THE NEED FOR A BETTER THEORY

IN THE NEXT FEW LECTURES WE INDICATE HOW OUR VIEW OF ELEMENTARY PARTICLE INTERACTIONS HAS BEEN GREATLY IMPROVED BY USE OF THE PRINCIPLE OF GAUGE INVARIANCE. THE MOTIVATION FOR THIS DEVELOPMENT WAS PRIMARILY THEORETICAL AND INTRODUCES IDEAS SOMEWHAT DETACHED FROM THE PREVIOUS MATERIAL OF THIS COURSE. AS THESE NOTIONS HAVE LED TO PREDICTIONS WHICH HAVE RECEIVED STRIKING EXPERIMENTAL CONFIRMATION WE HOPE TO GIVE SOME IMPRESSION OF WHAT THE IDEAS ARE, AS WELL AS THE FACTS THAT SUPPORT THEM.

REFERENCES: AN INTRODUCTORY WORK EMPHASIZING THE CONCEPT OF GAUGE INVARIANCE MORE THAN ITS APPLICATION TO EXPERIMENTAL PHYSICS IS:
'ELEMENTARY PRIMER FOR GAUGE THEORY' BY MORIYASU; WORLD SCIENTIFIC (1983)
BOOKS WITH GREATER EMPHASIS ON THE RELATION BETWEEN THEORY AND PHENOMENOLOGY INCLUDE:
'GAUGE THEORIES AND THE NEW PHYSICS' BY LEADER & PREDAZZI; CAMBRIDGE (1982)
'GAUGE THEORIES OF THE S., W., & E&M INTERACTIONS' BY QUIGG; BENJAMIN (1983)
'GAUGE THEORIES IN PARTICLE PHYSICS' BY AITCHISON & HEY; ADAM HILGER (1982)

1. TROUBLE WITH THE THEORY OF THE WEAK INTERACTION

WE HAVE ALREADY NOTED A POSSIBLE PROBLEM WITH THE FERMI THEORY OF THE WEAK INTERACTION IN LECTURE 3 p36 (SEE ALSO p350). THE CROSS-SECTION FOR, SAY, NEUTRINO-ELECTRON SCATTERING

$$\nu_e + e \to \nu_e + e \qquad \text{WILL BEHAVE LIKE}$$

$$\sigma \sim G^2 E_{cm}^2 \to \infty \quad \text{AT HIGH ENERGIES} \quad (\text{HEISENBERG}, 1936)$$

WHILE INFINITE CROSS SECTIONS DO OCCUR FOR LONG RANGE FORCES SUCH AS ELECTRICITY AND GRAVITY, THIS IS SURPRISING FOR THE WEAK FORCE WHICH WAS SUPPOSED TO BE A 'CONTACT' INTERACTION. WE CAN BE MORE PRECISE IF WE RECALL THE PARTIAL WAVE ANALYSIS OF SCATTERING CROSS SECTIONS (LECTURE 11, p206)

FOR A 'CONTACT' INTERACTION THERE CAN BE NO ORBITAL ANGULAR MOMENTUM INVOLVED. THAT IS, THE FERMI INTERACTION CAN ONLY BE S-WAVE (OR PERHAPS P-WAVE DUE TO THE AXIAL VECTOR CURRENT, AS ON p.312). THEN THE PARTIAL WAVE ANALYSIS TELLS US THAT

$$\sigma_{\text{ELASTIC}} \le \frac{4\pi}{k^2} \sum_l (2l+1) \to \frac{4\pi}{k^2} \sim \frac{\pi}{(E_{cm})^2} \text{ IF ONLY } l=0 \text{ IS POSSIBLE}$$

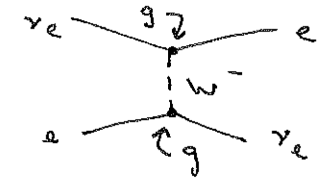
THIS LIMIT IS SOMETIMES CALLED THE S-WAVE UNITARITY BOUND

THE NEUTRINO SCATTERING CROSS SECTION REACHES THE BOUND WHEN

$$\sigma \sim G^2 E_{cm}^2 \sim \frac{1}{E_{cm}^2} \implies E_{cm}^4 \sim \frac{1}{G^2} \sim 10^{10} M_p^4 \quad \text{OR} \quad E_{cm} \sim 300\, M_p$$
$$\sim 300 \text{ GeV}$$

THIS ENERGY IS NOW ACHIEVABLE IN LABORATORY EXPERIMENTS.

WE HAVE ALSO ALREADY INDICATED A POSSIBLE RESOLUTION OF THIS PROBLEM : THE HYPOTHESIS OF THE $W^\pm$ BOSONS



IN THIS VIEW WE ESTIMATE

$$\frac{d\sigma}{d\Omega} \sim \frac{g^4 E_{cm}^2}{(q^2 - M_W^2)^2} \quad \text{NOTING THE W PROPAGATOR}$$

AGAIN WE IDENTIFY $G \sim \frac{g^2}{M_W^2}$ SO THAT FOR $E_{cm} \ll M_W$ OUR PREVIOUS ESTIMATE STILL HOLDS,

NOTE THAT $q^2 \sim -\frac{E_{cm}^2}{2}(1-\cos\theta) \Rightarrow dq^2 \sim E_{cm}^2\, d\Omega$

AND $$\sigma_{GL} = \int_{-E_{cm}^2}^{0} dq^2 \cdot \frac{d\sigma}{dq^2} \sim g^4 \int_{-E_{cm}^2}^{0} \frac{dq^2}{(q^2 - M_W^2)^2} \sim \frac{g^4}{M_W^2} \sim G^2 M_W^2$$

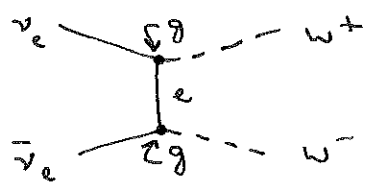
IF $E_{cm} \gg M_W$. THAT IS THE CROSS SECTION GOES TO A CONSTANT IN THE REVISED THEORY. FURTHERMORE, IN THE VERY HIGH ENERGY LIMIT THE DIFFERENTIAL CROSS SECTION VARIES AS

$$\frac{d\sigma}{d\Omega} \sim \frac{1}{(1-\cos\theta)^2}$$ WHICH IS CERTAINLY NOT PURE S-WAVE.

