## The Glashow - Weinberg - Salam Model

[This lecture largely follows the book of Aitchison & Hey]

### 1. Is high energy physics the same as solid state physics?

We remarked at the end of lecture 20 on a difficulty with the gauge theory approach. It appears that the quantum of the gauge invariant field must be massless, and so such theories could only apply to long range interactions like electromagnetism. We can display this difficulty semi-classically if we consider the wave equation for the vector potential:

$$\Box A_\mu - \partial_\mu (\partial_\nu A^\nu) = J_\mu \qquad \left(\Box \equiv \partial_\mu \partial^\mu\right)$$

Here we have not yet applied the usual gauge condition $\partial_\nu A^\nu = 0$. This wave equation is readily seen to be invariant under the gauge transformation $A_\mu \to A_\mu' = A_\mu + \partial_\mu \Omega$

For the case that the quantum of the field has mass we expect a wave equation of the form given by Yukawa (p.23)

$$(\Box + M^2) A_\mu - \partial_\mu (\partial_\nu A^\nu) = J_\mu$$

But now if we apply the gauge transformation the term $M^2 \partial_\mu \Omega$ spoils the invariance of the wave equation.

Anderson [P.R. 130, 149 (1963)] noted how in the phenomenon of superconductivity the interaction of the electromagnetic field with the Cooper pairs makes it appear that the photon has mass. Yet the theory of E&M remains gauge invariant. But the invariance principle becomes 'hidden', or 'spontaneously broken.'

These arguments provided an important clue for high energy physics, and mark a kind of conceptual unification of solid state physics and high energy physics. In the latter the vacuum takes on an analagous role to the ground state of a solid — which may be regarded as a revival of the notion of the ether. This time around the ether has the job of making the quanta of a gauge invariant theory appear to have a rather large mass.

We recall a few features of superconductivity. In a metal at low temperatures the conduction electrons form 'Cooper pairs' due to the very weak attractive potential provided by the lattice ions. The separation between the 2 electrons of a pair is quite large, $\sim 10^{-4}$ cm, and is certainly much larger than the separation between electrons belonging to different pairs. Then it is possible for the wave functions of a large number of pairs to be correlated, resulting in a coherent, macroscopic wave function describing all of the conduction electrons. The quantum mechanical

PROBABILITY CURRENT OF THE PAIRS MANIFESTS ITSELF AS AN ELECTRICAL CURRENT WHICH FLOWS WITHOUT APPARENT DRIVING FIELDS — THE SUPERCONDUCTING PHENOMENON.

SUPPOSE WE APPLY AN EXTERNAL MAGNETIC FIELD TO A SUPERCONDUCTING BLOCK OF METAL. IN THE STATIC LIMIT THE WAVE EQUATION FOR THE VECTOR POTENTIAL $\bar{A}$ REDUCES TO $\nabla^2 \bar{A} = -\bar{J}$ IF WE MAKE THE SPECIFIC CHOICE OF GAUGE CONDITION $\bar{\nabla} \cdot \bar{A} = 0$. THE PROBABILITY CURRENT FOR THE COOPER PAIRS, WHICH ARE BOSONS, CAN BE WRITTEN AS THAT DISCUSSED ON p. 174

$$J_\mu = i \left( \psi^* \partial_\mu \psi - (\partial_\mu \psi)^* \psi \right)$$

TO PRESERVE GAUGE INVARIANCE WE REPLACE $\partial_\mu$ BY $D_\mu = \partial_\mu + i e A_\mu$ AS DISCUSSED IN LECTURE 20. THEN THE ELECTRICAL CURRENT IS

$$\bar{J} = -\frac{ie}{2m} \left( \psi^* \bar{\nabla} \psi - (\bar{\nabla} \psi)^* \psi \right) - \frac{e^2}{m} \bar{A} |\psi|^2$$

WE HAVE RESTORED A NORMALISATION FACTOR $\sqrt{2m}$ FOR THE WAVE FUNCTION IN THE NON-RELATIVISTIC CASE. IF WE WRITE

$$\psi = |\psi| e^{i\theta} \qquad \text{THEN } |\psi| \sim \text{CONSTANT OVER THE SUPERCONDUCTOR}$$

AND $$\bar{J} = \frac{e}{m} |\psi|^2 \left( \bar{\nabla} \theta - e \bar{A} \right)$$

FOR CURRENT CONSERVATION TO HOLD, $\bar{\nabla} \cdot \bar{J} = 0$, SO THAT WITH $\bar{\nabla} \cdot \bar{A} = 0$ WE MUST HAVE $\bar{\nabla} \theta = $ CONSTANT, WHICH WE PLAUSIBLY SET TO ZERO FOR REASONABLE BOUNDARY CONDITIONS. NOW WE HAVE

$$\bar{J} = -\frac{e^2 |\psi|^2}{m} \bar{A} \quad \text{AND SO} \quad \nabla^2 \bar{A} = -\bar{J} = M^2 \bar{A} \quad \text{WITH} \quad M^2 = e^2 |\psi|^2/m$$

THIS IS EXACTLY THE FORM OF THE EQUATION FOR A STATIC YUKAWA POTENTIAL CORRESPONDING TO FIELD QUANTA OF MASS $M$.

THE PHYSICAL CONTENT OF THE EQUATION FOR $\bar{A}$ IS USUALLY ILLUSTRATED BY CONSIDERING THE MAGNETIC FIELD $\bar{B} = \bar{\nabla} \times \bar{A}$. THEN AMPERE'S LAW $\bar{\nabla} \times \bar{B} = \bar{J} \implies -\nabla^2 \bar{B} = \bar{\nabla} \times (\bar{\nabla} \times \bar{B}) = \bar{\nabla} \times \bar{J} = -M^2 \bar{\nabla} \times \bar{A} = -M^2 \bar{B}$

OR $\nabla^2 B = M^2 B \implies \bar{B}(x) = \bar{B}_0 e^{-Mx}$ INSIDE A SUPERCONDUCTOR WHICH OCCUPIES $x > 0$. THAT IS, $\bar{B}$ IS EXCLUDED FROM THE INTERIOR OF THE SUPERCONDUCTOR. (MEISSNER EFFECT).

