ELASTIC ELECTRON-PROTON SCATTERING, CONTINUED

THE DATA ARE RATHER WELL SUMMARIZED BY

$$G_E^P = \frac{G_M^P}{2.79} = \frac{G_M^n}{-1.91} = \frac{1}{\left(1 - \frac{q^2}{.71}\right)^2} \qquad G_E^n = 0 \qquad (q^2 < 0)$$

IT IS NOT NECESSARY A PRIORI THAT $G_E^P \approx G_M^P$, ALTHOUGH THIS IS NATURAL IN THE MODEL OF POINT-LIKE QUARKS.

IF WE SEEK A NON-RELATIVISTIC INTERPRETATION OF THE FORM FACTORS, WE INVERT THE FOURIER TRANSFORM TO FIND $\rho(r) \sim e^{-.84 r}$ OR $e^{-\frac{r}{.26}}$ FOR r IN FERMIS

= .84 GeV IN OUR UNITS WHERE $r \sim 1/\text{ENERGY}$

THEN $\langle r_p^2 \rangle^{1/2} = .81$ FERMIS — THE ELECTROMAGNETIC RADIUS OF THE PROTON

OF COURSE $\langle r_n^2 \rangle^{1/2} \sim 0$ FOR THE ELECTRIC FORM FACTOR.

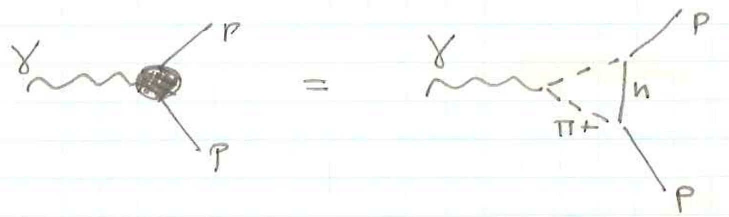
THE DISTRIBUTION $e^{-r/.26}$ IS CERTAINLY SUGGESTIVE OF A NON-RELATIVISTIC S-WAVE STATE. THE NEUTRON QUARK CONFIGURATION, udd, WOULD THEN GIVE $\langle r_n^2 \rangle = 0$, IF $M_u = M_d$.

WHEN DATA OF THIS SORT FIRST BECAME AVAILABLE IN THE 50'S, NOBODY WAS THINKING OF QUARKS. MESON PHYSICS WAS THE VOGUE. THE MODEL WAS

$$p \leftrightarrow n + \pi^+$$
$$n \leftrightarrow p + \pi^-$$

IT WAS THOUGHT THAT THE 'TRUE' PROTON MIGHT BE POINTLIKE, WHILE ORDINARY OBSERVERS SAW ONLY THE 'PION CLOUD' SURROUNDING THE PROTON.

THE PICTURE OF THE FORM FACTORS IS THEN



THIS SUGGESTS $\langle r_n^2 \rangle \approx - \langle r_p^2 \rangle$

THE NEUTRON FORM FACTOR RESULT WAS AN EARLY SIGN OF TROUBLE FOR THE MESON-CLOUD MODEL.

7. <u>THE VECTOR MESON HYPOTHESIS</u>

THE MESON-PHYSICS ARGUMENTS WERE CARRIED FURTHER BY ADDING <u>ISOSPIN</u>. THE PROTON WAS $I_z = 1/2$, THE NEUTRON WAS $I_z = -1/2$

HENCE WE MIGHT EXPECT THAT THE PROTON AND NEUTRON FORM FACTORS ARE RELATED BY

$$G^p_{E,M} = G^S_{E,M} + G^V_{E,M}$$

(NOT SELF-EVIDENT WITHOUT MORE KNOWLEDGE ABOUT ISOSPIN)

$$G^n_{E,M} = G^S_{E,M} - G^V_{E,M}$$

WHERE G^S AND G^V ARE THE <u>ISOSCALAR</u> AND <u>ISOVECTOR</u> FORM FACTORS.

$$G^S = \frac{G^p + G^n}{2} \qquad G_V = \frac{G^p - G^n}{2}$$

THE DATA SHOW THAT $G^S_E \sim G^V_E$

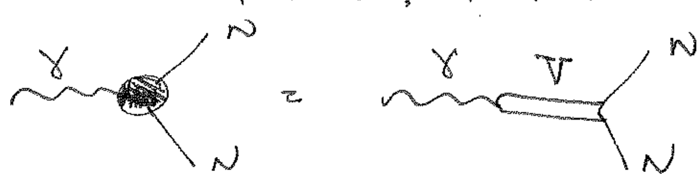
BUT IN THE MESON CLOUD PICTURE (TOP OF PAGE), A SPIN 1 PHOTON CAN COUPLE TO 2 PIONS ONLY IN AN $I=1$ (ISOVECTOR) STATE (DUE TO BOSE STATISTICS). HENCE WE WOULD EXPECT

$$G_E^S \sim 0$$

AND HENCE $G_E^N = -G_E^P$, AS EXPLAINED ABOVE ANOTHER WAY.

THE FACT THAT $G_E^S \neq 0$ LED Y. NAMBU. P.R. 106, 1366 (1957) TO POSTULATE THE EXISTENCE OF AN ISO SCALAR MESON (WHICH DECAYS LARGELY INTO 3 PIONS). FROM THE FITS TO THE FORM FACTOR, WE INFERRED THE MASS SHOULD BE $\sim \sqrt{.71} = .84\ GeV/c^2$. LIKEWISE, ONE EXPECTS AN ISO VECTOR MESON OF SIMILAR MASS TO EXPLAIN THE ISOVECTOR FORM FACTOR. BOTH MESONS WOULD HAVE SPIN 1. SUBSEQUENTLY THE $\rho(760)$ AND $\omega(780)$ MESONS WERE DISCOVERED, WITH $I = 1$ AND 0 RESPECTIVELY. THERE MUST BE SOME TRUTH TO NAMBU'S VIEWPOINT!

FRAZER AND FULCO, P.R.L. 2, 365 (1959) EXTENDED THE ARGUMENT TO THE SO-CALLED VECTOR DOMINANCE MODEL. IN THIS VIEW, THE PHOTON TURNS INTO A VECTOR MESON WHEN IN THE VICINITY OF A HADRON, IN PICTURES



WHERE V IS A VECTOR MESON.

IN FEYNMANS LANGUAGE, THE VECTOR MESON PROPAGATOR IS $\sim \dfrac{1}{q^2 - M_V^2}$. THE q^2 DEPENDENCE OF THE FORM FACTOR SHOULD BE ENTIRELY DUE TO THIS PROPAGATOR. BUT THE DATA IS MORE LIKE $\dfrac{1}{(a^2 - q^2)^2}$ FOR THE AMPLITUDE.

THE SUGGESTION IS THAT THERE ARE 2 VECTOR MESONS OF EACH TYPE, $I=0$ AND $I=1$, AND THE FORM FACTORS ARE THE DIFFERENCES BETWEEN PAIRS OF PROPAGATORS (DIPOLE MODEL). AN EXAMPLE OF SUCH A SCHEME IS THE FIT TO THE DATA

$$G_E^V = .62 + \frac{1.64}{1-\frac{q^2}{(.75)^2}} - \frac{1.75}{1-\frac{q^2}{(1.22)^2}}$$

$$G_E^S = -.10 + \frac{.96}{1-q^2/(.79)^2} - \frac{.36}{1-q^2/(1.03)^2}$$

THIS SUGGESTS $I=1$ MESONS OF $M \approx 750$ AND $1220\ \mathrm{MeV}/c^2$ AND $I=0$ MESONS WITH $M \approx 790$ AND $1030\ \mathrm{MeV}/c^2$. IT SO HAPPENS THAT THE $\rho'(1250)$ AND $\phi(1050)$ ARE GOOD CANDIDATES – ALTHOUGH THE EXACT STATUS OF THE $\rho'(1250)$ AS A PARTICLE IS SLIGHTLY DOUBTFUL.

