_\tau$ IS FAVORED DESPITE THE SMALL PHASE SPACE FACTOR $\left(1 - \frac{M_\tau^2}{M^2}\right)^2$

NUMERICALLY, $\Gamma_{F^+\to\tau^+\nu_\tau} \sim 2\times10^{-11}\text{ MeV} \sim 3\times10^{10}/\text{sec}$

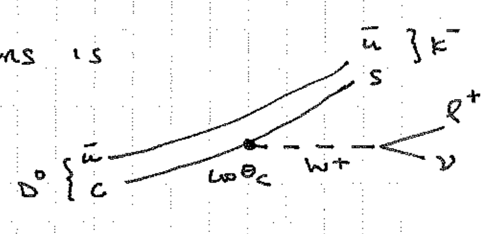
THIS RATE TURNS OUT TO BE A SMALL FRACTION OF THE TOTAL, AND THE LEPTONIC BRANCHING RATIOS HAVE NOT BEEN MEASURED YET.

b. SEMI-LEPTONIC DECAYS

WE CONSIDER DECAYS $\begin{Bmatrix} D \\ F \end{Bmatrix} \to \begin{Bmatrix} K \\ \pi \end{Bmatrix} + \ell\nu$

SUPPOSE THE ONLY QUARK TRANSITION THAT OCCURS IS

$$c \to \begin{Bmatrix} s \cos\theta_C \\ d \sin\theta_C \end{Bmatrix} + \ell\nu$$



AND THE OTHER QUARK ACTS AS A SPECTATOR.

THEN THE REACTION IS VERY MUCH LIKE $\mu \to \nu_\mu + e\nu_e$

RECALLING p 295 WE THEN ESTIMATE

$$\Gamma_{D\to K\ell\nu} \sim \left(\frac{M_c}{M_\mu}\right)^5 \Gamma_{\mu\to\nu_\mu e\nu_e} \sim \frac{G^2 M_c^5}{192\pi^3} \sim 2\times10^{11}/\text{sec}$$

USING $M_c \approx 1700$ MeV.

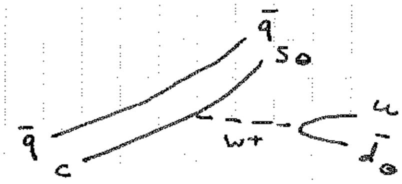
Also $\dfrac{\Gamma_{D\to\pi\ell\nu}}{\Gamma_{D\to K\ell\nu}} \sim \tan^2\theta_c \sim \dfrac{1}{20}$

Similar relations hold for the F.

c. Non-leptonic decays

We pursue the spectator quark idea further, and suppose the non-leptonic decays are due to transitions of the form