EVEN SO IT TURNS OUT THAT THE S-WAVE PART OF THIS CROSS SECTION EXCEEDS THE UNITARITY BOUND AT A SUPER HIGH ENERGY.

ANOTHER INDICATION THAT THE W BOSON IDEA DOESN'T SOLVE ALL OUR PROBLEMS COMES FROM CONSIDERING THE SOMEWHAT ACADEMIC REACTION

$$\nu \bar{\nu} \to W^+ W^-$$



IN THE HIGH ENERGY LIMIT WE MAY ESTIMATE THAT

$$\sigma \sim \frac{g^4}{E_{cm}^2}(\epsilon^+ \epsilon^-)^2 \quad \text{WHERE } \epsilon = \text{POLARIZATION OF THE SPIN 1 BOSON}$$

[FOR A MORE DETAILED DERIVATION USING DIRACOLOGY, SEE THE BOOK OF QUIGG, P 103]

A CONSIDERABLE COMPLICATION OF THE W BOSONS COMPARED TO THE PHOTON IS THAT THEY CAN HAVE LONGITUDINAL POLARIZATION, SINCE $M_W \neq 0$.

IN THE W REST FRAME THE POSSIBLE POLARIZATION VECTORS ARE

$$\epsilon_x^* = (0,1,0,0) \qquad \epsilon_y^* = (0,0,1,0) \qquad \text{AND} \qquad \epsilon_z^* = (0,0,0,1)$$

IN THE C.M. FRAME OF THE INTERACTION $\nu\bar{\nu} \to W^+W^-$ THE W'S ARE NOT AT REST. WE CHOOSE OUR Z-AXIS ALONG THE W DIRECTION OF MOTION, SO THAT THE 4-MOMENTUM OF THE W IS

$$k_\mu \approx \left(\frac{E_{cm}}{2}, 0, 0, \pm\frac{E_{cm}}{2}\right) \quad \text{IN THE HIGH ENERGY LIMIT } E_{cm} \gg M_W$$

THUS THE BOOST FROM THE W REST FRAME TO THE C.M. FRAME IS $\gamma = \dfrac{E_{cm}}{2M_W}$

THE POLARIZATION VECTORS IN THE C.M. FRAME ARE THEN

$$\epsilon_x^\pm = (0,1,0,0), \quad \epsilon_y^\pm = (0,0,1,0) \quad \text{AND} \quad \epsilon_z^\pm = (\beta\gamma, 0, 0, \pm\gamma)$$

WHERE $\pm$ REFER TO W^+ AND W^-. NOTE THAT $\epsilon_z^2 = \beta^2\gamma^2 - \gamma^2 = -1$; AND $\epsilon_\mu k^\mu = 0$

THEN $\epsilon^+\epsilon^- = \epsilon_x^+\epsilon_x^- + \epsilon_y^+\epsilon_y^- + \epsilon_z^+\epsilon_z^- = -2 + \beta^2\gamma^2 + \gamma^2 \approx \dfrac{E_{cm}^2}{2M_W^2}$

THE PRODUCT $\epsilon^+\epsilon^-$ IS DOMINATED BY THE LONGITUDINAL POLARIZATION TERM. SUCH A LARGE TERMS IS, OF COURSE, UNFAMILIAR FROM E & M WHERE IT IS CONVENIENTLY FORBIDDEN WHEN $M_{PHOTON} \to 0$.

$$\text{NOW} \quad \sigma_{\nu\bar{\nu}\to W^+W^-} \sim \frac{g^4}{E_{cm}^2}(\epsilon^+\epsilon^-)^2 \sim \frac{g^4 E_{cm}^2}{M_W^4} \sim G^2 E_{cm}^2$$

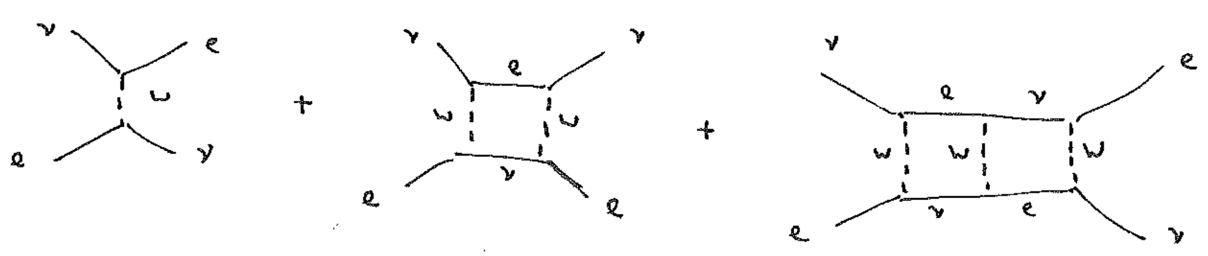
EXERCISE: BY CONSIDERING THE ν AND W SPINS SHOW THAT

$$\frac{d\sigma}{d\Omega} \sim \sin^2\theta \quad \text{FOR THE CASE OF LONGITUDINAL POLARIZATION OF THE W'S}$$

FROM THIS FORM FOR THE ANGULAR DISTRIBUTION WE INFER THAT AT MOST $l = 1$ AND 0 PARTIAL WAVES CONTRIBUTE. THEN A RISING CROSS SECTION $\sigma \sim E_{cm}^2$ AGAIN VIOLATES THE UNITARITY BOUND (AT $E_{cm} \sim 1000$ GEV)

WE MAY RECALL THAT FOR REAL PHOTONS THE LONGITUDINAL POLARIZATION STATES ARE EXCLUDED BY GAUGE INVARIANCE (P. 110). THE W BOSONS HAVE MASS SO THAT LONGITUDINAL POLARISATION IS NOT EXCLUDED. NONETHELESS IT REMAINS A CLUE THAT CONSIDERATIONS OF GAUGE INVARIANCE MAY HELP RESOLVE THE TROUBLES.