MORE IMPORTANT FOR US IS THE VECTOR POTENTIAL $\bar{A}$. THE KEY RESULT IS THAT THE GAUGE INVARIANT FORM OF THE CURRENT OF THE MACROSCOPIC WAVE FUNCTION $\psi$ HAS THE FORM $\bar{J} = -M^2 \bar{A}$. THIS IMPLIES THE WAVE EQUATION FOR $\bar{A}$ IS NOW

$$(\Box + M^2) \bar{A} = 0 \qquad \text{IN THE TIME VARYING CASE. THE PHOTON THEREFORE}$$

HAS APPARENT MASS $M$. WE SHOULD NOTE THAT OUR ANALYSIS WAS MADE WITH A SPECIAL CHOICE OF GAUGE: $\bar{\nabla} \cdot \bar{A} = 0$, SO WE MIGHT WONDER IF OUR

INTERPRETATION IS TRULY GAUGE INVARIANT. THE MECHANISM FOR GENERATING THE PHOTON MASS 'HIDES' THE GAUGE INVARIANCE IN A CERTAIN SENSE.

## 2. THE HIGGS MECHANISM AND HIDDEN GAUGE INVARIANCE

FOLLOWING HIGGS [P.R.L. 13, 508 (1964)] WE CAN GIVE A RELATIVISTIC VERSION OF THE ARGUMENT OF ANDERSON. WE WISH TO DEPART FROM THE CASE OF E&M TO CONSIDER OTHER FIELDS DESCRIBED BY A VECTOR POTENTIAL $A_\mu$, BUT WE SUPPOSE THE FORMALISM REMAINS SIMILAR.

TO GENERATE MASS FOR THE FIELD QUANTUM THERE MUST BE A 'BACKGROUND' WAVE FUNCTION $\phi$ WHICH FILLS THE UNIVERSE, AND INTERACTS WITH POTENTIAL $A_\mu$ VIA THE WAVE EQUATION

$$\Box A_\mu = -J_\mu \qquad \text{(WE ENFORCE THE GAUGE CONDITION } \partial_\nu A^\nu = 0\text{)}$$

THE CURRENT WHICH DESCRIBES THE INTERACTION IS

$$J_\mu = ig\left(\phi^* \partial_\mu \phi - (\partial_\mu \phi)^* \phi\right) - 2g^2 |\phi|^2 A_\mu$$

WHERE WE HAVE MADE THE REPLACEMENT $\partial_\mu \to D_\mu = \partial_\mu + ig A_\mu$ IN THE EXPRESSION ON p. 373.

$g \equiv$ 'CHARGE' OF THE POSTULATED INTERACTION

WE ALSO SUPPOSE THAT THE WAVE FUNCTION $\phi$ IS UNIFORM: $\phi = \dfrac{f}{\sqrt{2}} = \text{CONSTANT}$

THEN $\qquad J_\mu = -g^2 f^2 A_\mu = \Box A_\mu$

WE CAN THEN IDENTIFY THE MASS OF THE QUANTUM OF $A_\mu$ AS $\underline{M = gf}$.

IN THE VIEW OF HIGGS THE VACUUM IS NOT EMPTY, BUT MUST BE FILLED WITH THE CONSTANT SCALAR FIELD $\phi$. WE WILL NOT SUCCEED IN GIVING A PRECISE PHYSICAL INTERPRETATION TO $\phi$, BUT IT CERTAINLY INDICATES THAT THE 'VACUUM' IS AN ENTITY WITH CONSIDERABLE STRUCTURE.

FOR POSSIBLE COMFORT WE AGAIN REFER TO SOLID STATE PHYSICS AND REMARK ON THE FERROMAGNET. AT LOW TEMPERATURE THE UNPAIRED ELECTRON SPINS LINE UP ALONG SOME DIRECTION. THAT DIRECTION IS IN PRINCIPLE ARBITRARY ACCORDING TO ROTATIONAL INVARIANCE — BUT ONLY ONE DIRECTION IS POSSIBLE AT A TIME. IF YOU CANNOT HEAT UP THE FERROMAGNET, OR APPLY AN EXTERNAL $\vec{B}$ FIELD, THE ROTATIONAL INVARIANCE OF THE SYSTEM IS LESS THAN OBVIOUS.

HIGGS SUGGESTS SOMETHING SIMILAR FOR OUR VACUUM. WE WOULD LIKE TO CONFIRM THAT THE VACUUM IS REALLY GAUGE INVARIANT, ALTHOUGH THE PARTICULAR VALUE FOR THE FIELD $\phi$ OBSCURES THIS.

WE RECALL FROM LECTURE 20 THAT IF WE WISH PHYSICS TO BE INVARIANT UNDER THE GAUGE TRANSFORMATION

$$A_\mu \rightarrow A_\mu' = A_\mu + ig\,\partial_\mu \Omega$$

THEN THE WAVE FUNCTION $\phi$ OF THE SCALAR FIELD MUST ALSO TRANSFORM AS

$$\phi \rightarrow \phi' = e^{-ig\Omega}\phi$$

A PICTURE OF HOW THIS CAN BE CONSISTENT WITH THE HIGGS VACUUM IS PROVIDED BY YET ANOTHER ANALOGY TO SOLID STATE PHYSICS — THE GINZBURG-LANDAU MODEL. WE SUPPOSE THAT THE TOTAL INTERACTION OF THE FIELD $\phi$ WITH THE REST OF THE UNIVERSE CAN BE DESCRIBED BY A POTENTIAL

$$V(\phi) = -\frac{1}{2}\mu^2 |\phi|^2 + \frac{1}{2}\frac{\mu^2}{f^2}|\phi|^4$$

THIS INTERACTION IS INDEPENDENT OF THE PHASE OF $\phi$, ALLOWING THE LOCAL PHASE INVARIANCE NEEDED TO BE CONSISTENT WITH GAUGE INVARIANCE FOR $A_\mu$.

THE STATE $\phi = 0$ IS AN EXTREME OF THE POTENTIAL $V$, BUT IT IS NOT THE LOWEST ENERGY STATE WHICH WE IDENTIFY WITH THE VACUUM. WE HAVE AN INFINITE NUMBER OF CHOICES FOR THE VACUUM STATE AT THE POTENTIAL MINIMUM:

$$|\phi| \equiv \frac{f}{\sqrt{2}} = \text{CONSTANT}$$

PHASE OF $\phi$ ARBITRARY.