$$c \to s_\theta + W^+$$
$$\hookrightarrow u\,\bar{d}_\theta$$

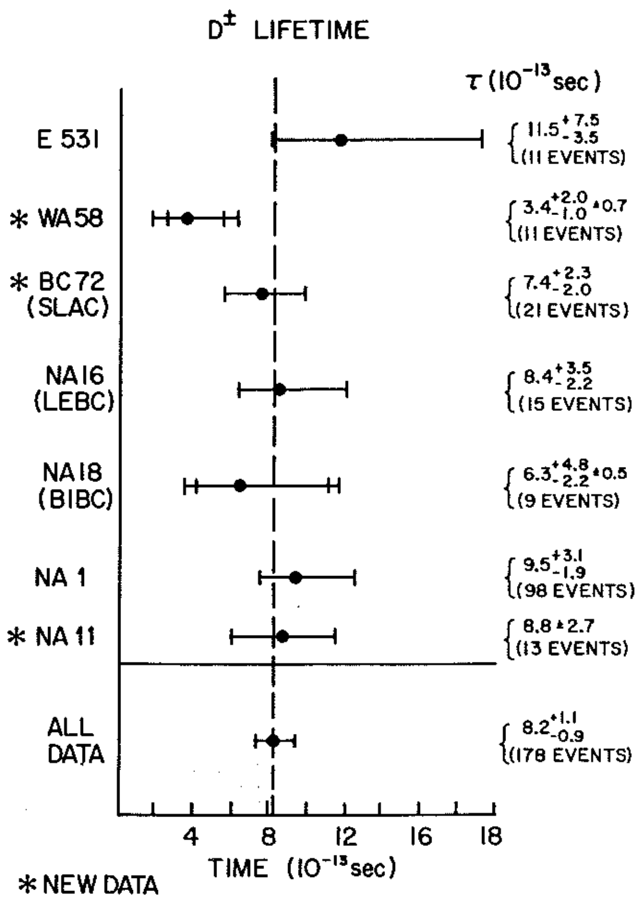


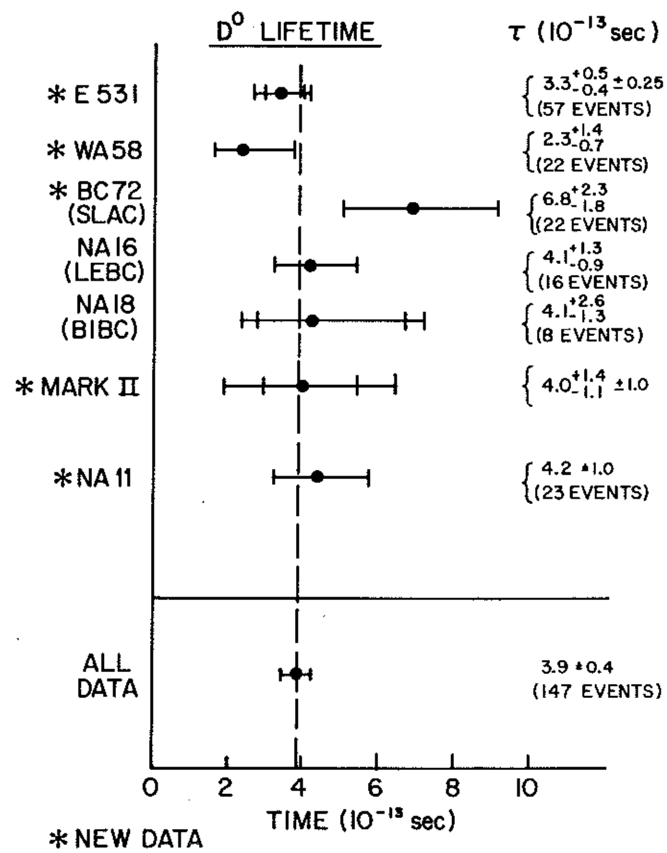
Taking into account the 3 colors of quarks, universality of the weak interaction then suggests that final states with hadrons + $\left\{\begin{array}{l} \ell\nu \\ \mu\nu \\ \text{hadrons} \end{array}\right\}$ occur in the ratio 1 : 1 : 3

[Recall the argument for τ meson decay p.260]

Then $\Gamma_{D\to \text{all}} \sim 5\,\Gamma_{D\to K\ell\nu} \sim 10^{12}/\text{sec}$

or lifetime $\sim 10^{-12}$ sec.





RECENT EXPERIMENTAL RESULTS INDICATE

$$\frac{\Gamma_{D^+ \to \text{hadrons} + e\nu}}{\Gamma_{D^+ \to \text{all}}} \sim 19 \pm 2\% \quad \text{but} \quad \frac{\Gamma_{D^0 \to he\nu}}{\Gamma_{D^0 \to \text{all}}} \sim 8 \pm 1\%$$

THE D^+ IS IN REASONABLE AGREEMENT WITH THE 1:1:3 HYPOTHESIS...

THE LIFETIME MEASUREMENTS ARE NOT YET TOO PRECISE:

$$\tau_{D^+} \sim 10.6 \pm 0.3 \times 10^{-13}\ \text{sec}$$

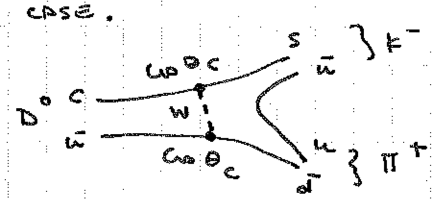
$$\tau_{D^0} \sim 4.8 \pm 0.3 \times 10^{-13}\ \text{sec}$$