THE IDEA THAT PHOTONS LIKE TO TURN INTO VECTOR MESONS HAS HAD A FAIR PHENOMENOLOGICAL SUCCESS IN OTHER PHOTON – HADRON INTERACTIONS, BUT HAS NOT LED TO FUNDAMENTAL INSIGHT INTO THE STRUCTURE OF MATTER.

8. TIME LIKE FORM FACTORS OF MESONS

THE VECTOR DOMINANCE IDEA HAS A READY APPLICATION TO THE FORM FACTORS OF THE $\pi^\pm$ AND $K^\pm$ MESONS.

HOWEVER, FOR REASONS UNKNOWN TO ME THE CONSPIRACY OF TWO VECTOR MESON PROPAGATORS TO CANCEL DOES NOT APPLY HERE. THE PION FORM FACTOR IS DOMINATED BY

Ph 529 LECTURE 7

THE SINGLE $\rho(760)$ PROPAGATOR $\Rightarrow F_\pi \sim \dfrac{1}{1-\frac{q^2}{.58}}$ $(q^2<0)$

RECALL THAT THE DATA ON P 71 IS WELL FIT BY $\dfrac{1}{1+Q^2/.68}$

IN REASONABLE AGREEMENT.

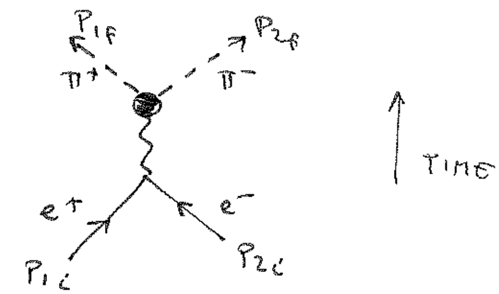
A MUCH MORE STRIKING TEST OF THE HYPOTHESIS COULD BE MADE IF $q^2>0$. THEN $F_\pi \to \infty$ FOR $q^2 \sim M_\rho^2$.

$q^2>0$ IS CALLED TIMELIKE. ($q^2<0$ IS SPACELIKE)

THIS CAN BE ARRANGED IN AN e^+e^- COLLIDING RING

$e^+e^- \to \gamma \to \pi^+\pi^-$ OR K^+K^- OR $\pi^+\pi^-\pi^0$

IN PICTURES:



FEYNMAN TELLS US WE CAN TREAT THIS PROCESS ALMOST IDENTICALLY TO THAT OF $e+\pi \to e+\pi$. THE MATRIX ELEMENT IS

$$M = \frac{e^2}{q^2}\left(\bar{v}_{1i}\,|\,\gamma_\mu\,|\,u_{2i}\right) \underline{P}_{f\mu}\; F(q^2)$$

WHERE NOW $q = P_{1i}+P_{2i} = P_{1f}+P_{2f}$ AND $\underline{P}_f = P_{1f}-P_{2f}$

THE INITIAL STATE POSITRON, P_{1i} IS TREATED FORMALLY LIKE A FINAL STATE PARTICLE IN THE MATRIX ELEMENT. WE WRITE $\bar{v}_{1i}$ RATHER THAN $\bar{u}_{1i}$ TO REMIND US. IN CALCULATING THE TRACE, WE MUST USE THE POSITRON PROJECTION OPERATOR $-\not{P}_{1i}+M_1$ NOT $\not{P}_{1i}+M_1$. THIS SIMPLY CHANGES THE SIGN OF THE TERM $q^2 P_f^2$ IN THE RESULT GIVEN ON P. 95.

$$\text{SO } |M|^2 = \frac{2e^4F^2}{q^4}\left[-\frac{q^2P_f^2}{2} + 2(P_{1i}P_f)(P_{2i}P_f)\right]$$

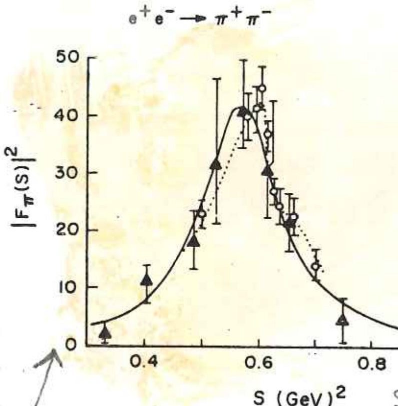
THE EXPERIMENT IS NATURALLY DONE IN THE C.M. FRAME, SO

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E^2}\,\frac{P_{1f}}{P_{1i}}\,|M|^2 \qquad E \equiv E_{1i} + E_{2i} = 2E_{1i}$$

NOTE THAT $P_{1f} \neq P_{1i}$ SINCE $M_{1i} \neq M_{1f}$. INSTEAD $E_{1f} = E_{1i}$

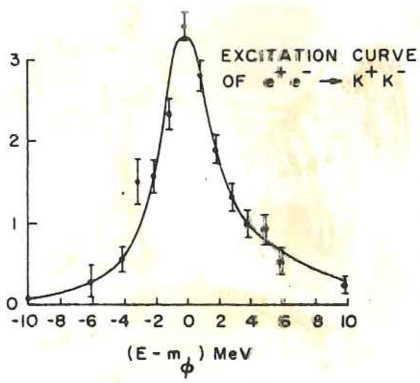
$$\frac{d\sigma}{d\Omega} = \alpha^2 \frac{P_{1f}^3}{E^5} \sin^2\theta \; F^2(q^2) \qquad \text{AND} \quad q^2 = E^2$$

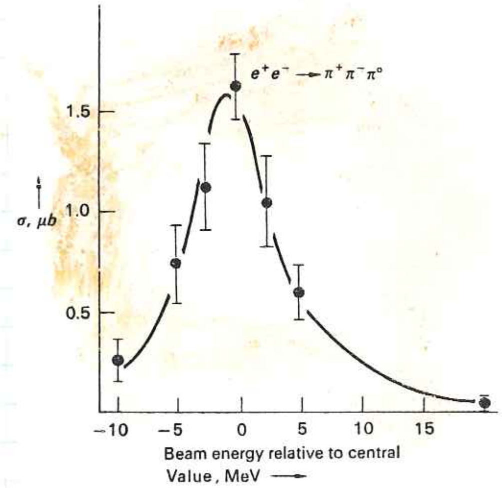
$$\sigma_{TOTAL} = \int \frac{d\sigma}{d\Omega} = \frac{8\pi\alpha^2}{3}\,\frac{P_{1f}^3}{E^5}\,F^2(E^2)$$



$S = E^2$

SHOULD BE 1 AT S = 0





BECAUSE OF THEIR RAPID DECAY RATES, THE VECTOR MESON PROPAGATORS ARE MODIFIED:

$$\frac{1}{q^2 - m^2} \longrightarrow \frac{1}{q^2 - m^2 + i m\Gamma}$$

SO $\quad F^2 \sim \dfrac{1}{(q^2-m^2)^2 + m^2\Gamma^2}$, THE FAMILIAR BREIT-WIGNER SHAPE.

(LECTURE 11)

IF WE BORROW A RESULT FROM PARTIAL WAVE ANALYSIS, WE MAY EXTRACT THE BRANCHING RATIO $\rho \rightarrow e^+e^-$.