THE VACUUM IS PHASE INVARIANT, BUT $\phi$ MUST TAKE ON ONE PARTICULAR PHASE, WHICH 'SPONTANEOUSLY BREAKS' THE PHASE SYMMETRY.

$$\phi = \phi_1 + i\phi_2$$
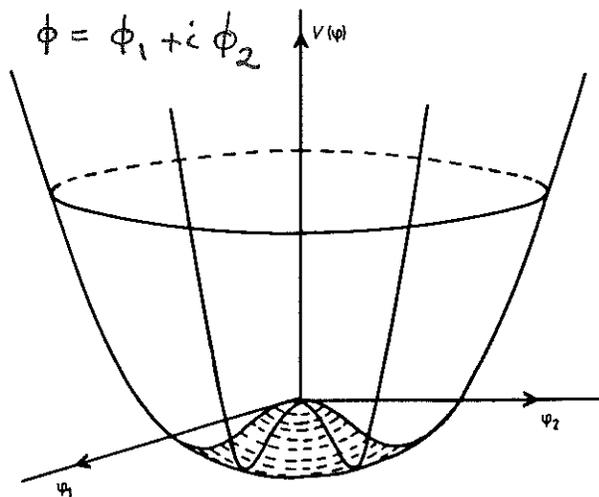


Figure 9.2 The Higgs potential

IN FACT WE CAN USE THE PHASE AND GAUGE INVARIANCE TO REDEFINE THE PHASE OF $\phi$ TO BE ZERO. FOR EXAMPLE, SUPPOSE

$$\phi = \frac{f}{\sqrt{2}} e^{ig\Omega(x)}$$

WHERE THE PHASE $\Omega(x)$ EVEN VARIES FROM PLACE TO PLACE. THE 'VACUUM CURRENT' ASSOCIATED WITH THIS FIELD IN ITS INTERACTION WITH $A_\mu$ IS

$$J_\mu = ig\left(\phi^* D_\mu \phi - (D_\mu \phi)^* \phi\right)$$

$$= ig\left(\phi^* \partial_\mu \phi - (\partial_\mu \phi)^* \phi\right) - 2g^2 |\phi|^2 A_\mu = -g^2 f^2 \left(A_\mu + \partial_\mu \Omega\right)$$

$A_\mu$ SATISFIES THE FULL WAVE EQUATION

$$\Box A_\mu - \partial_\mu(\partial_\nu A^\nu) = J_\mu = -g^2 f^2\left(A_\mu + \partial_\mu \Omega\right)$$

WHERE WE HAVEN'T YET MADE ANY PARTICULAR CHOICE OF GAUGE.

Then, as on p. 364, the physics is invariant under the combined transformations

$$\phi \rightarrow \phi' = e^{-ig\Omega}\phi = \frac{f}{\sqrt{2}}$$

$$A_\mu \rightarrow A_\mu' = A_\mu - \partial_\mu \Omega$$

So we find $\quad \Box A_\mu' - \partial_\mu(\partial_\nu A'^\nu) = -g^2 f^2 A_\mu'$

or $\quad (\Box + M^2) A_\mu' = \partial_\mu(\partial_\nu A'^\nu) \qquad$ with $M \equiv gf$

If we take the divergence of this equation we find

$$(\Box + M^2)(\partial^\mu A_\mu') = \Box(\partial_\nu A'^\nu) \qquad \Rightarrow \qquad \partial^\mu A_\mu' = 0$$

So in fact we now have $\quad (\Box + M^2) A_\mu' = 0$

The Higgs procedure may be summarized: We postulate a background scalar field whose wave function takes on a finite value in the vacuum state. The gauge invariance of the interaction of this field with the vector potential $A_\mu$ allows us to choose the phase of $\phi$ to be zero. In exchange we find that the quantum of field $A_\mu$ takes on mass

$$M = gf \qquad \text{where } g = \text{'charge' and } \phi = \frac{f}{\sqrt{2}}.$$

This prescription indicates how a gauge invariant interaction can have massive quanta and hence be a short range interaction.

3. $\underline{SU(2)_L \times U(1)_Y}$ UNIFICATION

In 1961 Glashow [Nuc. Phys. 22, 579 (1961)] suggested a multiplet scheme for leptons that provided a framework for the unification of the weak and electromagnetic interactions. After quarks were postulated in 1964 it was noted that they also fit into this scheme (although the details involving the Cabibbo angle were not worked out until 1970 by G.I.M.)

We have already remarked on the doublet organization of the weak interaction of quarks and leptons. In the V-A theory only left-handed particles interact, and weak transitions occur only between particular pairs of particles. Glashow assigned the <u>weak isospin</u> quantum number to the left-handed doublets as follows

$$I = \frac{1}{2} \qquad I_3 = \begin{cases} \frac{1}{2} \\ -\frac{1}{2} \end{cases} \qquad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L \quad \cdots$$

The weak isospin is not physically related to isospin of the strong interaction, but of course the formal algebra is the same. Right handed particles do not interact weakly. These particles were assigned to a series of weak isospin singlets:

$$I = 0, \quad I_3 = 0 : \quad \bar{e}_R, \quad \bar{\mu}_R, \quad u_R, \quad d_{\theta R}, \quad c_R, \quad s_{\theta R} \quad \cdots$$

WHILE THIS ORGANIZATION IS A USEFUL REMINDER OF THE WAY
THE V-A INTERACTION WORKS, IT CANNOT BE AN EXACT SYMMETRY,
AS THE MEMBERS OF A GIVEN WEAK ISOSPIN MULTIPLET HAVE QUITE
DIFFERENT MASSES. NONETHELESS GLASHOW FELT THERE WAS ENOUGH
INTEREST IN THE WEAK ISOSPIN SCHEME TO TRY TO RELATE IT TO ELECTRICITY.
HE NOTED THAT EACH MULTIPLET COULD BE ASSIGNED AN ADDITIONAL
QUANTUM NUMBER  $Y \equiv$ WEAK HYPERCHARGE  SUCH THAT ELECTRICAL
CHARGE IS GIVEN BY

$$Q = e\left(I_3 + \frac{Y}{2}\right)$$

THIS RECALLS THE GELL-MANN-NISHIJIMA RELATION OF $SU(3)$   (p.189)

THE TABLE SUMMARIZES THE
ASSIGNMENTS OF $Y$ NEEDED TO MAKE
THE ARITHMETIC CONSISTENT.