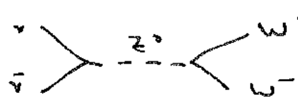
ANOTHER KIND OF PROBLEM OCCURS IF WE CONSIDER HIGHER ORDER WEAK INTERACTIONS INVOLVING W BOSONS. FOR EXAMPLE $\nu_e \to \nu_e$

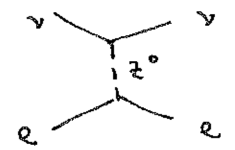


EVEN THO THE COUPLINGS ARE WEAK, DETAILED CALCULATION SHOWS THAT THE HIGHER ORDER DIAGRAMS IMPLY EVEN LARGER CROSS SECTIONS THAT FOR THE LOWEST ORDER CASE.

THIS RESULT IS IN CONTRAST TO THE SITUATION IN ELECTROMAGNETISM WHERE HIGHER ORDER DIAGRAMS GIVE ONLY SMALL CORRECTIONS. THE TECHNICAL REASON FOR THE GOOD BEHAVIOR IN QED IS THAT THERE ARE ALWAYS SEVERAL KINDS OF DIAGRAMS IN EACH ORDER, AND THESE LARGELY CANCEL ONE ANOTHER IN A MOST CONVENIENT MANNER. IT MAY BE WORTH REMARKING THAT THE GAUGE INVARIANCE OF THE ELECTROMAGNETIC INTERACTION REQUIRES THE EXISTENCE OF MANY DIAGRAMS IN EACH ORDER.

FOR THE WEAK INTERACTION WHOLE NEW CLASSES OF DIAGRAMS ARE POSSIBLE WHEN WE CONSIDER THE WEAK NEUTRAL CURRENTS DUE TO Z^0 EXCHANGE:

$\nu\bar{\nu} \to W^+W^-$ 

; $\nu e \to \nu e$ 

ETC...

A PARTICULAR COMBINATION OF THE $W^\pm$ AND Z^0 BOSONS WITH THE PRINCIPLE OF GAUGE INVARIANCE HAS BEEN SET FORTH BY GLASHOW, WEINBERG & SALAM. THIS IMPROVED THEORY RESOLVES ALL OF THE DIFFICULTIES MENTIONED ABOVE, AS WELL AS MAKING MANY DETAILED PREDICTIONS WHICH ARE EXPERIMENTALLY VERIFIABLE. IN ORDER TO OBTAIN SOME APPRECIATION OF THE POINT OF VIEW OF THIS THEORY WE MAKE A SIZABLE DIGRESSION ABOUT THE CONCEPT OF GAUGE INVARIANCE.

2. GAUGE INVARIANCE IN CLASSICAL PHYSICS

WE ARE FAMILIAR WITH THE CONCEPT OF GAUGE INVARIANCE IN ELECTROMAGNETISM. THE 4 POTENTIAL $A_\mu = (\phi, \bar{A})$ CAN BE ALTERED TO $A_\mu + \partial_\mu \Omega$ WITHOUT CHANGING THE FIELDS $\bar{E}$ OR $\bar{B}$ (OR, MORE DIRECTLY, THE FIELD TENSOR $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$). THE NAME 'GAUGE' INVARIANCE FOR THIS RESULT SEEMS A BIT ARBITRARY AS THERE DOESN'T SEEM TO BE ANY GAUGES OR METERS INVOLVED. BUT IN FACT THERE IS A HISTORICAL RELATION BETWEEN GAUGE INVARIANCE AND CERTAIN GEOMETRICAL IDEAS ADVANCED BY WEYL IN 1919.

WEYL SOUGHT TO PROVIDE A LINK BETWEEN ELECTROMAGNETISM AND THE THEN RECENT SUCCESS OF THE GEOMETRICAL APPROACH TO GRAVITATION — GENERAL RELATIVITY. HE NOTED THAT A KEY FEATURE OF GENERAL RELATIVITY IS THAT THE BEHAVIOR OF NATURE IS 'SIMPLE' ONLY IN A LIMITED REGION OF SPACE AND TIME AROUND A GIVEN OBSERVER. THIS IS UNLIKE SPECIAL RELATIVITY FOR WHICH THE OBSERVER'S COORDINATE SYSTEM CAN BE THOUGHT OF EXTENDING OVER ALL SPACE AND TIME IN A UNIFORM MANNER. BUT IN THE GENERAL CASE ONE MUST HAVE ADDITIONAL RULES AS TO HOW TO CONNECT THE DESCRIPTION OF SPACE-TIME BETWEEN TWO NEIGHBORING REGIONS. THESE RULES OF CONNECTION ARE TO A LARGE EXTENT THE CONTENT OF THE THEORY OF GENERAL RELATIVITY.

THIS SUGGESTS A POSSIBLE APPROACH TO PHYSICS, AMONG OTHERS. ONE FIRST ESTABLISHES THE LOCAL BEHAVIOR OF NATURE IN A LIMITED REGION, AND THEN DEVELOPS PROCEDURES TO EXTEND THIS UNDERSTANDING TO AN EVER LARGER DOMAIN.

WEYL CONSIDERED THE POSSIBILITY THAT WHILE THE MEASURE, OR SCALE, OF LENGTH MIGHT BE WELL DEFINED IN A LOCAL REGION, IT ACTUALLY VARIES FROM PLACE TO PLACE. HE FOUND THAT THE FORMALISM TO DESCRIBE THIS INCLUDES A SORT OF INVARIANCE UNDER A TRANSFORMATION LIKE $A_\mu \to A_\mu + \partial_\mu \Omega$

TO SEE THIS WE CONSIDER A SCALAR FUNCTION $f(X)$ WHERE $X = X_\mu$ = POSITION IN SPACE TIME. IF THE MEASURE OF SIZE IS THE SAME EVERYWHERE, CHANGES OF f WITH POSITION ARE DESCRIBED BY

$$f(x+dx) = f(x) + \partial_\mu f(x)\, dx^\mu$$

SUPPOSE HOWEVER THAT THE RULE FOR MEASURING SIZE VARIES FROM PLACE TO PLACE IN A SMOOTH MANNER. WE DESCRIBE THIS BY A SCALE FACTOR $S(x)$, TAKEN TO BE 1 AT OUR STARTING POINT x. AT A NEIGHBORING POINT $x+dx$ THE SCALE FACTOR IS

$$S(x+dx) = 1 + \partial_\mu S\, dx^\mu$$

THEREFORE THE MEASURED VALUE OF f AT $x+dx$ SHOULD BE GIVEN IN TERMS OF QUANTITIES MEASURED AT x AS

$$f(x+dx) = (1 + \partial_\mu S\, dx^\mu)(f(x) + \partial_\mu f\, dx^\mu)$$

$$= f(x) + (\partial_\mu + \partial_\mu S) f\, dx^\mu \quad + O(dx^2)$$

WE NOTE THE APPEARANCE OF $D_\mu \equiv \partial_\mu + \partial_\mu S$ WHICH MIGHT BE CALLED THE 'SCALE COVARIANT' DERIVATIVE. USE OF D_μ RATHER THAN THE ORDINARY DERIVATIVE OPERATOR ∂_μ ALLOWS US TO MAKE THE CONNECTION BETWEEN THINGS AT x AND $x+dx$.