NAMELY $\quad \sigma_{TOT} = \dfrac{\pi}{E_{1i}^2}\,\dfrac{2J+1}{(2S_{1i}+1)(2S_{2i}+1)}\,\dfrac{\Gamma_t \Gamma_{e^+e^-}}{(E-M_\rho)^2 + \Gamma_t^2/4}$ $\quad$ NON-RELATIVISTIC

THE MAXIMUM CROSS SECTION OCCURS WHEN $E = M_V$

$$\sigma_{MAX} = \frac{12\pi}{M_\rho^2}\frac{\Gamma_{ee}}{\Gamma_t} = \frac{\pi\alpha}{3M_\rho^2}F_\pi^2(M_\rho)$$

NOTING $P_{if} \approx E_i = M_V/2$

$$\text{SO} \quad \frac{\Gamma_{e^+e^-}}{\Gamma_t} = \frac{\alpha^2}{36}F_\pi^2(M_\rho) \sim 1.5\times10^{-6}\,F^2(M_\rho)$$

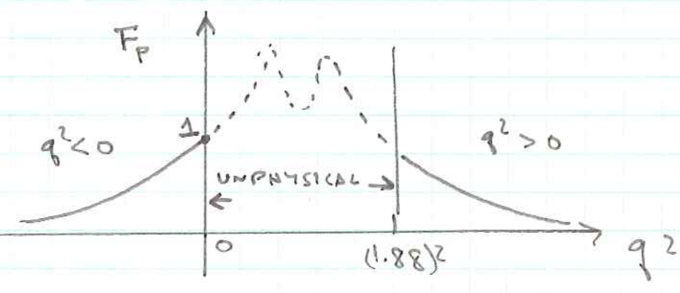
FROM THE DATA ON P 106., $F_\pi^2(M_\rho) \sim 40$ SO

$$\frac{\Gamma_{\rho\to e^+e^-}}{\Gamma_{\rho\to ALL}} = B_{\rho\to e^+e^-} \sim 6\times10^{-5}$$

9. TIME-LIKE FORM FACTORS OF NUCLEONS

IN THE TIME-LIKE REGION, $q^2_{MIN} = (2M_P)^2 \sim (1.88)^2$

THIS MEANS THAT THE REGION WHERE THE PROTON FORM FACTOR SHOULD BLOW UP IS NOT KINEMATICALLY ACCESSIBLE



SO NOT MUCH CAN BE LEARNED THIS WAY.

THE FIRST EXPERIMENTS TO EXPLORE $q^2 > (2M_P)^2$ WERE OF THE TYPE $\bar{p}p \to e^+e^-$, AN ANTIPROTON BEAM STRIKING A HYDROGEN TARGET. NO EVENTS WERE SEEN. MORE RECENTLY A TRICKLE OF DATA HAS EMERGED FROM e^+e^- & $\bar{p}p$ STORAGE RINGS. C.F. BAGLIN ET AL., PHYS. LETT. 163B, 400 (1985).

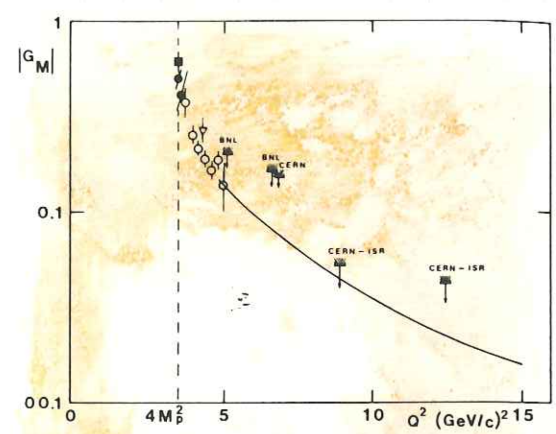
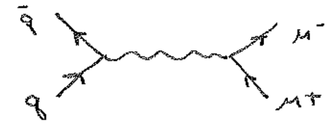


Fig. 1. The proton magnetic form factor $|G_M|$ plotted versus Q^2. ▲ ADONE [5], ○ DCI [6], ● Mulhouse–Strasbourg–Torino [7], ■ LEAR [8]. The experimental points and the CERN [3] and BNL [4] upper limits correspond to $|G_M| = |G_E|$. The solid curve is the $1/Q^4$ prediction of $|G_M|$ normalized to the experimental value at 5 (GeV/c)^2.

10. ANNIHILATION OF SPIN 1/2 PARTICLE-ANTIPARTICLE PAIRS INTO LEPTONS

LATER IN THE COURSE WE CONSIDER THE REACTION $\bar{q}q \to \mu^+\mu^-$

q = QUARK, μ = MUON.



SO WE RECORD THE CROSS SECTION FOR SUCH A PROCESS, IN THE C.M. FRAME

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E_i^2}\frac{p_f}{p_i}\Bigg[\underbrace{(F_1+2M_iF_2)}_{G_M}{}^2\left(1+\cos^2\theta+\frac{m_f^2}{E_i^2}\sin^2\theta\right) + \underbrace{\left(\frac{m_f}{E_i}F_1+2E_iF_2\right)}_{\frac{m_f}{E_i}G_E}{}^2\left(\sin^2\theta+\frac{m_f^2}{E_i^2}\cos^2\theta\right)\Bigg]$$

M_i AND p_i REFER TO THE INITIAL STATE PARTICLE, WHICH IS THE ONE WITH THE FORM FACTORS F_1, F_2.

THIS CAN BE DERIVED FROM THE 4 VECTOR FORM OF THE MATRIX ELEMENT IN SECTION 6 OF THIS LECTURE, NOTING A FEW SIGN CHANGES BECAUSE OF THE ANTIPARTICLE PROJECTION OPERATORS.

FOR MASSLESS, POINTLIKE PARTICLES, THIS REDUCES TO $(F_1 \to Q_q,\ F_2 \to 0)$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 Q_q^2}{16E_i^2}\left(1+\cos^2\theta\right), \quad \sigma = \frac{4\pi}{3}\frac{\alpha^2}{s}Q_q^2 \quad (s = E_{cm}^2 = 4E_i^2).$$

THIS IS TO BE COMPARED WITH $\quad \dfrac{d\sigma}{d\Omega} = \dfrac{\alpha^2}{32E_i^2}\sin^2\theta \quad$ (P106)

IF THE HADRONS WERE SPIN-0, AND POINTLIKE

NOTE: FOR SMALL ENERGY $E_i = \sqrt{s}/2$, WE FIND

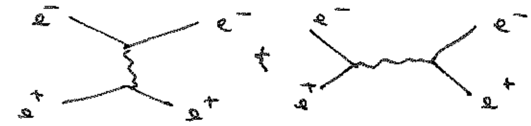
$$\sigma = \frac{4\pi}{3}\frac{\alpha^2}{s}Q_q^2\frac{\sqrt{1-\frac{4M_e^2}{s}}}{\sqrt{1-\frac{4M_q^2}{s}}}\left(1+\frac{2M_e^2}{s}\right)\left(1+\frac{2M_q^2}{s}\right)$$

NOTE ALSO THAT $\quad V_e = \dfrac{P_f}{E_i} = \sqrt{1-\dfrac{4M_e^2}{s}}, \quad V_q = \dfrac{P_i}{E_i} = \sqrt{1-\dfrac{4M_q^2}{s}}$

11. BHABHA SCATTERING - POINTLIKE NATURE OF THE ELECTRON

THE REACTION $e^+e^- \to e^+e^-$ HAS BEEN STUDIED IN COLLIDING BEAM MACHINES AT ENERGIES UP TO 20 GeV PER PARTICLE. THE AGREEMENT WITH EXPERIMENT WITH THE DIRAC THEORY IS EVIDENCE OF THE POINTLIKE NATURE OF THE ELECTRON DOWN TO A DISTANCE $r \sim \frac{1}{20\text{ GeV}} \sim \frac{1}{100}$ FERMI

IN THE 1 PHOTON APPROXIMATION THERE ARE 2 DIAGRAMS FOR THE ABOVE PROCESS, WHICH INTERFERE:



$\longrightarrow$ TIME

THE FIRST DIAGRAM IS OFTEN CALLED MØLLER SCATTERING. WE CAN VERIFY FROM p 98 & 80 THAT FOR A DIRAC ELECTRON $(F_1 = 1,\ F_2 = 0)$ THE CROSS SECTION IN THE C.M. FRAME IS
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E_{BEAM}^2}\,\frac{1+\cos^4\theta/2}{\sin^4\theta/2}$$
IF $E_{BEAM} \gg m_e$.