WE SAY THAT ISOSPIN IS ASSOCIATED
WITH AN $SU(2)$ GROUP OF TRANSFORMATIONS
IN A 3 DIMENSIONAL INTERNAL ISOSPIN
SPACE. HYPERCHARGE IS ASSOCIATED
WITH A $U(1)$ GROUP OF TRANSFORMATIONS
IN A ONE DIMENSIONAL INTERNAL SPACE

$$\phi \rightarrow U\phi = e^{i\alpha Y}\phi \qquad 0 \le \alpha \le 2\pi$$

**Table 11.1** Weak isospin and hypercharge assignments for fermions.

|  | $I$ | $I_3$ | $y$ | $Q$ |
|---|---|---|---|---|
| $\nu_e, \nu_\mu$ | 1/2 | 1/2 | −1 | 0 |
| $e_L, \mu_L$ | 1/2 | −1/2 | −1 | −1 |
| $e_R, \mu_R$ | 0 | 0 | −2 | −1 |
| $u_L, c_L$ | 1/2 | 1/2 | 1/3 | 2/3 |
| $(d_C)_L, (s_C)_L$ | 1/2 | −1/2 | 1/3 | −1/3 |
| $u_R, c_R$ | 0 | 0 | 4/3 | 2/3 |
| $(d_C)_R, (s_C)_R$ | 0 | 0 | −2/3 | −1/3 |

SO GLASHOW'S MULTIPLET SCHEME IS OFTEN CALLED $SU(2)_L \times U(1)_Y$

WHAT GOOD DOES THIS FANCY TERMINOLOGY DO FOR US? OUR GOAL
IS TO GO BEYOND CLASSIFICATION SCHEMES TO AN UNDERSTANDING OF THE
DYNAMICS OF THE WEAK INTERACTION. IN 1961 IT WAS NOT YET CLEAR HOW
TO DO THIS. THERE WERE SUGGESTIONS THAT THE $SU(2)_L$ GROUP
IMPLIES THE EXISTENCE OF 3 WEAK BOSONS $W^+, W^-$ AND $W^0$, ALL
OBEYING V-A COUPLING (BLUDMAN, 1958); OR THAT THE $W^0$ BOSON
IS REALLY THE PHOTON - WHICH WOULD UNIFY THE WEAK AND E&M INTERACTIONS
(SCHWINGER, 1957). THE MORE SUCCESSFUL UNDERSTANDING COMES
FROM WEINBERG [P.R.L. <u>19</u>, 1264 (1967)] AND SALAM [p367, ELEMENTARY
PARTICLE THEORY, ED. BY SVARTHOLM, STOCKHOLM (1968)] WHO
ASSOCIATED 4 BOSONS WITH THE $SU(2)_L \times U(1)_Y$ SCHEME OF
GLASHOW. INVOKING THE HIGGS MECHANISM THEY SHOWED HOW 3
OF THESE BOSONS CAN BE MASSIVE WHILE THE 4RTH REMAINS MASSLESS,
AND THE WHOLE THEORY IS GAUGE INVARIANT IN THE BARGAIN.

## 4. THE STANDARD MODEL

THE PRESCRIPTION OF WEINBERG AND SALAM COMBINES THE
VARIOUS PROCEDURES WE HAVE SKETCHED IN THE PREVIOUS $1\frac{1}{2}$ LECTURES.
WE FIRST REQUIRE THAT THE WAVE FUNCTION OF A STATE      OF
WEAK ISOSPIN AND HYPERCHARGE SHOULD OBEY LOCAL PHASE INVARIANCE.
SO WE CONSIDER TRANSFORMATIONS:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \rightarrow \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L' = e^{\frac{ig\vec{\alpha}\cdot\vec{\tau}}{2}} e^{\frac{ig'\theta Y}{2}} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

ACTS ON WEAK ISOSPIN SPACE                    ACTS ON WEAK HYPERCHARGE
                                                          SPACE

We anticipate that the constants $g$ and $g'$ act as the 'charges' of the weak isospin and weak hypercharge interactions.

For local phase invariance to hold we must have a gauge invariant interaction which can affect the 'phases' $\bar{\chi}$ and $\Theta$. We characterize this interaction by the way it modifies the derivative operator to become the gauge covariant derivative. From Lecture 20 we can 'guess' that the desired form is

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu + i\frac{g}{2} W_\mu^i \sigma^i + i\frac{g'}{2} Y B_\mu$$

We introduce four potentials $W_\mu^1$, $W_\mu^2$, $W_\mu^3$ and $B_\mu$. These describe fields in ordinary space, as well as particular interactions with the internal spaces of isospin and hypercharge. The factors $\sigma^i$ and $Y$ indicate the various couplings of the potentials to the members of the $SU(2)_L \times U(1)_Y$ multiplets. (The factors $\frac{1}{2}$ are a standard convention.) The potentials $W_\mu^i$ and $B_\mu$ obey a gauge invariance whose form we indicated in examples in Lecture 20, but which is not explicitly needed here. (The weak force is so short in range that we don't bother too much with the physical interpretation of the weak electric and magnetic fields.) But the gauge invariance does suggest that the quanta of all 4 kinds of potentials should be massless.

For this model to simulate experiment we need 1 massless quantum, the photon, and 3 massive quanta, now called the $W^+$, $W^-$ and $Z^0$. Weinberg and Salam utilized the Higgs mechanism to arrange this, via an extension of the argument of Section 2. There a background scalar field $\phi$ was introduced, and then gauge invariance was used to transform away the phase of $\phi$ in exchange for generating a mass for the field quantum. Here we need to generate 3 masses, for the $W^+$, $W^-$ and $Z^0$. This suggested to our experts that there might be 2 (complex) scalar fields, with a total of 4 real components. If 3 of these components can be transformed away then 3 mass terms will appear in the wave functions for the potentials.

A pair of scalar fields fits very nicely into a weak isospin doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \text{with} \quad I_\phi = \frac{1}{2} \qquad Y_\phi = 1$$

Here $+$ and $0$ refer to the electric charge of the scalar fields.

Again it is imagined that the interaction of the fields $\phi$ with the rest of the universe leads to a vacuum ground state with a non-zero 'background' field. This leaves an ether of $\phi$ filling the universe. This might be somewhat distressing as the field $\phi$ is electrically charged, while the vacuum appears to be electrically neutral.

As the theory is gauge invariant we are free to transform to a more convient representation. This requires the simultaneous transformation of the scalar fields $\phi$, the $SU(2)_L \times U(1)_Y$ multiplets, and the potentials $W_\mu^i$, $B_\mu$. In particular

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow \phi' = e^{i\frac{g}{2}\vec{\alpha}\cdot\vec{\sigma}} e^{i\frac{g'}{2}Y\Theta} \phi$$

We cannot make $\phi'$ vanish completely as the vacuum strength of the field $\phi'$ is related to the $W^\pm$ boson masses. But (we claim) we can choose the four transformation functions $\alpha^i$ & $\Theta$ so that

$$\phi'^+ = 0 \qquad \text{and} \qquad \phi'^0 = \frac{f}{\sqrt{2}} \qquad \text{with } f \text{ real.}$$

In the following we drop the ' and deal only with the fields and particles after the appropriate gauge transformation has been made.