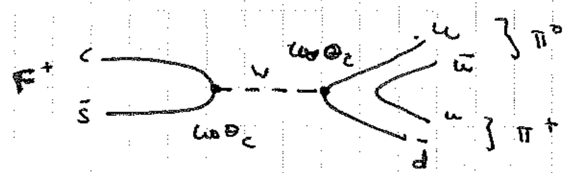
$$\tau_{F^+} \sim 4.5 \pm 0.3 \times 10^{-13}\ \text{sec}$$

THESE LIFETIMES ARE LONG ENOUGH THAT THE PARTICLES TRAVEL A MEASURABLE DISTANCE BEFORE DECAY, THANKS TO THE RELATIVISTIC INCREASE IN THE LIFETIME OF A MOVING PARTICLE. $\Rightarrow$ CAN MEASURE LIFETIME BY WATCHING TRACK LENGTH BEFORE DECAY.

WE FIND ROUGH AGREEMENT WITH OUR ESTIMATE OF 10^{-12} SEC. HOWEVER THE RESULT $\dfrac{\tau_{D^+}}{\tau_{D^0}} = \dfrac{\Gamma_{D^0}}{\Gamma_{D^+}} \sim 2$ IS STRIKING.

ALSO $\dfrac{\Gamma_{F^+}}{\Gamma_{D^+}} \sim 3 \pm 1.5$. IT IS NOTABLE THAT IN D^0 AND F^+ DECAYS THERE ARE ADDITIONAL POSSIBLE DIAGRAMS BESIDES THE SPECTATOR CASE.





(THESE DIAGRAMS ARE SUPPRESSED AT LEAST BY $\sin\theta_c$ IN AMPLITUDE FOR THE D^+ CASE. TRY IT.)

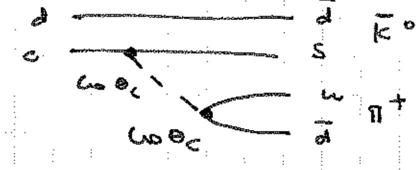
WHILE THIS ARGUMENT MAY NOT PREDICT EXACTLY A FACTOR OF 2 BETWEEN Γ_{D^0} AND Γ_{D^+} IT DOES INDICATE WHY $\Gamma_{D^0} > \Gamma_{D^+}$

EXERCISE: SHOW $\dfrac{\Gamma_{D^0 \to K^- \pi^+}}{\Gamma_{D^0 \to \bar{K}^0 \pi^0}} = 4$ (DATA: 1.36 ± 0.73)
IF ONLY CONSIDER THE W-EXCHANGE DIAGRAM AS SHOWN

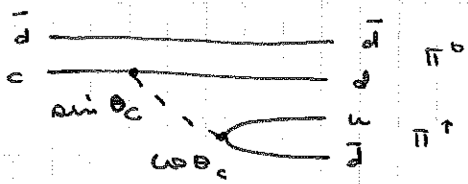
NOTE THAT THIS IS <u>NOT</u> A RESULT OF THE $\Delta I = \frac{1}{2}$ RULE! ALSO NOTE THAT BOTH FINAL STATES CAN BE REACHED BY CABIBBO ALLOWED SPECTATOR TYPE DIAGRAMS.

EXERCISE: USE THE SPECTATOR DIAGRAM TO PREDICT SOME PROMINENT 2 BODY HADRONIC DECAY MODES OF THE F^+

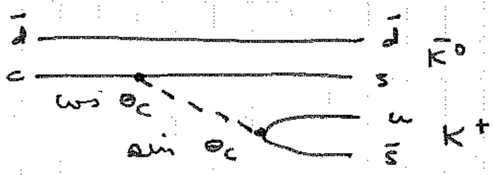
Ratios among a few particular decays of the D^+ can be estimated in the Cabibbo model



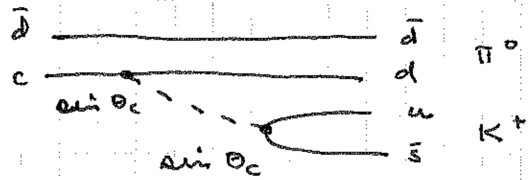
$$\Gamma_{D^+ \to \bar{K}^0 \pi^+} \sim \cos^4\theta_C$$



$$\Gamma_{D^+ \to \pi^0 \pi^+} \sim \cos^2\theta_C \sin^2\theta_C$$



$$\Gamma_{D^+ \to \bar{K}^0 K^+} \sim \sin^2\theta_C \cos^2\theta_C$$



$$\Gamma_{D^+ \to \pi^0 K^+} \sim \sin^4\theta_C$$

Of these only $\Gamma_{D^+ \to \pi^+ \bar{K}^0}$ is well measured, with a branching ratio of $2.8 \pm 0.4\,\%$