ON p 108 WE FOUND FOR THE 2ND DIAGRAM: $\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E_{BEAM}^2}\left(1+\cos^2\theta\right)$

THE FULL RESULT INCLUDING THE INTERFERENCE TERM IS
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E_{BEAM}^2}\left(\frac{1+\cos^4\theta/2}{\sin^4\theta/2} - \frac{2\cos^4\theta/2}{\sin^2\theta/2} + \frac{1+\cos^2\theta}{2}\right)$$

BARBER ET AL, PHYS. REV. LETT 43, 1915 (1979).

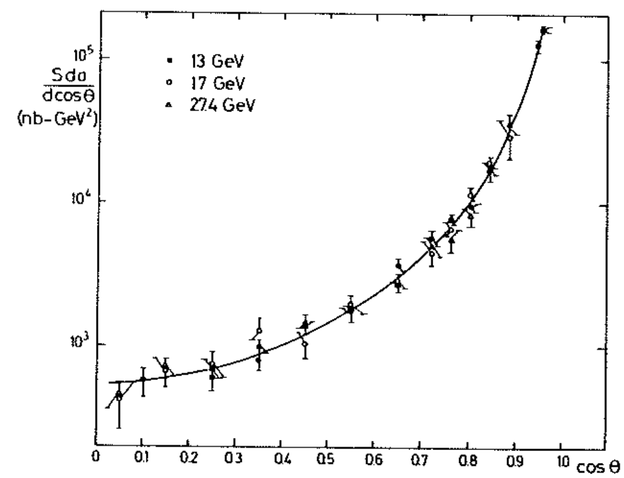


FIG. 1. The data for Bhabha scattering at $\sqrt{s}$ = 27.4 GeV together with our earlier work compared with the predictions of QED.

12. A DIGRESSION ON PHOTON POLARIZATION, AND HELICITY CONSERVATION

SO FAR WE HAVE BEEN ABLE TO IGNORE THAT FACT THAT A PHOTON HAS SPIN 1. REAL PHOTONS CAN OCCUR IN ONLY 2 OF THE 3 POSSIBLE SPIN STATES: $J_z = \pm 1$, IF z IS ALONG THE DIRECTION OF THE PHOTON. FOR VIRTUAL PHOTONS - SUCH AS THOSE EXCHANGED IN ELECTRON-HADRON SCATTERING - THIS RESTRICTION IS REMOVED. HOWEVER IN THE LIMIT OF HIGH ENERGY, THE TRANSVERSE NATURE OF THE PHOTONS IS RE-ASSERTED EVEN FOR VIRTUAL PHOTONS, DUE TO HELICITY CONSERVATION OF THE ELECTROMAGNETIC INTERACTION.

WARNING: TRANSVERSE REFERS TO THE CLASSICAL E&M PICTURE THAT $\bar{E}$ IS $\perp$ TO DIRECTION OF PROPAGATION. THE $J_z = \pm 1$ SPIN STATES ARE CALLED TRANSVERSE IN THAT $J_z = 1 \Leftrightarrow$ RHC (RIGHT HANDED CIRCULAR POLARIZATION); $J_z = -1 \Leftrightarrow$ LHC AND $x = \frac{1}{\sqrt{2}}(\text{RHC} + \text{LHC})$; $y = -\frac{i}{\sqrt{2}}(\text{RHC} - \text{LHC})$

$J_z = 0$ IS CALLED LONGITUDINAL IN THAT IT CORRESPONDS TO $\bar{E}$ ALONG $\hat{z}$ = DIR. OF MOTION.

13. WHY REAL PHOTONS CAN HAVE ONLY 2 SPIN STATES

WE GIVE A SEMI-CLASSICAL ARGUEMENT. IT IS SOMETIMES CLAIMED THAT THE TRUE ARGUMENT HAS TO DO WITH PROPERTIES OF THE LORENTZ GROUP.

THE QUANTUM MECHANICAL WAVE FUNCTION OF THE PHOTON IS A 4-VECTOR, CORRESPONDING TO THE CLASSICAL 4-VECTOR POTENTIAL OF E&M. WE WILL CALL THIS $\phi_\mu = \epsilon_\mu e^{-ikx}$, FOR PLANE WAVES.

WHERE ϵ_μ IS A 4 VECTOR WHICH WILL DESCRIBE THE PHOTON POLARIZATION.

THE 4-POTENTIAL OBEYS THE WAVE EQUATION

$$\Box^2 \phi_\mu = \frac{j_\mu}{\epsilon_0}$$

TO DERIVE THIS EQUATION CLASSICALLY, ONE INVOKES THE LORENTZ CONDITION

$$\bar{\nabla}\cdot\bar{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$$

OR $\partial_\mu \phi^\mu = 0$ IN 4-VECTOR NOTATION

WE CARRY THIS CONDITION OVER INTO QUANTUM MECHANICS

WHICH IMPLIES $\epsilon_\mu k^\mu = 0$

THIS REDUCES THE NUMBER OF POSSIBLE POLARIZATION STATES FROM 4 TO 3.

THIS CONDITION IS SUPPOSED TO HOLD WHETHER THE PHOTON IS REAL OR VIRTUAL.

CONSIDER A PHOTON MOVING ALONG THE Z AXIS.

THEN THE MOMENTUM 4-VECTOR IS $K_\mu = (\omega, 0, 0, K)$

IF THE PHOTON IS MASSLESS, $\omega = K$. OTHERWISE $\omega^2 = K^2 + m^2$

INVOKING THE LORENTZ CONDITION WE MAY CHOOSE 3 POLARIZATION STATES:

$$\left.\begin{aligned} \epsilon_\mu(1) &= (0, 1, 0, 0) \\ \epsilon_\mu(2) &= (0, 0, 1, 0) \end{aligned}\right\} \text{TRANSVERSE}$$

$$\epsilon_\mu(3) = (K, 0, 0, \omega) \quad - \text{LONGITUDINAL}$$

WE CAN GET RID OF $\epsilon(3)$ USING <u>GAUGE INVARIANCE</u>, IF $\omega = K$.