To see what masses have been given the potentials $W_\mu^i$ and $B_\mu$ we must examine the couplings of these to the currents of the scalar field $\phi$. If the currents can be written in the form

$$J_\mu = -M^2 A_\mu$$

Then we identify $M$ with the mass generated for the field quanta by the background interaction. As we have four potentials there will be 4 different currents which we label

$$J_\mu^i \quad i=1,2,3 \qquad \text{and} \qquad J_\mu^Y$$

corresponding to weak isospin and weak hypercharge. To distinguish the functional dependence of the 4 currents we need a slight generalisation of the prescription of section 2. The current of a given potential must carry the 'charge' and internal coupling associated with that potential. In our case this is

$\frac{g\sigma^i}{2}$ for isospin current $i$, and $\frac{g'Y}{2}$ for the hypercharge current

Then we generalize from $\qquad J_\mu = i\left( \phi^* e D_\mu \phi - (D_\mu \phi^*) e \phi \right)$

To $\qquad J_\mu^i = i\left( \phi^* \frac{g}{2}\sigma^i D_\mu \phi - (D_\mu \phi)^* \frac{g\sigma^i}{2} \phi \right)$

And $\qquad J_\mu^Y = i\left( \phi^* \frac{g'Y}{2} D_\mu \phi - (D_\mu \phi)^* \frac{g'Y}{2} \phi \right)$

where $\qquad D_\mu = \partial_\mu + i\frac{g}{2}\sigma^i W_\mu^i + i\frac{g'}{2} Y B_\mu \qquad$ as on p. 378

The order of the operators $\sigma^i$ is significant.

Because the scalar field $\phi$ has been transformed to a constant value $\phi = \begin{pmatrix} 0 \\ f/\sqrt{2} \end{pmatrix}$ the derivatives $\partial_\mu$ do not contribute anything to the currents $J_\mu$.

THUS   $J_\mu^i = -\frac{g^2}{4}(\sigma^i \sigma^j + \sigma^j \sigma^i) W_\mu^j |\phi|^2 - \frac{g'}{4} Y B_\mu (\phi^* \frac{g \sigma^i}{2} \phi)$

$\qquad = -\frac{g^2}{2} |\phi|^2 W_\mu^i - \frac{gg'}{2} Y B_\mu (\phi^* \sigma^i \phi)$   USING $\sigma^i \sigma^j + \sigma^j \sigma^i = 2 \delta^{ij}$

ALSO   $\phi = \binom{0}{f/\sqrt{2}}$ ; $|\phi|^2 = \frac{f^2}{2}$ ; $\phi^* \sigma^1 \phi = \phi^* \sigma^2 \phi = 0$ ; $\phi^* \sigma^3 \phi = -\frac{f^2}{2}$

$\qquad$ AND   $y(\phi) = +1$   AS ON P.379

SO   $J_\mu^i = -\frac{g^2 f^2}{4} W_\mu^i + g g' \frac{f^2}{4} B_\mu \delta^{i3}$

SIMILARLY   $J_\mu^Y = -g' Y (\phi^* \frac{g}{2} \sigma^i \phi) W_\mu^i - \frac{g'^2}{2} Y^2 |\phi|^2 B_\mu$

$\qquad = g g' \frac{f^2}{4} W_\mu^3 - \frac{g'^2 f^2}{4} B_\mu$

THIS IS STILL A BIT COMPLICATED.  AT LEAST THE CASES OF $W^1$ AND $W^2$ ARE SIMPLE:

$$J_\mu^{1,2} = -\left(\frac{gf}{2}\right)^2 W_\mu^{1,2}$$

So WE IDENTIFY   $M_W = \frac{gf}{2}$   AS THE MASS OF THE QUANTA

ASSOCIATED WITH POTENTIALS $W_\mu^1$ AND $W_\mu^2$. IT IS CONVENIENT TO WRITE   $W_\mu^\pm = \frac{W_\mu^1 \pm i W_\mu^2}{\sqrt{2}}$   SO THAT POTENTIALS $W_\mu^\pm$ RAISE OR LOWER WEAK ISOSPIN BY 1 UNIT. THE CORRESPONDING QUANTA $W^+$ AND $W^-$ HAVE OFTEN BEEN REFERRED TO WHEN ILLUSTRATING THE U-A THEORY.

$\qquad$ THE OTHER 2 CURRENTS YIELD A VITAL RESULT IF WE SUPPOSE THE OBSERVED FIELDS ARE CERTAIN LINEAR COMBINATIONS OF $W_\mu^3$ AND $B_\mu$. IN PARTICULAR WE DEFINE

$$A_\mu = \frac{g B_\mu + g' W_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$Z_\mu = \frac{-g' B_\mu + g W_\mu^3}{\sqrt{g^2 + g'^2}}$$

THEN   $J_\mu(A) = \frac{1}{\sqrt{g^2+g'^2}} (g J_\mu^Y + g' J_\mu^3) = 0$

WHILE   $J_\mu(Z) = \frac{1}{\sqrt{g^2+g'^2}} (-g' J_\mu^Y + g J_\mu^3) = -\frac{g^2 + g'^2}{4} f^2 Z_\mu$

Hence the quantum of the potential $A_\mu$ remains massless even after the Higgs trick is applied! We note that this is the photon.

We also identify

$$M_Z = \frac{\sqrt{g^2+g'^2}}{2} f = \frac{\sqrt{g^2+g'^2}}{g} M_W$$

It is appropriate now to introduce the standard notation

$$\tan\Theta_W \equiv g'/g \qquad \Theta_W = \text{Weinberg angle}$$

$$\cos\Theta_W = g/\sqrt{g^2+g'^2}$$

$$\sin\Theta_W = g'/\sqrt{g^2+g'^2}$$

Then $\quad M_Z = \dfrac{M_W}{\cos\Theta_W} \qquad$ A famous result

We may also write

$$A_\mu = \cos\Theta_W B_\mu + \sin\Theta_W W_\mu^3 \qquad \Longrightarrow \qquad B_\mu = \cos\Theta_W A_\mu - \sin\Theta_W Z_\mu$$

$$Z_\mu = -\sin\Theta_W B_\mu + \cos\Theta_W W_\mu^3 \qquad\qquad W_\mu^3 = \sin\Theta_W A_\mu + \cos\Theta_W Z_\mu$$

It is useful to rewrite the covariant derivative in terms of $A_\mu$ and $Z_\mu$!

