Rain and Relativity

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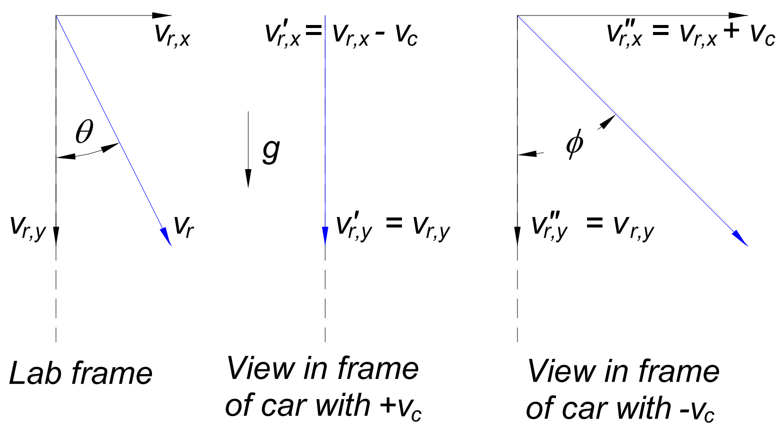
1 Problem

Rain is falling steadily with speed v_r in the lab frame, making angle θ to the vertical. When an observer drives in a car with horizontal speed v_c (in the plane of vector $\mathbf{v}_r$) the rain appears to be falling vertically. When the observer drives in the opposite direction with speed v_c the rain appears to make angle ϕ to the vertical. What are θ and v_r in terms of ϕ and v_c?

2 Solution

We analyze this problem by consider two frames in addition to the lab frame: the $'$ frame which has horizontal speed v_c with respect to the lab frame, and the $''$ frame which has horizontal speed $-v_c$ with respect to the lab frame, where the rain in the lab frame has positive horizontal (x) component.



The components of the rain-velocity vectors $\mathbf{v}'_r$ and $\mathbf{v}''_r$ are given by the Galilean transformations,

$$v'_{r,x} = v_{r,x} - v_c, \qquad v''_{r,x} = v_{r,x} + v_c, \tag{1}$$

$$v'_{r,y} = \quad v_{r,y} \quad = v''_{r,y}. \tag{2}$$

Then,

$$v'_{r,x} = 0 \qquad \Rightarrow \qquad v_{r,x} = v_r \sin\theta = v_c, \qquad v''_{r,x} = 2v_c = 2v_r \sin\theta, \tag{3}$$

and.

$$v''_{r,y} = v_{r,y} = v_r \cos\theta, \tag{4}$$

so that.

$$\tan\phi = \frac{v''_{r,x}}{v''_{r,y}} = \frac{2v_r\sin\theta}{v_r\cos\theta} = 2\tan\theta, \tag{5}$$

$$\tan\theta = \frac{\tan\phi}{2}\,. \tag{6}$$

Also,

$$\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} = \frac{\tan\phi}{\sqrt{4+\tan^2\phi}}\,, \tag{7}$$

so,

$$v_r = \frac{v_c}{\sin\theta} = v_c\frac{\sqrt{4+\tan^2\phi}}{\tan\phi}\,. \tag{8}$$