RECALL THAT GAUGE INVARIANCE MEANS THAT THE PHYSICS IS NOT CHANGED BY THE TRANSFORMATION $\phi_\mu' = \phi_\mu - \partial_\mu \Omega$

SO LONG AS $\Box^2 \Omega = 0$. (RECALL $\bar{E} = -\bar{\nabla}\phi - \frac{\partial \bar{A}}{\partial t}$, $\bar{B} = \bar{\nabla} \times \bar{A}$)

FOR EXAMPLE, CONSIDER $\Omega = i e^{-iKx}$. CERTAINLY $\Box^2 \Omega = 0$

THEN $\partial_\mu \Omega = K_\mu e^{-iKx}$

HENCE WE COULD USE NEW POLARIZATION STATES

$$\epsilon_\mu' = \epsilon_\mu - K_\mu$$

IN PARTICULAR, $\epsilon_\mu'(3) = (K - \omega, 0, 0, \omega - K)$

IF THE PHOTON <u>IS MASSLESS</u>, $\omega = K$ AND $\epsilon_\mu'(3) = 0$.

ONLY 2 PHYSICALLY MEANINGFUL POLARIZATION STATES REMAIN!

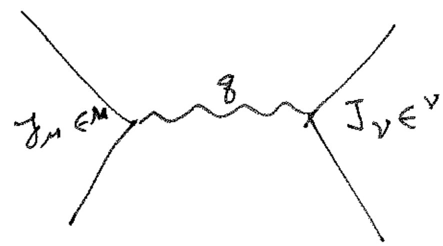
FOR A MASSIVE VIRTUAL PHOTON, NO GAUGE TRANSFORMATION CAN REMOVE THE 3RD STATE, AND THE SPIN BEHAVIOR IS LIKE THAT OF AN ORDINARY SPIN-1 PARTICLE.

14. POLARIZATION AND THE PHOTON PROPAGATOR

SHOULD THERE BE SOME EVIDENCE OF THE PHOTON POLARIZATION IN THE PHOTON PROPAGATOR $\left(\frac{1}{q^2}\right)$? AFTER ALL, THE PHOTON IS THE QUANTUM OF THE 4-VECTOR POTENTIAL A_μ, AND THE VERTEX

REPRESENTS THE INTERACTION $\gamma_\mu A^\mu$. FOR A SINGLE PHOTON, THE POLARIZATION ϵ_μ IS THE 4-VECTOR ASPECT OF THE POTENTIAL, SO THE VERTEX STRENGTH SHOULD REALLY BE $\gamma_\mu \epsilon^\mu$.

IN A VIRTUAL PHOTON EXCHANGE DIAGRAM, WE HAVE A TERM $J_\nu \epsilon^\nu$ AT THE OTHER VERTEX. THE ENTIRE MATRIX ELEMENT IS THEN



$$\sum_{\text{ALL } \epsilon \text{ STATES}} (\gamma^\mu | \epsilon_\mu) \frac{1}{q^2} (\epsilon_\nu | J^\nu)$$

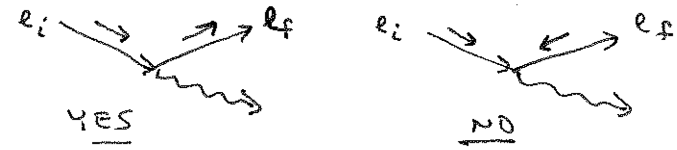
WE SUM OVER ALL POSSIBLE POLARIZATIONS SINCE WE DON'T OBSERVE THE VIRTUAL PHOTON. BUT $\sum_{\text{ALL } \epsilon} |\epsilon_\mu)(\epsilon_\nu| = \delta_{\mu\nu}$

SO WE COULD WRITE THE PHOTON PROPAGATOR AS $\frac{\delta_{\mu\nu}}{q^2}$

CLEARLY THIS MAKES NO DIFFERENCE TO OUR PREVIOUS CALCULATIONS, BUT ITS COMFORTING TO SEE EXPLICIT NOTICE OF THE PHOTON POLARIZATION. THIS ALSO SUGGESTS THAT WE CAN INTERPRET THE VERTEX DIAGRAM AS MEANING THE CURRENT γ_μ EMITS A PHOTON OF POLARIZATION $\epsilon_\nu \approx \gamma^\mu \delta_{\mu\nu} = \gamma_\nu$. SO BY EXAMINING THE COMPONENTS OF γ_μ IN DETAIL, WE CAN DETERMINE THE VIRTUAL PHOTON POLARIZATION, IF DESIRED.

15. HELICITY CONSERVATION IN THE HIGH ENERGY LIMIT

WE NOW GIVE SOME DETAILS IN SUPPORT OF THE CONCEPT OF HELICITY CONSERVATION, FIRST MENTIONED IN LECTURE 3. THE CLAIM IS THAT IN THE HIGH ENERGY LIMIT ($E \gg M$) THE PHOTON-SPIN 1/2 PARTICLE COUPLING PRESERVES HELICITY:



IN THIS LIMIT, POSITIVE AND NEGATIVE HELICITY ELECTRONS (PROTONS, QUARKS...) ARE ESSENTIALLY DIFFERENT PARTICLES, IN THAT NO COMMUNICATION IS POSSIBLE BETWEEN THEM VIA ELECTROMAGNETIC INTERACTIONS!

WE NOW EXAMINE THE VERTEX FACTOR $(\bar{u}_f|\gamma_\mu|u_i)$ IN DETAIL.

WE NEED TO DISPLAY THE EXPLICIT FORM OF THE HELICITY SPINORS. RECALL THAT HELICITY = SPIN COMPONENT ALONG DIRECTION OF MOTION. IN THE PICTURES ABOVE, e_i HAS HELICITY $+1/2$, WHILE IN THE 2ND PICTURE e_f HAS HELICITY $-1/2$.

IT MAY BE USEFUL TO BUILD THE 4-SPINORS OUT OF 2 COMPONENT SPINORS AS ON P. 86. IF $\hat{p}$ = UNIT VECTOR ALONG THE DIRECTION OF MOTION, THEN HELICITY STATES ARE EIGENSTATES OF THE OPERATOR $\vec{\sigma}\cdot\hat{p}$, WHERE $\vec{\sigma}$ = 2X2 PAULI MATRICES.

A SIMPLE CASE IS $\hat{p} = \hat{z}$, FOR WHICH $\chi_{+1/2} = \begin{pmatrix}1\\0\end{pmatrix}$ AND $\chi_{-1/2} = \begin{pmatrix}0\\1\end{pmatrix}$ ARE THE HELICITY 2-SPINOR STATES.

IN THE FOLLOWING WE MEASURE ALL SPINOR COMPONENTS ALONG THE Z-AXIS EVEN WHEN $\hat{p} \neq \hat{z}$ – IN ORDER TO CALCULATE $(\bar{u}_f|\gamma_\mu|u_i)$ ETC.

IN GENERAL $\vec{\sigma}\cdot\hat{p} = \begin{pmatrix}\cos\theta & \sin\theta\, e^{-i\varphi}\\ \sin\theta\, e^{i\varphi} & -\cos\theta\end{pmatrix}$ WHERE θ, φ ARE THE SPHERICAL COORDS OF $\hat{p}$ ABOUT THE Z AXIS. THE 2-SPINOR EIGENSTATES ARE READILY SEEN TO BE

$$\chi_{+1/2} = \begin{pmatrix}\cos\theta/2\; e^{-i\phi/2}\\ \sin\theta/2\; e^{i\phi/2}\end{pmatrix} \qquad \chi_{-1/2} = \begin{pmatrix}-\sin\theta/2\; e^{-i\phi/2}\\ \cos\theta/2\; e^{i\phi/2}\end{pmatrix}$$

THE 4-SPINORS OF DEFINITE HELICITY ARE THEN (FOR $\phi = 0$) (SEE P. 86)

$$u_+(p) = \sqrt{E+M}\begin{pmatrix}\cos\theta/2\\ \sin\theta/2\\ \frac{p}{E+M}\cos\theta/2\\ \frac{p}{E+M}\sin\theta/2\end{pmatrix} \quad \& \quad u_-(p) = \sqrt{E+M}\begin{pmatrix}-\sin\theta/2\\ \cos\theta/2\\ \frac{p}{E+M}\sin\theta/2\\ -\frac{p}{E+M}\cos\theta/2\end{pmatrix}$$

Ph 529 Lecture 7

We can now verify the claim of helicity conservation.