$$D_\mu = \partial_\mu + i\frac{g}{2}\sigma^i W_\mu^i + i\frac{g'}{2}Y B_\mu$$

$$= \partial_\mu + \frac{ig}{2}(\sigma^1 W_\mu^1 + \sigma^2 W_\mu^2) + \frac{ig}{2}\sigma^3(\sin\Theta_W A_\mu + \cos\Theta_W Z_\mu) + \frac{ig'}{2}Y(\cos\Theta_W A_\mu - \sin\Theta_W Z_\mu)$$

$$\underset{\Big\uparrow\, g\tan\Theta_W}{= \partial_\mu + \frac{ig}{2}(\sigma^1 W_\mu^1 + \sigma^2 W_\mu^2) + ig\sin\Theta_W A_\mu\left(\frac{\sigma^3}{2}+\frac{Y}{2}\right) + \frac{ig}{\cos\Theta_W}Z_\mu\left(\cos^2\Theta_W\frac{\sigma^3}{2} - \sin^2\Theta_W\frac{Y}{2}\right)}$$

If this current acts on a particle of weak isospin $I$, then $\frac{\sigma^3}{2} \to I_3$

We also note that $\quad W_\mu^\pm \equiv \dfrac{W_\mu^1 \pm i W_\mu^2}{\sqrt{2}}$ , while $\sigma^\pm = \dfrac{\sigma^1 \pm i\sigma^2}{2}$

Then $\quad D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}}(\sigma^+ W^- + \sigma^- W^+) + ig\sin\Theta_W A_\mu\underset{\Big\uparrow\, Q/e}{\left(I_3 + \frac{Y}{2}\right)} + \frac{ig}{\cos\Theta_W}Z_\mu\left(\cos^2\Theta_W I_3 - \sin^2\Theta_W \frac{Y}{2}\right)$

This relation, properly interpreted, contains the information as to the couplings of the various quanta to the quarks and leptons. For now we just note that the Gell-Mann-Nishijima relation $Q = e(I_3 + Y/2)$ tells us that the potential $A_\mu$ does indeed couple to electric charge, and thus is completely consistent with being identified as the electromagnetic potential. We obtain the important constraint

$$e = g\sin\Theta_W = gg'/\sqrt{g^2+g'^2}$$

Fifty years after Weyl's first efforts the unification of two interaction is achieved in a gauge invariant theory!

WE HAVE INTRODUCED 3 PARAMETERS: THE 'CHARGES' $g$ AND $g'$, AND THE STRENGTH OF THE BACKGROUND FIELD $f$. WE HAVE ONE RELATION AMONG THE CHARGES:

$$e = g \sin \theta_W$$

A SECOND RELATION IS READILY OBTAINED BY COMPARING THE WEINBERG-SALAM VIEW WITH THE V-A THEORY. FOR EXAMPLE, CONSIDER $\mu \to e^- \bar{\nu}_e \nu_\mu$

$$\mathcal{M}_{V-A} = \frac{G}{\sqrt{2}} (\bar{u}_{\nu_\mu} | \gamma_\alpha (1-\gamma_5) | u_\mu)(\bar{v}_{\bar{\nu}} | \gamma^\alpha (1-\gamma_5) | u_e)$$

IN THE WEINBERG-SALAM MODEL THE COUPLING IS ONLY TO LEFT HANDED PARTICLES:
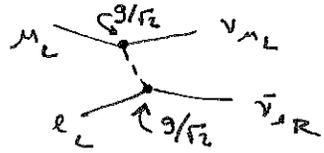


$$\mathcal{M}_{W-S} = \frac{g}{\sqrt{2}} (\bar{u}_{\nu_L} | \gamma_\alpha | u_{\mu_L}) \frac{1}{q^2 - M_W^2} (\bar{u}_{\bar{\nu}_R} | \gamma^\alpha | u_{e_L}) \frac{g}{\sqrt{2}}$$

NOW $|u_L) = \frac{1-\gamma_5}{2} |u)$  USING THE HELICITY PROJECTION OPERATOR INTRODUCED ON P. 116

THUS $(\bar{u}_{\nu_L} | \gamma_\alpha | u_{\mu_L}) = (\bar{u}_\nu | (\frac{1+\gamma_5}{2}) \gamma_\alpha (\frac{1-\gamma_5}{2}) (u_\mu) = \frac{1}{2} (\bar{u}_\nu | \gamma_\alpha (1-\gamma_5) | u_\mu)$

SO FOR SMALL $q^2$,

$$\mathcal{M}_{W-S} = \frac{g^2}{8 M_W^2} (\bar{u}_{\nu_\mu} | \gamma_\alpha (1-\gamma_5) | u_\mu)(\bar{v}_{\bar{\nu}} | \gamma^\alpha (1-\gamma_5) | u_e)$$

THUS $\quad \frac{G}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{1}{2f^2} \quad$ WITH $M_W = \frac{gf}{2}$

THUS $\quad f = \frac{1}{\sqrt{G\sqrt{2}}} \sim 246$ GeV  FOR WHAT IT'S WORTH.

$$M_W = \frac{gf}{2} = \frac{ef}{2 \sin \theta_W} = \frac{\sqrt{4\pi\alpha} \, f}{2 \sin \theta_W} = \frac{37.3 \text{ GeV}}{\sin \theta_W} \qquad \left( \alpha = \frac{e^2}{4\pi} \right)$$

AGAIN $\quad M_Z = M_W / \cos \theta_W$

IN THE NEXT LECTURE WE WILL REVIEW HOW SEVERAL EXPERIMENTS IN THE 1970'S LED TO A MEASUREMENT THAT $\sin^2 \theta_W = 0.23 \pm .01$

THEN THE PREDICTION IS $\quad M_W = 78 \pm 2$ GeV

$$M_Z = 89 \pm 2 \text{ GeV}$$

OF COURSE, THE MOST DRAMATIC EVIDENCE FOR THE WEINBERG-SALAM MODEL IS THAT GIVEN IN THE NEXT SECTION.