15. Bottom Meson Decays and the Kobayashi - Maskawa Matrix

With the bottom quark identified as having $Q = -1/3$ it is tempting to place it in a 3rd quark doublet along with the hypothetical top quark $\begin{pmatrix} t \\ b \end{pmatrix}$. However we might expect the weak interactions of the bottom quark are somehow twisted together with those of the d and s quarks, in the manner of Cabibbo. This possibility was explored by Kobayashi & Maskawa [Prog. Theor. Phys. Japan <u>49</u>, 652 (1973)]. They supposed that the weak charged current interactions couple the u, c & t quarks to the d_θ, s_θ & b_θ quarks, where the latter are obtained by a unitary transformation of d, s & b.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} U \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.

IN GENERAL A 3x3 UNITARY MATRIX HAS 9 REAL PARAMETERS (i.e. 9 COMPLEX MATRIX ELEMENTS, AND 9 RELATION $U^{+} = U^{-1}$). IN THE PRESENT CASE THE MATRIX U CONNECTS (d, s, b) TO (u, c, t) TO YIELD THE WEAK CURRENT

$$J_\alpha^{\text{weak}} = (\bar{u}, \bar{c}, \bar{t})\left(\gamma_\alpha (1-\gamma_5)\right) U \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

HOWEVER THE PHASES OF THE 6 QUARK STATES ARE UNOBSERVABLE, SO WE CAN MAKE 5 RELATIVE PHASE ADJUSTMENTS AS WE LIKE. IN THIS WAY WE CAN REFINE THE MATRIX ELEMENTS OF U, USING UP 5 OF THE 9 FREE PARAMETERS. FURTHERMORE WE CAN MAKE THE ADJUSTMENT SO AS TO MAKE 5 OF THE 9 MATRIX ELEMENTS REAL. THE STANDARD PRESCRIPTION IS INDICATED ON p 322.

THE RESULT THAT MATRIX U IS NOT REAL IMPLIES IT CONTAINS A T VIOLATING TRANSITION SOMEWHERE.

$$\langle i | U | f \rangle \xrightarrow{\;T\;} \langle f | U^{+} | i \rangle$$

$$U_{if} \quad = \quad U^{+}_{fi} = U^{*}_{if}$$

IF T INVARIANCE HOLDS

T VIOLATION IN TURN IMPLIES CP VIOLATION, GIVEN CPT INVARIANCE, WHICH WAS THE ORIGINAL MOTIVATION OF KOBAYASHI & MASKAWA IN INVENTING THEIR MATRIX.

ON COMPARISON WITH THE CABIBBO MODEL WE SEE THAT

$$V_{us} = \sin\theta_C \approx \sin\theta_{12}\cos\theta_{13} \sim \sin\theta_{12}\cos\theta_{23} \approx V_{cd}$$

THIS INDICATES THAT $\cos\theta_{13} \sim 1 \sim \cos\theta_{23}$. THAT IS, IF EITHER θ_{13} OR θ_{23} WERE BIG, THE CABIBBO MODEL WOULDN'T HAVE WORKED WELL, AND MIGHT NOT EVEN HAVE BEEN DISCOVERED.

FROM NUCLEAR β DECAY ONE LEARNS THAT $V_{ud} = \cos\theta_{12} = .975 \pm .001$

FROM THE STRANGENESS CHANGING DECAYS OF K MESONS AND HYPERONS ONE INFERS $V_{us} = \sin\theta_{12}\cos\theta_{13} = .219 \pm .002$

THE MATRIX ELEMENT V_{cd} IS APPARENTLY BEST MEASURED IN CHARM PRODUCTION BY NEUTRINO BEAMS (LECTURE 19)

$$V_{cd} \approx \sin\theta_{12}\cos\theta_{23} \sim .22 \pm .02$$

THESE RESULTS INDICATE θ_{13} AND θ_{23} ARE SMALL, BUT DON'T GIVE ACCURATE PREDICTIONS.