The current is $e(\bar{u}_f(p_f)|\gamma_\mu|u_i(p_i))$. We will go back to basics and evaluate the matrix element for various initial and final spin states (the way people did before Feynman!)

Use the lab frame, and, importantly, helicity spin states

Referring to p 113, the initial spinors are $(\theta_i = 0,\ \varphi_i = 0)$

$$u_+(p_i) = \sqrt{E_i+m}\begin{pmatrix} 1 \\ 0 \\ p_i/E_i+m \\ 0 \end{pmatrix} \qquad u_-(p_i) = \sqrt{E_i+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -p_i/E_i+m \end{pmatrix}$$

The final spinors, for scattering angle θ, $\phi = 0$ are

$$u_+(p_f) = \sqrt{E_f+m}\begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \\ \frac{p_f}{E_f+m}\cos\theta/2 \\ \frac{p_f}{E_f+m}\sin\theta/2 \end{pmatrix} \qquad u_-(p_f) = \sqrt{E_f+m}\begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \\ \frac{p_f}{E_f+m}\sin\theta/2 \\ -\frac{p_f}{E_f+m}\cos\theta/2 \end{pmatrix}$$

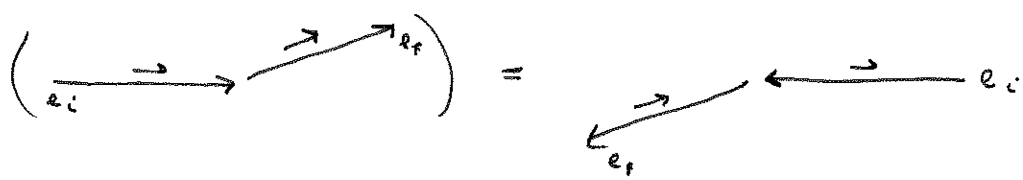
There are 4 matrix elements to consider

Helicity conserving: $(\bar{u}_+(f)|\gamma_\mu|u_+(i))$ and $(\bar{u}_-(f)|\gamma_\mu|u_-(i))$

Helicity flip: $(\bar{u}_-(f)|\gamma_\mu|u_+(i))$ and $(\bar{u}_+(f)|\gamma_\mu|u_-(i))$

Due to parity conservation, only 2 of these are independent

PARITY



After a rotation by 180° ⊥ to paper, we find $(\bar{u}_+(f)|\gamma_\mu|u_+ i) = (\bar{u}_-(f)|\gamma_\mu|u_-(i))$

etc. Recall that parity leaves the direction of spin unchanged, i.e. it flips helicity.

WORKING OUT THE VARIOUS MATRIX ELEMENTS, WE FIND

$$\gamma_0(++) = \sqrt{(E_i+m)(E_f+m)}\ \cos\theta/2 \left(1 + \frac{p_i p_f}{(E_i+m)(E_f+m)}\right)$$

$$\gamma_1(++) = \text{"} \quad \sin\theta/2 \left(\frac{p_i}{E_i+m} + \frac{p_f}{E_f+m}\right)$$

$$\gamma_2(++) = \text{"} \quad i\sin\theta/2 \left(\quad \text{"} \quad\right)$$

$$\gamma_3(++) = \text{"} \quad \cos\theta/2 \left(\quad \text{"} \quad\right)$$

AND

$$\gamma_0(+-) = \text{"} \quad \sin\theta/2 \left(-1 + \frac{p_i p_f}{(E_i+m)(E_f+m)}\right)$$

$$\gamma_1(+-) = \text{"} \quad \cos\theta/2 \left(\frac{p_i}{E_i+m} - \frac{p_f}{E_f+m}\right)$$

$$\gamma_2(+-) = \text{"} \quad -i\cos\theta/2 \left(\quad \text{"} \quad\right)$$

$$\gamma_3(+-) = \text{"} \quad -\sin\theta/2 \left(\quad \text{"} \quad\right)$$

$$\gamma_\mu(++) \equiv (\bar{u}_+(f)|\gamma_\mu|u_+(i)) \quad \text{ETC.}$$

IN GENERAL BOTH THE HELICITY CONSERVING CURRENT $\gamma_\mu(++)$ AND THE HELICITY FLIP CURRENT $\gamma_\mu(+-)$ ARE NON-VANISHING.

BUT IN THE HIGH ENERGY LIMIT THAT BOTH THE INITIAL AND FINAL PARTICLES ARE RELATIVISTIC, $E_i \gg m$, $E_f \gg m$

WE SEE THAT $\gamma_\mu(+-) \longrightarrow 0$

$$\gamma_\mu(++) \rightarrow 2\sqrt{E_i E_f}\left(\cos\theta/2,\ \sin\theta/2,\ i\sin\theta/2,\ \cos\theta/2\right)$$

ONLY THE HELICITY CONSERVING CURRENT SURVIVES!

16. THE HELICITY PROJECTION OPERATOR

HAVING VERIFIED HELICITY CONSERVATION BY 'BRUTE FORCE', IT MAY ALSO BE WORTH INDICATING HOW IT FITS INTO FEYNMAN'S MORE ELEGANT VIEWPOINT. THE KEY IS TO FIND THE HELICITY PROJECTION OPERATOR, WHICH GIVES +1 WHEN APPLIED TO A POSITIVE HELICITY SPINOR, AND 0 WHEN APPLIED TO A NEGATIVE HELICITY SPINOR. YOU MAY WISH TO RECALL THE PARTICLE AND ANTI-PARTICLE PROJECTION OPERATORS, P 93.

Ph 529 LECTURE 7

IT TURNS OUT THAT THE COMPLEXITIES OF RELATIVITY ARE SUCH THAT THE HELICITY PROJECTION OPERATOR IS SIMPLE ONLY IN THE HIGH ENERGY LIMIT – WHICH IS WHAT WE ARE MAINLY INTERESTED IN ANYWAY!

IN THIS LIMIT WE SEE FROM p 113 THAT THE HELICITY SPINOR STATES ARE

$$u_+ \rightarrow \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \\ \cos\theta/2 \\ \sin\theta/2 \end{pmatrix} \qquad u_- \rightarrow \begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \\ \sin\theta/2 \\ -\cos\theta/2 \end{pmatrix}$$

WE MUST LOOK FOR SOME DIRAC MATRIX WHICH TRANSFORMS THESE IN SIMPLE WAYS. FOR INSTANCE $\Gamma u_\pm = \pm u_\pm$ WOULD BE NICE.

ONE READILY CONCLUDES $\Gamma = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ DOES THE JOB.