## 5. DISCOVERY OF THE $W^{\pm}$ AND $Z^0$ BOSONS

EVIDENCE FOR THE REACTION    $\bar{P} P \rightarrow W^{\pm} + X$    IS GIVEN    IN

| | | | |
|---|---|---|---|
| ARNISON ET AL | PHYS. LETT. | 122B, | 103 (1983) |
| BANNER ET AL | PHYS. LETT. | 122B, | 476 (1983) |
| ARNISON ET AL | PHYS. LETT. | 129B, | 273 (1983) |
| ARNISON ET AL | PHYS. LETT. | 134B, | 467 (1984) |

AND FOR    $\bar{P} P \rightarrow Z^0 + X$    IN

| | | | |
|---|---|---|---|
| ARNISON ET AL | PHYS. LETT. | 126B, | 398 (1983) |
| BAGNAIA ET AL | PHYS. LETT | 129B, | 130 (1983) |
| ARNISON ET AL | PHYS. LETT. | 147B, | 241 (1984) |

THE $W^{\pm}$ IS DETECTED BY ITS DECAY TO $e^{\pm} \nu$. THE ELECTRON IS OBSERVED, AND THE $\nu$ IS INFERRED BY MEASURING THE NET TRANSVERSE ENERGY BALANCE

$\bar{P} \longrightarrow \quad \longleftarrow P$

$\nu \Rightarrow$ MISSING ENERGY

$E_{\nu_{||}}$

$E_{\nu_T}$



**a**    RUN 2958 Evt/1279    **b**    RUN 4017 Evt 838

$E_T$ max 23 7GeV    $E_T$ max 25 1GeV

$E_T$ max 25 5GeV    $E_T$ max 32 7GeV

$E_T$ max 0 5GeV    $E_T$ max 19GeV
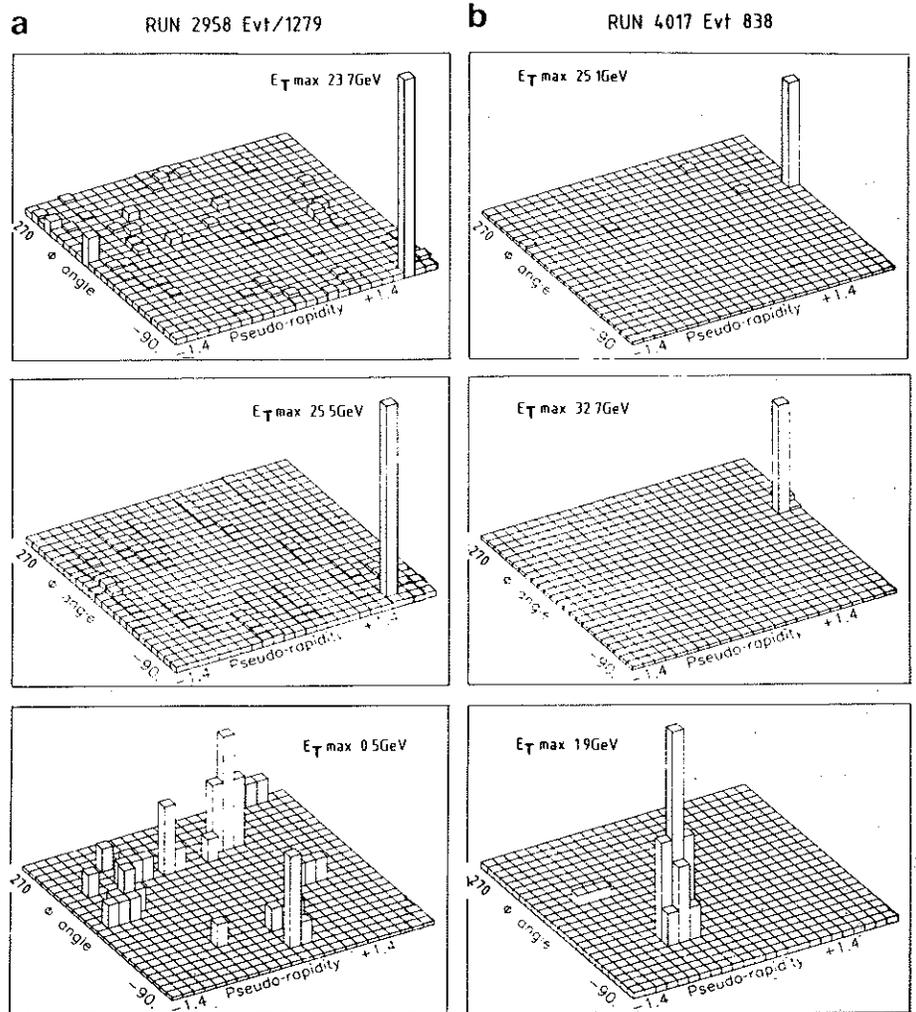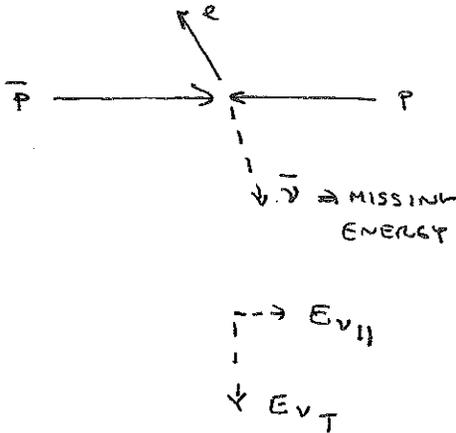
Fig. 7. The energy deposited in the cells of the central calorimetry and the equivalent plot for track momenta in the central detector for the two events of fig. 6. The top diagram shows the electromagnetic cells, the middle shows the central detector tracks, and the bottom plot, with a very much increased sensitivity, shows the energy in the hadron calorimeter. The plots reveal no hadronic energy behind the electron and no jet structure; (a) high-multiplicity; (b) low-multiplicity.

IF THE W IS PRODUCED
NEARLY AT REST (LAB FRAME
≈ C.M. FRAME IN THIS
EXPERIMENT) THE e AND ν
SHOULD LIE IN A PLANE
CONTAINING THE BEAM
AXIS. FURTHER, THE
TRANSVERSE COMPONENTS
OF THE e AND ν ENERGIES
SHOULD BE EQUAL
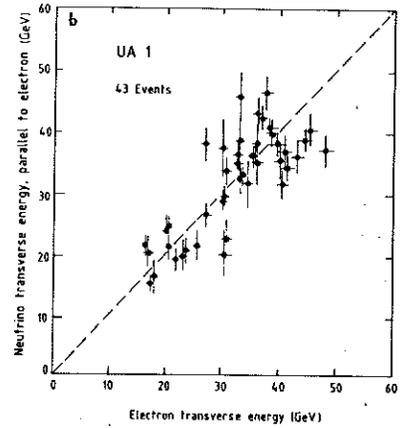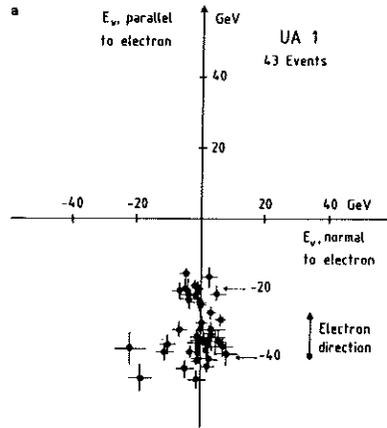
Fig. 3. (a) Two-dimensional plot of the transverse components of the missing energy (neutrino momentum). Events have been ro-
tated to bring the electron direction pointing along the vertical axis. The striking back-to-back configuration of the electron—
neutrino system is apparent. (b) Correlation between the electron and neutrino transverse energies. The neutrino component along
the electron direction is plotted against the electron transverse energy.