BUT WE CAN EXTRACT A QUALITATIVE IMPRESSION REGARDING BOTTOM MESON DECAY:

Ph 529 Lecture 17

GIVEN THE APPARENT SMALLNESS OF $\sin\theta_{13}$ AND $\sin\theta_{23}$ WE CAN WRITE

$$V_{CKM} \approx \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\exp(-i\delta_{13}) \\ -s_{12}c_{23} & c_{12}c_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}\exp(i\delta_{13}) & -c_{12}s_{23} & c_{13}c_{23} \end{pmatrix}$$

PERHAPS AN EVEN MORE USEFUL APPROXIMATION IS THAT DUE TO WOLFENSTEIN [P.R.L. 51, 1945, (1983).]

$$V_{CKM} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

THE DATA SUMMARIZED ON P. 323 LEAD TO $\underline{\lambda = 0.22 \pm 0.02}$

THE OTHER PARAMETERS, $A, \rho, \& \eta$ ARE BEST APPROACHED VIA BOTTOM MESON DECAYS.

A DECAY $b \to c$ DEPENDS ON $V_{cb} \approx A\lambda^2$

WHILE $b \to u$ DEPENDS ON $V_{ub} \approx A\lambda^3(\rho - i\eta)$

THE $b \to c$ TRANSITION DOMINATES b DECAYS, SO A MEASUREMENT OF THE B-MESON LIFETIME WILL TELL US ABOUT A.

THE FIRST MEASUREMENTS WERE $\tau_B = 1.8 \pm 0.6 \times 10^{-12}$ sec FERNANDEZ PRL 51, 1022 (1983)
1.2 ± 0.5 " LOCKYER PRL 51 1316 (1983)

AND THE PRESENT VALUE IS $\tau_B \sim 1.23 \pm 0.10$

NOTE THAT $\tau_B > \tau_D$, EVEN THOUGH A DIMENSIONAL ESTIMATE WOULD SUGGEST $\frac{\tau_B}{\tau_D} \sim \left(\frac{M_D}{M_B}\right)^5 \sim \left(\frac{1}{3}\right)^5 \sim \frac{1}{250}$

THE RELATIVELY LONG LIFE OF B MESONS MEANS. THEY TRAVEL A DISCERNABLE DISTANCE FROM THEIR PRODUCTION POINT BEFORE DECAY $\Rightarrow$ GOOD EXPERIMENTAL ACCESSIBILITY.

TO EXTRACT THE VALUE OF A, WE USE THE COUNTING ARGUMENT OF THE SPECTATOR MODEL:

$$b \to c + W^-$$
$$\hookrightarrow (e\nu)$$
$$(\mu\nu)$$
$$(\tau\nu)$$
$$3(u\, d_\theta)$$
$$3(c\, s_\theta)$$

SO, FOR EXAMPLE, WE EXPECT $\frac{\Gamma_{B\to Xe\nu}}{\Gamma_{B\to ALL}} = \frac{1}{9}$ DATA: $11.6 \pm 0.5\%$

RECALL FROM p 321 THAT FOR D MESONS A SIMILAR ARGUMENT WORKED BEST FOR THE D^+ MESON:

$$\frac{\Gamma_{D^+\to X e\nu}}{\Gamma_{D^+\to ALL}} \approx \frac{1}{5}$$

So

$$\frac{\tau_{D^+}}{\tau_B} = \frac{\Gamma_{B\to ALL}}{\Gamma_{D^+\to ALL}} = \frac{9\,\Gamma_{B\to Xe\nu}}{5\,\Gamma_{D^+\to Xe\nu}} \approx \frac{9\,(M_B)^5\,|V_{cb}|^2}{5\,(M_{D^+})^5\,|V_{cs}|^2}$$

← REST OF MATRIX ELEMENT SAME FOR B & D !