THIS MATRIX ALREADY HAS A NAME! IT IS $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$

IT IS WORTH NOTING A FEW FACTS ABOUT γ_5:

$$\gamma_5^2 = 1 \;;\quad \overline{\gamma_5} = -\gamma_5 \;;\quad \gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$$

IN ANY CASE, WE SEE THAT

$\dfrac{1+\gamma_5}{2}$ = POSITIVE HELICITY OPERATOR IN THE HIGH ENERGY LIMIT

$\dfrac{1-\gamma_5}{2}$ = NEGATIVE HELICITY OPERATOR "

WE CAN NOW QUICKLY RECONFIRM HELICITY CONSERVATION, NOTING

$$|u_+) = \left(\frac{1+\gamma_5}{2}\right)|u_+) \quad \text{so} \quad (\bar{u}_+| = (\bar{u}_+|\overline{\left(\frac{1+\gamma_5}{2}\right)} = (u_+|\left(\frac{1-\gamma_5}{2}\right)$$

THUS
$$(\bar{u}_+|\gamma_\mu|u_-) = (\bar{u}_+|\left(\frac{1-\gamma_5}{2}\right)\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)|u_-) = (\bar{u}_+|\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\left(\frac{1-\gamma_5}{2}\right)|u_-)$$

$$= 0 \;! \quad \text{SINCE } \gamma_5^2 = 1$$

BUT $(\bar{u}_+|\gamma_\mu|u_+)$ DOES NOT VANISH.

WE SHOULD REMARK ON THE ANTI-PARTICLE HELICITY SPINORS.

ON PP 85-86 WE STATED THAT $\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ IS THE SPIN <u>DOWN</u> ANTI PARTICLE AT REST, WHILE $\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$ IS SPIN <u>UP</u>. IN THE HIGH ENERGY LIMIT WE EXPECT THESE TO TRANSFORM TO (THEY MUST BE ORTHOGONAL TO u_+ & u_-)

$$v_- \sim \begin{pmatrix}-\cos\theta/2\\-\sin\theta/2\\\cos\theta/2\\\sin\theta/2\end{pmatrix} \quad \text{AND} \quad v_+ \sim \begin{pmatrix}-\sin\theta/2\\\cos\theta/2\\-\sin\theta/2\\\cos\theta/2\end{pmatrix}$$

NOW $\gamma_5 v_\pm = \mp v_\pm$

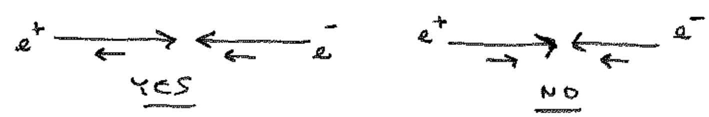
SO $\frac{1-\gamma_5}{2}$ PROJECTS OUT POSITIVE HELICITY ANTIPARTICLES

$\frac{1+\gamma_5}{2}$ . . . . . NEGATIVE . . . . . .

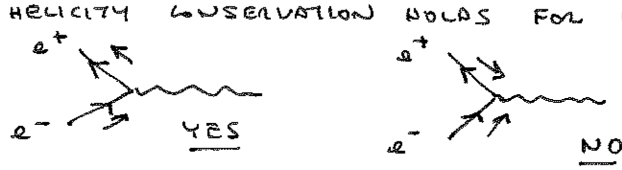
(OR MORE PRECISELY, $\left(\frac{1-\gamma_5}{2}\right)\left(\frac{-\not{p}+m}{2}\right)$ PROJECTS + HELICITY ANTI PARTICLES ETC).

THE SIGN REVERSAL HAS AN IMPORTANT CONSEQUENCE FOR THE HELICITY CONSERVATION ARGUMENT: NAMELY $(\bar{v}_+|\gamma_\mu|u_+) = 0$; $(\bar{v}_-|\gamma_\mu|u_+) \neq 0$

IN PARTICLE-ANTIPARTICLE ANNIHILATION THE HELICITIES MUST BE OPPOSITE (IN THE HIGH ENERGY LIMIT)



(RECALL OUR LABELLING OF ANTIPARTICLES WITH AN ARROW OPPOSITE TO THEIR MOTION. WITH THIS CONVENTION 'HELICITY' CONSERVATION HOLDS FOR BOTH PARTICLES AND ANTIPARTICLES)



17. ILLUSTRATIONS OF HELICITY CONSERVATION

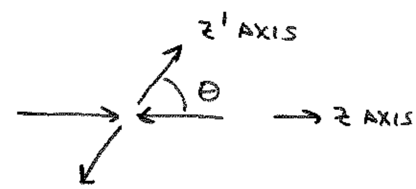
SOME FEATURES OF THE CROSS SECTIONS DERIVED IN LECTURES 6 AND 7 CAN BE READILY UNDERSTOOD AS CONSEQUENCES OF HELICITY CONSERVATION. WE OF COURSE CONSIDER ONLY THE HIGH ENERGY LIMIT.

THE SIMPLEST CASE IS e^+e^- ANNIHILATION INTO A PAIR OF SPIN 0, OR SPIN 1/2 PARTICLES. WE WORK IN THE C.M. FRAME.

$$e^+ \xrightarrow{\leftarrow} \bullet \xleftarrow{\leftarrow} e^- \qquad J_z = \pm 1$$

AS JUST REMARKED ON p118, HELICITY CONSERVATION AS APPLIED TO A PARTICLE-ANTIPARTICLE PAIR TELLS US THEIR SPINS MUST LINE UP, AS SHOWN (OR BOTH OPPOSITE TO THAT SHOWN). HENCE THE INITIAL STATE MUST HAVE $J_z = \pm 1$ ONLY. THE PHOTON IS THEN SAID TO BE TRANSVERSE (EVEN THO IT IS ACTUALLY AT REST!)

WE NOW INQUIRE AS TO THE ANGULAR DISTRIBUTION OF THE FINAL STATE



FOR THE FINAL STATE WE MEASURE SPINS AND HELICITIES ALONG THE z' AXIS, AT ANGLE θ TO THE z AXIS. FOR SPIN-ZERO PARTICLES, CERTAINLY $J_{z'} = 0$, WHILE FOR A PAIR OF SPIN 1/2 PARTICLE & ANTIPARTICLE, WE MUST HAVE $J_{z'} = \pm 1$ BY THE SAME HELICITY-CONSERVATION ARGUMENT.

THE ANGULAR DISTRIBUTION CAN NOW BE DETERMINED BY PROJECTING THE INITIAL SPIN STATE ONTO THE FINAL SPIN STATE. SINCE WE HAVE 1 PHOTON ALL BY ITSELF IN THE INTERMEDIATE STATE, WE MUST HAVE $J=1$. ∴ WE NEED THE SPIN 1 ROTATION MATRIX

(CF. FEYNMAN RED BOOK VOL III CH 17)

R^1	1	0	−1
1	$\frac{1+\cos\theta}{2}$	$+\frac{\sin\theta}{\sqrt{2}}$	$\frac{1-\cos\theta}{2}$
0	$-\frac{\sin\theta}{\sqrt{2}}$	$\cos\theta$	$+\frac{\sin\theta}{\sqrt{2}}$
−1	$\frac{1-\cos\theta}{2}$	$-\frac{\sin\theta}{\sqrt{2}}$	$\frac{1+\cos\theta}{2}$

FOR THE SPIN 0 FINAL STATE WE GET

$$\left(R'_{10}\right)^2 + \left(R'_{-10}\right)^2 = \sin^2\theta$$

WHILE FOR THE SPIN ½ PARTICLE FINAL STATE

$$\left(R'_{11}\right)^2 + \left(R'_{1-1}\right)^2 + \left(R'_{-11}\right)^2 + \left(R'_{-1-1}\right)^2 = 1 + \cos^2\theta$$

(ALL INITIAL STATES, 1 & −1 ARE EQUALLY LIKELY; WE SQUARE BEFORE ADDING BECAUSE WE CAN DISTINGUISH THE VARIOUS SPIN CASES IN PRINCIPLE)

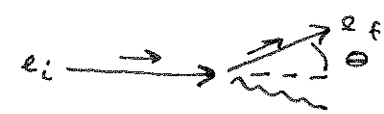
THESE ARE THE ANGULAR DISTRIBUTIONS NOTED ON p108!