SUPPOSE INSTEAD THAT THE RULE FOR SCALE CHANGES WERE TO TAKE THE PRODUCT OF TWO SCALE FACTORS $S(x)\Omega(x) \equiv S'(x)$.

THEN $S'(x+dx) = 1 + (\partial_\mu S + \partial_\mu \Omega)\, dx + O(dx)^2$ FOR $S(x) = 1$

OR $\partial_\mu S' = \partial_\mu S + \partial_\mu \Omega$ AND $D'_\mu = D_\mu + \partial_\mu \Omega$

THE FORM OF THE PROCEDURE FOR RESCALING QUANTITIES FROM PLACE TO PLACE IS INVARIANT UNDER THE CHANGE $\partial_\mu S \to \partial_\mu S + \partial_\mu \Omega$.

THIS SUGGESTED TO WEYL THAT PERHAPS THE ELECTROMAGNETIC POTENTIAL A_μ COULD BE IDENTIFIED WITH $\partial_\mu S$. IF SO, ELECTRICITY COULD BE TIED TO GEOMETRY.

THIS PARTICULAR VERSION OF SCALE OR 'GAUGE' INVARIANCE HAS

NOT SURVIVED, ALTHOUGH THE NAME IS STILL IN USE TODAY. IN THE NEXT SECTION WE SEE HOW THE GAUGE INVARIANCE OF ELECTROMAGNETISM IS RELATED TO THE PHASE INVARIANCE OF THE WAVE FUNCTION IN QUANTUM MECHANICS.

3. GAUGE INVARIANCE IN QUANTUM MECHANICS

THE SCHRÖDINGER EQUATION FOR A CHARGED PARTICLE IN AN ELECTROMAGNETIC FIELD IS

$$\left[\frac{\left(-i\bar{\nabla}-e\bar{A}\right)^2}{2m}+e\phi\right]\psi = i\frac{\partial\psi}{\partial t}$$

CAN WE REPLACE A_μ BY $A_\mu + \partial_\mu \Omega(x)$ WITHOUT CHANGING THE PHYSICS, AS IS THE CASE IN CLASSICAL E & M? FOCK & LONDON (1927) NOTED THAT THE FORM OF THE SCHRÖDINGER EQUATION IS UNCHANGED PROVIDED WE ALSO REPLACE ψ BY $e^{-ie\Omega}\psi$. THEN SINCE IT IS WELL KNOWN THAT THE ABSOLUTE PHASE OF THE WAVE FUNCTION IS NOT OBSERVABLE, OTHER PHYSICS RESULTS ARE UNCHANGED ALSO.

TO SKETCH THIS IT IS USEFUL TO INTRODUCE THE NOTATION

$$D_\mu = \partial_\mu + ieA_\mu = \text{'GAUGE COVARIANT' DERIVATIVE}$$

USING THIS FANCY NOTATION, THE SCHRÖDINGER EQUATION BECOMES

$$\frac{\left(-i\bar{\nabla}+ie\bar{A}\right)^2}{2m}\psi = i\left(\frac{\partial}{\partial t}+ie\phi\right)\psi \quad \text{OR} \quad \frac{\left(-i\bar{D}\right)^2}{2m}\psi = iD_0\psi$$

IF WE TRANSFORM $A_\mu \to A_\mu' = A_\mu + \partial_\mu\Omega$ AND $\psi \to \psi' = e^{-ie\Omega}\psi$

THEN $D_\mu \to D_\mu' = \partial_\mu + ieA_\mu + ie\partial_\mu\Omega$

AND
$$D_\mu'\psi' = \left(\partial_\mu + ieA_\mu + ie\partial_\mu\Omega\right)e^{-ie\Omega}\psi = e^{-ie\Omega}\left(\partial_\mu + ieA_\mu\right)\psi$$
$$= e^{-ie\Omega}D_\mu\psi$$

LIKEWISE $\bar{D}'^2\psi' = e^{-ie\Omega}\bar{D}^2\psi$, SO CLEARLY $\frac{\left(-i\bar{D}'\right)^2}{2m}\psi' = iD_0'\psi'$ AS DESIRED.

SIMILARLY QUANTITIES LIKE $\psi^*\psi$ AND $\psi^* D_\mu\psi$ ARE INVARIANT UNDER THE COMBINED GAUGE AND PHASE TRANSFORMATION.

MORE IMPORTANT FOR US HERE IS TO VIEW THE ARGUMENT IN REVERSE.