WITH $V_{cs} \simeq 1$ WE HAVE

$$|V_{cb}|^2 \approx A^2\lambda^4 = \frac{5}{9}\left(\frac{M_{D^+}}{M_B}\right)^5 \frac{\tau_{D^+}}{\tau_B}$$

$$= \frac{5}{9}\left(\frac{1869}{5278}\right)^5 \frac{1.06\times10^{-12}}{1.23\times10^{-12}} = 0.0027$$

NOW $\lambda^4 = (0.22)^4 = 0.0023$ SO $A \simeq 1$

ANALYSES WITH SMALL CORRECTION FACTORS GIVE $\underline{A = 0.85 \pm 0.1}$

KNOWLEDGE OF V_{ub} COMES VIA DECAYS $b \to u + W^-$, $W^- \to e\nu$

WHICH IS RARER THAN $b \to c + W^-$, $W^- \to e\nu$ BUT THE ELECTRONS CAN HAVE HIGHER ENERGY IN THE $b \to u$ TRANSITION.

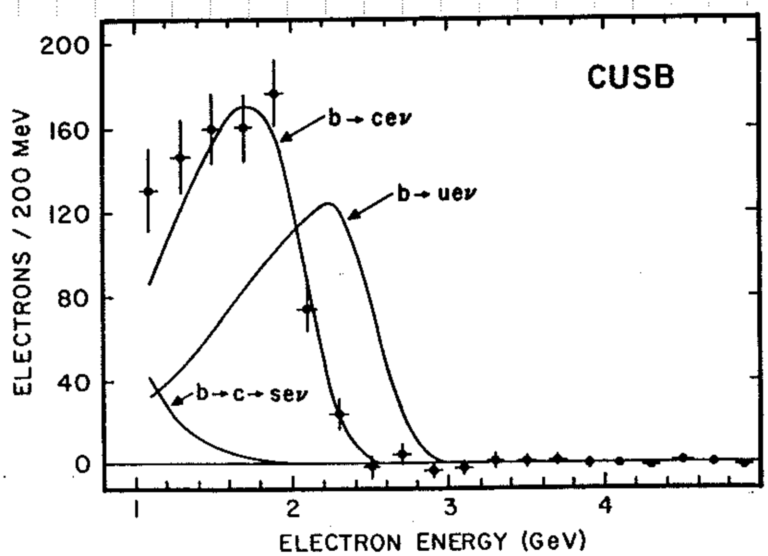
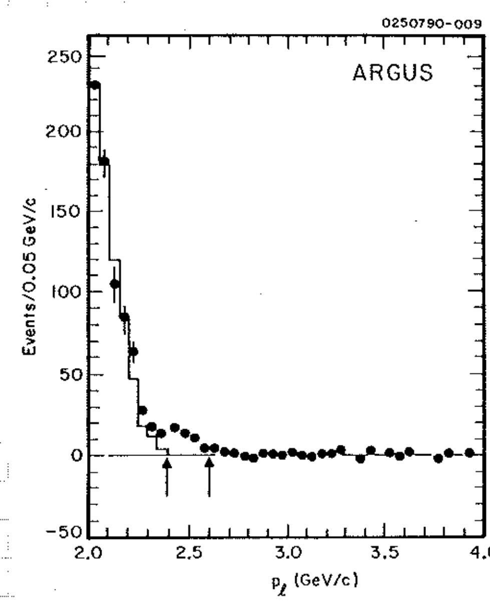


Fig. 16: Electron momentum spectrum from CUSB. The b→ueν curve shows the spectrum predicted by Altarelli et al. for semileptonic decay proceeding via b→u.



FROM RECENT EXPERIMENTS SUCH AS ARGUS, IT IS CLAIMED THAT $\left|\frac{V_{ub}}{V_{cb}}\right| \sim 0.11 \pm 0.05$

IN THE WOLFENSTEIN PARAMETRIZATION $\frac{V_{ub}}{V_{cb}} = \lambda(\rho - i\eta)$

SO WE INFER $\underline{\sqrt{\rho^2+\eta^2} \sim 0.5 \pm 0.25}$ USING $\lambda = 0.22$

SEPARATE VALUES FOR ρ & η ARE NOT KNOWN AT PRESENT, BUT ADDITIONAL INFORMATION IS AVAILABLE FROM B^0-$\bar{B}^0$ MIXING, AND CP VIOLATION....