WE NOW TURN TO ELASTIC SCATTERING OF AN ELECTRON OFF A SPIN 0, AND SPIN ½ PARTICLE, IN THE LAB FRAME. ON pp 96 & 99 WE FOUND

$$\frac{d\sigma}{d\Omega} \sim \left.\frac{d\sigma}{d\Omega}\right|_{\text{RUTHERFORD}} \cdot \cos^2\theta/2 \qquad \text{SPIN 0 TARGET}$$

$$\frac{d\sigma}{d\Omega} \sim \left.\frac{d\sigma}{d\Omega}\right|_{\text{RUTHERFORD}} \cdot \left(\cos^2\theta/2 - \frac{q^2}{2M_2^2}\sin^2\theta/2\right) \qquad \text{POINTLIKE SPIN ½ TARGET}$$

THE $\cos^2\theta/2$ FACTOR IS READILY EXPLAINED AS DUE TO HELICITY CONSERVATION:



LAB VIEW

IN THE HELICITY-CONSERVING CASE, THE ELECTRON SPIN IS ROTATED THRU θ.

THE SPIN ½ ROTATION MATRIX IS

$R^{1/2}$	$1/2$	$-1/2$
$1/2$	$\cos\theta/2$	$-\sin\theta/2$
$-1/2$	$\sin\theta/2$	$\cos\theta/2$

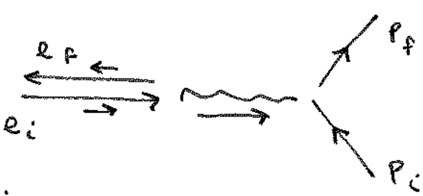
WE NEED $\left(R^{1/2}_{\frac12\frac12}\right)^2 + \left(R^{1/2}_{-\frac12-\frac12}\right)^2 = \cos^2\theta/2$

(THE HELICITY FLIP CASE IS NOT RULED OUT BY ANGULAR-MOMENTUM CONSERVATION AS CLAIMED BY PERKINS, UNLESS YOU ALREADY KNOW THAT THE VIRTUAL PHOTON CANNOT HAVE $J_z = 0$)

A 'PSEUDO-' ARGUMENT FOR THE SPIN 1/2 TARGET IS AS FOLLOWS. THE $\sin^2\theta/2$ TERM IS DUE TO THE MAGNETIC DIPOLE-DIPOLE INTERACTION OF THE SPIN 1/2 PARTICLES. WE ALREADY ARGUED ON p 99 THAT THIS TERM SHOULD BE ~ q^2/M^2 TIMES THE CHARGE INTERACTION TERM. NOW SUPPOSEDLY A DIPOLE-DIPOLE INTERACTION FAVORS SPIN FLIPS. SO IT SHOULD BE SUPPRESSED AT $\theta = 0$ WHILE STRONG AT $\theta = 180°$ IF WE ARE TO CONSERVE HELICITY. THE ANGULAR TERM $\sin^2\theta/2$ DOES THIS NICELY.

AS A LAST CONSIDERATION WE INQUIRE ABOUT THE POLARIZATION OF THE VIRTUAL PHOTON IN ELASTIC ELECTRON-HADRON SCATTERING. IN GENERAL THE PHOTON POLARIZATION 4-VECTOR HAS ALL 4 COMPONENTS NON-VANISHING. RECALL THAT ON p 112 WE ARGUED THAT THE ELECTRON CURRENT (IN MOMENTUM SPACE) CAN BE THOUGHT OF AS THE POLARIZATION OF THE RADIATED PHOTON. THEN ON p 115 WE EVALUATED THE CURRENT J_μ EXPLICITLY.

HOWEVER, WE CAN ALWAYS MAKE A LORENTZ TRANSFORMATION OF ANY PARTICULAR SCATTERING EVENT SUCH THAT THE ELECTRON APPEARS TO HAVE SUFFERED A 180° SCATTER!



IN THIS FRAME THE TARGET IS NOT AT REST INITIALLY. AS THE SCATTERING IS ELASTIC, $\bar{P}_{e_f} = -\bar{P}_{e_i} \Rightarrow E_f = E_i \Rightarrow$ THIS IS A <u>BREIT</u> <u>FRAME</u> (p. 78)

IN FACT THIS IS WHAT MOST PEOPLE MEAN BY <u>THE</u> BREIT FRAME.

HELICITY CONSERVATION AND ANGULAR MOMENTUM CONSERVATION TELL US THAT THE VIRTUAL PHOTON CAN HAVE HELICITY ±1 ONLY. SO IN THE BREIT FRAME, THE PHOTON APPEARS TO BE TRANSVERSE IN THE HIGH ENERGY LIMIT. THIS ADDS TO THE UTILITY OF THE BREIT FRAME IN ALLOWING A SIMPLE PICTURE OF THE SCATTERING PROCESS.

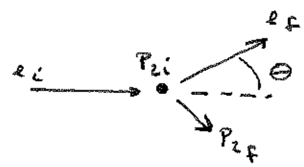
18. A Footnote on Calculating the Matrix Element

We have used Feynman's trace method to calculate spinor matrix elements. Before 1948 people used more 'brute-force' techniques, which however sufficed for most simple (= low order) problems. In section 15. (p115) we found an explicit form for the electron current in the high energy limit: $\mathcal{J}_\mu(++) \to 2\sqrt{E_i E_f}\,(\cos\theta/2,\ \sin\theta/2,\ i\sin\theta/2,\ \cos\theta/2)$

We can use this to complete a brute-force calculation.

An easy case is electron - spin 0 elastic scattering. On p 92 we saw that the matrix element is $M = \dfrac{e^2 F(q^2)}{q^2}\,\mathcal{J}_\mu P_2^\mu$

$P_2 = P_{2i} + P_{2f}$. We work in the lab frame



$e_i \approx (E_i, 0, 0, E_i) \qquad e_f \approx (E_f,\ E_f\sin\theta,\ 0,\ E_f\cos\theta)$

$P_{2i} = (M_2, 0, 0, 0) \qquad P_{2f} = e_i + P_{2i} - e_f \approx (E_i + M_2 - E_f,\ -E_f\sin\theta,\ 0,\ E_i - E_f\cos\theta)$

$$\text{so } M = 2\frac{e^2 F\sqrt{E_iE_f}}{q^2}\Big(\cos\theta/2\,(E_i + 2M_2 - E_f) + E_f\sin\theta\sin\theta/2 - (E_i - E_f\cos\theta)\cos\theta/2\Big)$$

$$= 4\frac{e^2 M_2 F\sqrt{E_iE_f}}{q^2}\cos\theta/2$$

$$|M|^2 = \frac{16\, e^4 M_2^2 F^2 E_i E_f \cos^2\theta/2}{q^4} = -4\frac{e^4 M_2^2 F^2 \cos^2\theta/2}{q^2\sin^2\theta/2} \quad \text{using } q^2 = -4E_iE_f\sin^2\theta/2$$

Just as found on p. 96.

This case was perhaps easier by 'brute force'! Already for the case of electron - pointlike spin 1/2 scattering, 'brute force' is about as much work as Feynman's method...

(Actually we should have averaged the $|M|^2$ for currents $\mathcal{J}_\mu(++)$ and $\mathcal{J}_\mu(--)$. But they are the same, so the above result holds.)