WE UNDERSTAND READILY THAT ALL PHYSICAL RESULTS OF QUANTUM MECHANICS ARE INVARIANT UNDER THE <u>GLOBAL</u> PHASE TRANSFORMATION

$$\psi \to e^{i\theta}\psi \quad \text{WHERE } \theta = \text{CONSTANT}$$

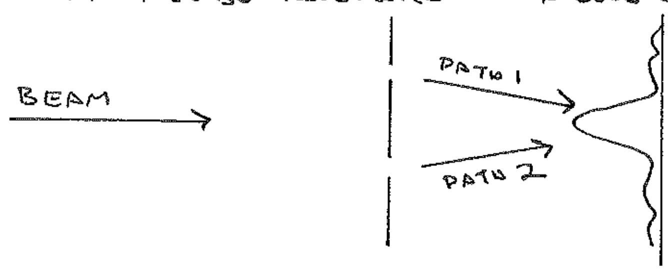
BUT IS QUANTUM MECHANICS INVARIANT UNDER THE BROADER TRANSFORMATION $\psi \to e^{i\Theta(x,t)}\psi$ WHERE $\Theta(x,t)$ VARIES FROM PLACE TO PLACE? IT IS NOT UNREASONABLE TO HOPE SO. THE GLOBAL TRANSFORMATION WITH Θ = CONSTANT MUST BE APPLIED INSTANTANEOUSLY THROUGHOUT THE ENTIRE UNIVERSE. IT MAKES MORE SENSE IF WE CAN APPLY A LOCAL CHANGE OF PHASE $\Theta(x,t)$, WHILE SENDING A MESSAGE TO OUR MORE DISTANT COLLEAGUES THAT THEY SHOULD CHANGE THEIR PHASES ALSO. BUT THE LAWS OF PHYSICS SHOULD NOT BE DIFFERENT DURING THE TIME WHILE THE MESSAGE IS BEING SENT.

SUPPOSE WE MAKE THE LOCAL PHASE TRANSFORMATION ON A FREE PARTICLE WHICH OBEYS THE SCHRODINGER EQUATION $\frac{(-i\vec{\nabla})^2}{2m}\psi = i\frac{\partial\psi}{\partial t}$

NOTE THAT $\partial_\mu\psi \to \partial_\mu e^{i\Theta}\psi = e^{i\Theta}\partial_\mu\psi + i(\partial_\mu\Theta)\psi$

IF $\partial_\mu\Theta \neq 0$ THE SCHRÖDINGER EQUATION IS NOT INVARIANT UNDER THE PHASE TRANSFORMATION!

THIS MAY ALSO BE ILLUSTRATED WITH A DOUBLE SLIT EXPERIMENT



IF WE CHANGE THE PHASE OF THE WAVE FUNCTION ALONG PATH 1 BUT NOT ALONG PATH 2 THE INTERFERENCE PATTERN CHANGES. THUS IN GENERAL WE ARE NOT FREE TO MAKE A LOCAL PHASE CHANGE IF WE DESIRE THE PHYSICAL RESULTS TO BE UNAFFECTED.

HOWEVER WE CAN HAVE LOCAL PHASE INVARIANCE IF THE PARTICLE IS NOT FREE BUT RATHER IT OBEYS CERTAIN RULES AS IT MOVES FROM PLACE TO PLACE. NAMELY WE MODIFY THE SCHRODINGER EQUATION TO READ

$$\frac{(-i\vec{D})^2}{2m}\psi = iD_0\psi \qquad \text{WHERE} \quad D_\mu = \partial_\mu + ieA_\mu$$

THEN IF WE WRITE $\Theta = e\Omega$, PHYSICS IS INVARIANT UNDER THE COMBINED TRANSFORMATIONS

$$\psi \to e^{-ie\Omega}\psi \qquad \text{AND} \qquad A_\mu \to A_\mu + \partial_\mu\Omega$$

THE DEMAND FOR LOCAL PHASE INVARIANCE REQUIRES THE PARTICLE TO UNDERGO AN INTERACTION WHICH MUST HAVE THE FORM OF ELECTROMAGNETISM. IN PRINCIPLE WE DIDN'T NEED TO KNOW ABOUT ELECTRICITY IN ADVANCE. THE IMPLEMENTATION OF A LOCAL INVARIANCE PRINCIPLE LEADS TO A SPECIFIC FORM OF A PARTICLE INTERACTION WHICH IS DESCRIBED BY A FIELD OBEYING GAUGE INVARIANCE.

We have a glimpse of a powerful procedure which may yield clues as to the form of other interactions besides electromagnetism. But this procedure is not self-evident: 50 years elapsed between Weyl's conjecture and the successful formulation of the Glashow-Weinberg-Salam model.

4. The Aharanov-Bohm Effect

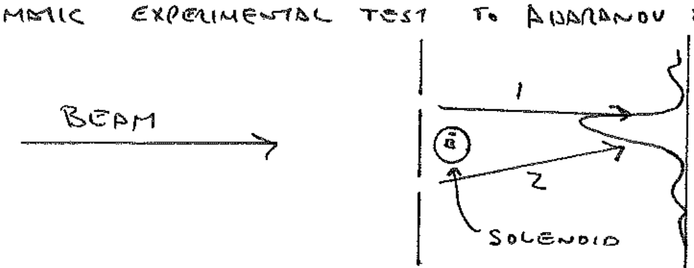
Our analysis of local phase invariance indicates that the electromagnetic vector potential plays a more fundamental role in quantum mechanics than in classical physics. Indeed the success of the local gauge theories has caused a major shift in attitude whereby the fields $\bar{E}$ and $\bar{B}$ are regarded as secondary concepts, with A_μ providing the most relevant insight into the microworld.

[It is amusing to note that the vector potential, called the 'electrotonic force' by Faraday, played a larger role in Maxwell's own view than in subsequent discussion of classical E & M. Faraday's Law:

$$\oint \bar{E}\cdot d\bar{l} = -\frac{1}{c}\frac{\partial}{\partial t}\oint \bar{A}\cdot d\bar{l}.$$

In this formulation the induced EMF is due to local changes in $\bar{A}$, not due to distant changes in magnetic flux.]

From the example of the double slit experiment we see that local phase changes in a particle's wave function can have physical consequences. But to have such changes we must have an interaction of the particle with a vector potential. This suggested a dramatic experimental test to Aharanov & Bohm [P.R. 115, 485 (1959)]



For an ideal solenoid $\oint \bar{A}\cdot d\bar{l} = \int \bar{B}\cdot d\bar{s} \Rightarrow A_\phi = \frac{BR^2}{2r}$ for $r > R_{solenoid}$

So $A_\phi \neq 0$ anywhere, even tho $\bar{B}=0$ outside the solenoid. Classically we would expect no effects outside the solenoid, but in Q.M. this is no longer so. The phase of the wave function along paths 1 & 2 differs $\Rightarrow$ shift of the interference pattern!