THE MASS OF THE W CAN BE MEASURED BY
NOTING THAT FOR W's AT REST, $E_e = \dfrac{M_w}{2}$.

So THE TRANSVERSE ENERGY OBEYS  $E_{e_T} \leq \dfrac{M_w}{2}$

SLIGHTLY MORE SOPHISTICATED VERSIONS OF THIS
ANALYSIS YIELD

$M_w \approx 81 \pm 1.5 \pm 3$  GeV   BASED ON 52 EVENTS

↑ SYSTEMATIC ERROR ON
THE ENERGY SCALE.
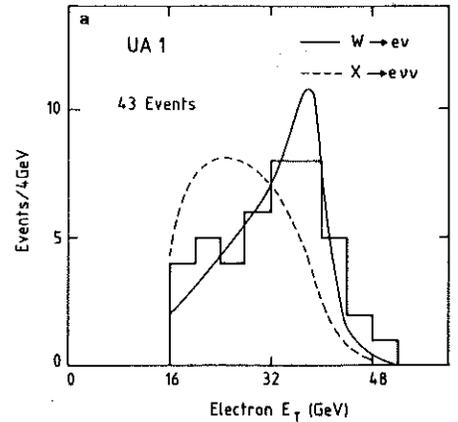
A TEST OF THE LEFT-HANDED COUPLING
IS PROVIDED BY THE ANGULAR DISTRIBUTION
OF THE ELECTRON WITH RESPECT TO THE
PROTON IN W⁻ DECAY
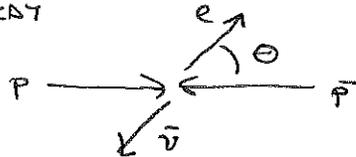
THE QUARK REACTION IS  $d\bar{u} \to \bar{W} \to e^- \bar{\nu}$
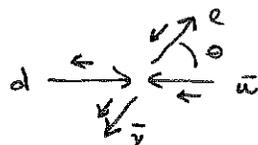
THE LEFT-HANDED COUPLING TELLS US THAT

$\dfrac{d\sigma}{d\Omega} \sim (1 + \cos\theta)^2$  AS ON P 349.
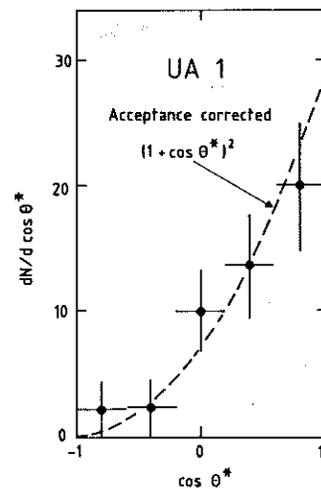
Fig. 8. The angular distribution of the electron emission angle
$\theta^*$ in the rest frame of the W after correction for experimental
acceptance. Only those events in which the electron charge is
determined and the kinematic ambiguity (see text) has been
resolved have been used. The later requirement has been cor-
rected for in the acceptance calculation.

THE EXPERIMENTAL DETECTION OF THE REACTION   $\bar{p}p \rightarrow Z^0 + X$
$\hookrightarrow e^+e^-$

IS MORE STRAIGHTFORWARD, ALTHOUGH THE RATE IS SOMEWHAT LESS THAN FOR W PRODUCTION. SO 13 EVENTS HAVE BEEN SEEN SO FAR
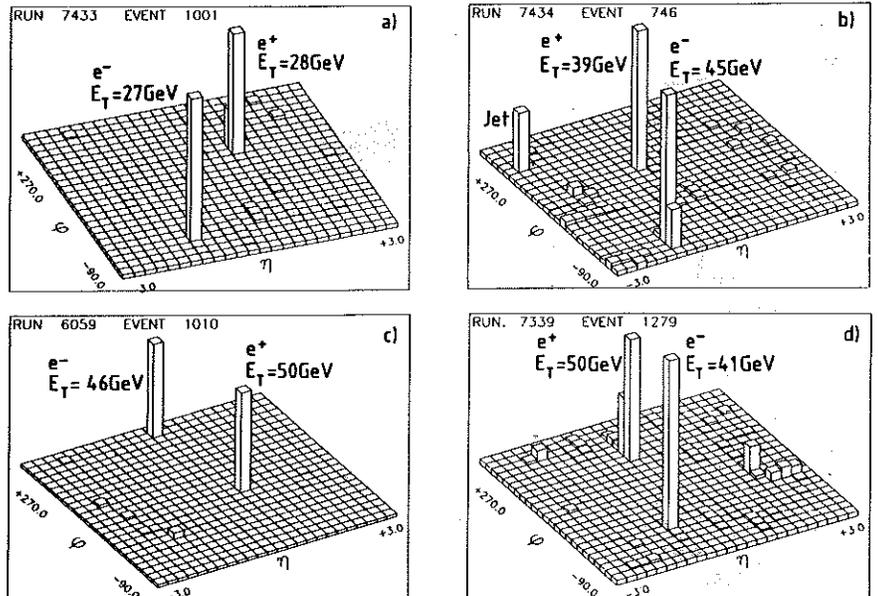


Fig. 3. Electromagnetic energy depositions at angles >5° with respect to the beam direction for the four electron pairs.

AS BOTH ELECTRONS ARE OBSERVED A FAIRLY GOOD DETERMINATION IS MADE OF THE $Z^0$ MASS!

$$M_{Z^0} = 92 \pm 1.3 \pm 1.4 \; GeV$$

THESE VALUES FOR $M_W$ AND $M_Z$ IMPLY   $\sin^2\Theta_W = 0.227 \pm .010$ WHICH IS REMARKABLY CONSISTENT WITH EARLIER MEASUREMENTS OF $\Theta_W$.

A TANTALIZING FACT IS THAT 3 EVENTS HAVE BEEN OBSERVED OF THE TYPE  $Z^0 \rightarrow e^+e^-\gamma$  WITH $E_\gamma > 30 \; GeV$.  SUCH A LARGE RATE OF EXTRA PHOTONS IS NOT EXPECTED AS A SIMPLE RADIATIVE CORRECTION TO $Z^0 \rightarrow e^+e^-$ DECAY ....