[Classical paradox: Suppose we alter the current of the solenoid so that $\dot{B}_{in}$ = constant. Then $\bar{B}_{outside} = 0$ still, but $\bar{E}_{out} \neq 0$]

We sketch the formalism. Suppose we know the wave function $\psi_0(x,t)$ along some path in the absence of the vector potential $\bar{A}$.

Then $\frac{(-i\bar{\nabla})^2}{2m}\psi_0 = i\frac{\partial\psi_0}{\partial t}$

NOW WE SUPPOSE $\bar{A} \neq 0$ BUT $\bar{B} = \bar{\nabla} \times \bar{A} = 0$ ALONG THE PATH. THEN THE LORENTZ FORCE VANISHES SO THE PARTICLE'S MOTION IS NOT AFFECTED (CLASSICALLY). BUT ITS WAVE FUNCTION BECOMES

$$\psi(x) = e^{iS}\psi_0 \qquad \text{WHERE} \qquad S = e\int^x d\bar{x}\cdot\bar{A}$$

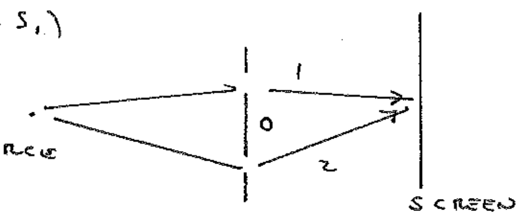
ψ IS READILY VERIFIED TO BE A SOLUTION OF THE SCHRÖDINGER EQUATION OF AN INTERACTING CHARGE

$$\frac{(-i\bar{\nabla} - e\bar{A})^2}{2m}\psi = i\frac{\partial\psi}{\partial t} \qquad \text{NOTING } \bar{\nabla}\cdot\bar{A} = 0$$

THEN IN THE DOUBLE SLIT EXPERIMENT WITH $\bar{A} = 0$ WE HAVE $\psi_{screen} = \psi_{10} + \psi_{20}$ ADDING THE WAVE FUNCTIONS FOR THE 2 PATHS.

WHEN $\bar{A} \neq 0$, $\psi_{screen} \sim \psi_{10} + \psi_{20}\, e^{i(S_2 - S_1)}$

WHERE $S_2 - S_1 = e\int_{LOOP} d\bar{x}\cdot\bar{A} = e\,\Phi_{MAG} \neq 0$



THE PHASE DIFFERENCE $S_2 - S_1$ ALLOWS QUANTITATIVE PREDICTION OF THE SHIFT OF THE INTERFERENCE PATTERN, AS HAS BEEN VERIFIED EXPERIMENTALLY [CHAMBERS, P.R.L. 5, 3 (1960)].

IF WE ACCEPT THAT THE BASIS OF THE ELECTROMAGNETIC FIELD CONCEPT IS TO ELIMINATE ACTION AT A DISTANCE, THEN CLEARLY THE PARTICLE INTERACTS DIRECTLY WITH $\bar{A}$ AND NOT $\bar{E}$ OR $\bar{B}$ IN THE AHARANOV-BOHM EXPERIMENT. THIS INSIGHT IS A RESULT OF QUANTUM MECHANICS; IT REMAINS THAT ALL CLASSICAL EFFECTS CAN BE UNDERSTOOD IN TERMS OF LOCAL $\bar{E}$ AND $\bar{B}$ FIELDS WITHOUT EXPLICIT MENTION OF $\bar{A}$ IF SO DESIRED.

5. YANG-MILLS GAUGE THEORY

THE FIRST APPLICATION OF THE LOCAL GAUGE INVARIANCE CONCEPT OUTSIDE ELECTROMAGNETISM WAS MADE BY YANG & MILLS [P.R. 96, 196 (1954)]. THEY WERE INTERESTED IN THE NUCLEAR FORCE, WHICH WAS KNOWN TO BE 'CHARGE INDEPENDENT'. MORE PRECISELY THE SYMMETRY OF ISOSPIN SEEMS TO BE WELL OBSERVED IN THE STRONG INTERACTION. PERHAPS ISOSPIN INVARIANCE PLAYS A ROLE IN THE STRONG INTERACTION SIMILAR TO THAT OF PHASE INVARIANCE IN THE CASE OF E&M.

SO FAR AS THE STRONG INTERACTION IS CONCERNED THE DISTINCTION BETWEEN A NEUTRON AND PROTON SEEMS SOMEWHAT ARBITRARY. (THE YEAR IS 1954; QUARK FLAVOR WAS NOT YET BEEN POSTULATED) IF WE IGNORE CHARGE WE COULD INTERCHANGE LABELS OF PROTON AND NEUTRON AND OUR DISCUSSION OF EXPERIMENTAL RESULTS ON NUCLEON-NUCLEON SCATTERING WOULD NOT BE AFFECTED IN ANY FUNDAMENTAL WAY. IF WE ARE FREE TO LABEL THE 2 KINDS OF NUCLEONS AS PROTON AND NEUTRON WHICHEVER WAY WE LIKE, THEN SOMEONE ELSE AT ANOTHER

PLACE OUGHT TO HAVE SUCH FREEDOM ALSO. BUT IN ORDER TO OBTAIN CONSISTENT RESULTS IN BOTH PLACES, THE FORM OF THE NUCLEAR INTERACTION IS NOT ARBITRARY – IT MUST SATISFY BOTH ISOSPIN INVARIANCE AND SOME FORM OF GAUGE INVARIANCE. UNDERSTANDING THIS CONSTRAINED FORM OF THE NUCLEAR FORCE WAS THE GOAL OF YANG & MILLS.

A FIRST STEP IS TO CAST ISOSPIN INVARIANCE INTO A FORM SIMILAR TO PHASE INVARIANCE OF THE WAVE FUNCTION. WE AGAIN INTRODUCE THE ISOSPIN DOUBLET: $\psi = \begin{pmatrix} p \\ n \end{pmatrix}$. AND CONSIDER UNITARY TRANSFORMATION OF THIS

$$\psi' = U\psi \quad \text{or} \quad \begin{pmatrix} p' \\ n' \end{pmatrix} = U\begin{pmatrix} p \\ n \end{pmatrix} \qquad \text{WHERE } U \text{ IS A } 2\times2 \text{ MATRIX}$$

[AS IN LECTURE 10, P178 WE CAN RESTRICT OURSELVES TO THE $SU(2)$ SUBGROUP OF TRANSFORMATIONS WHICH HAVE $\det U = 1$]

A STANDARD PROCEDURE IN GROUP THEORY IS TO CHARACTERIZE TO FORM OF U BY FIRST CONSIDERING TRANSFORMATIONS WHICH DIFFER ONLY SLIGHTLY FOR THE UNIT MAXTRIX

$$U = 1 + i\epsilon \qquad \text{WHERE } 1 \text{ AND } \epsilon \text{ AND } 2\times2 \text{ MATRICES}$$

MATRIX ϵ CAN ALWAYS BE WRITTEN AS $\vec{\epsilon}\cdot\vec{\sigma}$

WHERE $\vec{\epsilon} = (\epsilon_1, \epsilon_2, \epsilon_3)$ = 3 SMALL NUMBERS ; σ_i ARE THE PAULI SPIN MATRICES

THE GENERAL TRANSFORMATION U CAN BE BUILT UP OUT OF PRODUCTS OF THE 'INFINITESIMAL' TRANSFORMATION $1 + i\vec{\epsilon}\cdot\vec{\sigma}$

$$U = (1 + i\vec{\epsilon}\cdot\vec{\sigma})^n = e^{i n \vec{\epsilon}\cdot\vec{\sigma}} \equiv e^{i\vec{\alpha}\cdot\vec{\sigma}}$$

IT IS WORTH RECALLING THE VIEW THAT LABELS P AND N REFER TO DIRECTIONS OF A POINTER IS AN INTERNAL 3-DIMENSIONAL SPACE ATTACHED TO THE NUCLEON.

POINTER UP $\rightarrow$ P

POINTER DOWN $\rightarrow$ N

THE GENERAL TRANSFORMATION U IS THOUGHT OF AS A KIND OF ROTATION BY ANGLES α_1, α_2 AND α_3 ABOUT THE 3 AXES OF THE INTERNAL SPACE. THE USUAL TECHNICALITY WITH ROTATIONS IS THAT THE ORDER OF ROTATIONS ABOUT AXES 1, 2, & 3 MATTERS. THAT IS

$$\sigma_i\sigma_j = \sigma_j\sigma_i + 2\epsilon_{ijk}\sigma_k \qquad \text{or} \qquad [\sigma_i, \sigma_2] = 2i\,\epsilon_{ijk}\sigma_k$$

PEOPLE SAY THAT THE $SU(2)$ GROUP IS 'NON-ABELIAN' MEANING EXACTLY THAT THE 'GENERATORS' σ_i OF THE ROTATIONS ARE NON-COMMUTATIVE. THE THEORY THAT YANG & MILLS PROCEEDED TO CONSTRUCT IS THEREBY CALLED A 'NON-ABELIAN GAUGE THEORY'.

THE NEXT STEP IS TO REQUIRE THE THEORY TO BE INVARIANT UNDER LOCAL PHASE CHANGES (= ROTATIONS IN ISOSPIN SPACE)

SUCH AS $\psi \to e^{ig\bar{\alpha}(x)\cdot\bar{\sigma}}\psi$

WE HAVE INTRODUCED A CONSTANT FACTOR g WHICH WE ANTICIPATE WILL PLAY A ROLE SIMILAR TO THAT OF CHARGE IN THE E & M CASE. THE SUGGESTION IS THAT THE PHASE INVARIANCE REQUIRES THE EXISTENCE OF A NUCLEAR INTERACTION WHICH MODIFIES THE VARIATION OF THE DEPENDENCE OF NUCLEON WAVE FUNCTION ON POSITION. FORMALLY WE KEEP TRACK OF THIS MOST SIMPLY BY ALTERING THE DERIVATIVE OPERATOR

$$\partial_\mu \to D_\mu = \partial_\mu + ig\bar{A}_\mu\cdot\bar{\sigma} = \partial_\mu + ig A_\mu^i \sigma_i$$

(THROUGHOUT THE REMAINING DISCUSSION, INDICES μ & ν REFER TO ORDINARY SPACE, WHILE i & j REFER TO ISOSPIN SPACE. A BAR OVER A SYMBOL, SUCH AS $\bar{\sigma}$ REFERS TO A 3-VECTOR IN ISOSPIN SPACE)

IN THE DEFINITION OF D_μ YANG & MILLS INTRODUCE 3 DIFFERENT VECTOR POTENTIALS A^1_μ, A^2_μ AND A^3_μ

THESE PLAY A DUAL ROLE. THEY ARE THE POTENTIALS OF THE NUCLEAR FORCE FIELD IN ORDINARY SPACE, AND THEY COUPLE TO 3 PARTICULAR TRANSFORMATIONS IN ISOSPIN SPACE

WITH SOME EFFORT WE CAN VERIFY THAT PHASE INVARIANCE HOLDS FOR THE INFINITESIMAL TRANSFORMATION $e^{ig\bar{\epsilon}\cdot\bar{\sigma}} \sim 1 + ig\bar{\epsilon}\cdot\bar{\sigma}$

PROVIDED THE POTENTIALS A^i_μ OBEY A SPECIFIC GAUGE INVARIANCE RELATION. NAMELY

IF $\psi \to \psi' = (1 + ig\bar{\epsilon}\cdot\sigma)\psi$

AND $\bar{A}_\mu \to \bar{A}'_\mu = \bar{A}_\mu - \partial_\mu\bar{\epsilon} - g\bar{\epsilon}\times\bar{A}_\mu$

THEN THE NEW $D'_\mu = \partial_\mu - ig\bar{A}'_\mu\cdot\bar{\sigma}$ SATISFIES

$$D'_\mu\psi' = (1 + ig\bar{\epsilon}\cdot\bar{\sigma})D_\mu\psi$$

ACCORDING TO THE ANALYSIS IN SECTION 3 THE LAST RELATION IS WHAT IS NEEDED FOR THE PHASE INVARIANCE TO HOLD. THAT IS, $\psi^*\psi$, AND $\psi^* D_\mu\psi$ ETC ARE INVARIANT ....

[TO SHOW THE ABOVE RELATIONS ARE CONSITENT RECALL THAT
$$(\bar{a}\cdot\bar{\sigma})(\bar{b}\cdot\bar{\sigma}) = (\bar{a}\cdot\bar{b}) + i\,\bar{\sigma}\cdot\bar{a}\times\bar{b} \qquad (P\ 88)$$
]

SEE SEC. 8.5 OF THE BOOK OF AITCHISON & HEY FOR DETAILS...

THE POTENTIALS A^i_μ LEAD TO A DESCRIPTION OF THE NUCLEAR FORCE FIELD IN ORDINARY SPACE. CHAPTERS 3-5 OF THE BOOK BY MORIYASU EXPLORE THE NATURE OF THESE FIELDS IN SOME DETAIL USING A GEOMETRIC APPROACH. HERE WE SIMPLY TAKE A SUGGESTIVE SHORT-CUT. SOMEONE NOTICED THAT WE CAN EXPRESS THE ELECTROMAGNETIVE FIELD TENSOR $F_{\mu\nu}$ INTERMS OF THE GAUGE COVARIANT DERIVATIVE:

$$\begin{aligned} F_{\mu\nu} &= \frac{1}{ie}\left[D_\nu, D_\mu\right] \\ &= \frac{1}{ie}\left[\partial_\nu + ieA_\nu, \partial_\mu + ieA_\mu\right] \\ &= \partial_\nu A_\mu - \partial_\mu A_\nu + ie\left[A_\nu, A_\mu\right] \end{aligned}$$

FOR E & M THE LAST TERM CONVENIENTLY VANISHES.

IN THE YANG-MILLS CASE THERE ARE 3 KINDS OF FIELDS $F^i_{\mu\nu}$ $i = 1,2,3$ EACH ACTING A DIFFERENT WAY IN ISOSPIN SPACE, IN ADDITION TO THE MORE USUAL FIELD BEHAVIOR IN ORDINARY SPACE. WE MAKE THE 'GUESS'

$$\begin{aligned} \sum_i F^i_{\mu\nu}\sigma_i &= \frac{1}{ig}\left[D_\nu, D_\mu\right] \\ &= \frac{1}{ig}\left[\partial_\nu + igA^i_\nu\sigma_i, \partial_\mu + igA^j_\mu\sigma_j\right] \\ &= \partial_\nu A^i_\mu\sigma_i - \partial_\mu A^i_\nu\sigma_i + igA^i_\nu A^j_\mu \underbrace{\left[\sigma_i, \sigma_j\right]}_{2i\epsilon_{ijk}\sigma_k} \end{aligned}$$

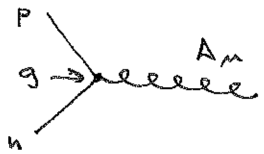
FOR A SINGLE ISOSPIN FIELD WE HAVE

$$F^i_{\mu\nu} = \partial_\nu A^i_\mu - \partial_\mu A^i_\nu - 2g\,\epsilon_{ijk}A^j_\nu A^k_\mu$$

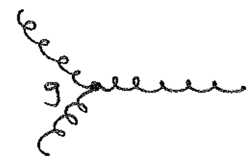
THIS SKETCH ALREADY REVEALS ONE IMPORTANT FEATURE OF THE NON-ABELIAN GAUGE THEORY. THE FIELDS $F^i_{\mu\nu}$ DON'T OBEY THE SUPERPOSITION PRINCIPLE, IN THAT THE 3RD TERM ABOVE IS A PRODUCT OF THE POTENTIALS. HENCE THE FIELDS $F^i_{\mu\nu}$ ARE NOT IMMEDIATELY INTUITIVE IN TERMS OF OUR EXPERIENCE WITH CLASSICAL E & M.

IN THE QUANTISED VERSION OF THE FIELD THEORY WE DESCRIBE THE EXTRA COMPLEXITY BY SAYING THAT THE QUANTA OF A^i_μ CARRY CARRY THE CHARGE g WHICH CAUSES THE NUCLEAR FORCE BETWEEN n AND p. THAT IS, THE FIELD ACTS AS A SOURCE OF MORE FIELD, A NON-LINEAR EFFECT.

ONE DRAWS DIAGRAMS IN THE QUANTISED FIELD CASE:



AND ALSO



THE 2ND TYPE OF DIAGRAM DOES NOT OCCUR IN E & M ACCORDING TO CHARGE CONJUGATION INVARIANCE (p.172)

WE NOTE THAT THE FIRST DIAGRAM IMPLIES THAT THE QUANTA OF THE STRONG FORCE MUST CARRY ELECTRIC CHARGE. THIS MAY OR MAY NOT BE A GOOD THING. IT WOULD SUGGEST SOME SORT OF UNITY BETWEEN THE STRONG AND ELECTROMAGNETIC FORCES...

HOWEVER THERE IS ONE SERIOUS DRAWBACK OF THE ORIGINAL YANG-MILLS THEORY. THE QUANTA OF THE FIELD MUST BE MASSLESS TO MAINTAIN THE GAUGE INVARIANCE OF THE FIELD. THIS IS NOT SELF EVIDENT FROM THE PRECEDING DISCUSSION. ONE APPEALS TO THE HAMILTONIAN OR LAGRANGIAN OF THE NUCLEAR FIELD WHICH INCLUDES A TERM $F_{\mu\nu}F^{\mu\nu}$ DUE TO THE FIELD ENERGY, AND APPARENTLY

A TERM $M^2 A_\mu A^\mu$ IF THE QUANTA HAVE MASS. WHILE $F_{\mu\nu}F^{\mu\nu}$ IS INVARIANT UNDER THE GAUGE TRANSFORMATION MENTIONED ABOVE, THE TERM $A_\mu A^\mu$ CLEARLY IS NOT. BUT IF THE QUANTA ARE MASSLESS THE FORCE WILL BE LONG RANGE - QUITE UNLIKE EXPERIMENTAL FACT....

THE YANG-MILLS THEORY SHOWED HOW LOCAL INVARIANCE PRINCIPLES MIGHT ALLOW ONE TO GUESS THE FORM OF PARTICLE INTERACTIONS. BUT A MAJOR MYSTERY REMAINS HOW TO PRODUCE A SHORT RANGE INTERACTION IN A GAUGE INVARIANT THEORY.