Moffatt (Nature **404**, 833 - 834) discusses the ever-intriguing motions of a rolling disk, as it whirrs and shudders to a horizontal stop. The paper assumes air damping only. The slow-varying mathematical solution leads to stoppage in finite time, as the disk approaches its final singular configuration. A predicted rolling-speed singularity is preempted, the paper claims, by its incorrect prediction of eventual tensional contact.

The slow-varying viscous theory may be appropriate for very flat disks, on very sturdy flat surfaces, for a range of angles. But it is not so general or surprising as one may infer. And other mechanisms explain similar phenomenology in more common circumstances.

Regarding generality, the paper does not apply to the two examples mentioned, coins and a toy (Fig. 2 therein) by lack of the conformal contact that the theory demands (Fig. 1 therein): a coin by its raised images and generally rough edges, and the toy by its concave support bowl (Fig. 2 of the paper). One can roll almost any disk, including the toy and a coin, on a variety of nominally flat surfaces and note that the settling time differs by factors of two or so (although the disk parameters and air viscosity vary by far less than 1% between observations). Thus the viscous mechanism can't generally be the primary damping.

The paper checks the validity of its approximate solution at the time of predicted contact loss by the consistency check of slow variation. This is not sufficient. At the predicted termination time, no-longer-negligible torque from the air pressure has already changed the unperturbed dynamics (eq. 1 therein) by order 1.

Also, the flatness assumption becomes highly stringent at the named final frequency of 500 Hz. A surface undulation of, say, $10^{-7}$ m over 1/4 revolution would lead to bouncing contact.

A non-fluid-based theory not only gives finite settling time but also predicts that it should depend on the support surface, as observed with common disks. Rolling contact is dissipative (from inelastic contact deformation, micro-slip, and radiated vibrations). Assume, as one common class of rolling loss approximations, dissipation proportional to (contact speed)^beta, (typically, 1 <= beta <= 3). In a slow-varying small-angle theory like in the paper, this gives a finite-time singularity; and the slow-varying singular approximation will be inaccurate near stoppage for many reasons (one possibility being neglected fluid effects).

Although always intriguing, finite-time singularities within approximate theories are not so rare in classical mechanics. The “Painleve paradox" describes a smoothly sliding rod that eventually jambs with infinite acceleration; the rolling of a french curve (with zero curvature at the tail) on a flat surface leads to infinite forces as a singular configuration is approached; a very well-known simple bouncing ball model predicts an infinite number of bounces in finite time; and Newton's Principia describes the finite-time singularity as two gravitationally attracted particles fall in to one another.

A final open question concerns the “classical" solution quoted for a rolling disk. Really there is a three-parameter family of classical solutions, and even a two-parameter family of simply-expressed circular rolling motions. Without supporting comments, the paper selects for perturbation a one-parameter subset of rolling solutions: the ones shared with a frictionless disk. Play with coins, toys, *etc*., indeed shows that these seem to be about the right subset for final shuddering during real rolling of many disks. The possible dynamical attraction to this special one-parameter family of solutions seems likely (to me) to follow from dissipative side-slip effects in the rolling contact, a significant contact mechanism neglected in the paper. Without considering such effects there is no a-priori reason to expect that even a slowly damped perfect-rolling ideal disk should start at, or stay near, the assumed one-parameter family of solutions.

Andy Ruina
Professor,
Theoretical and Applied Mechanics and Mechanical Engineering
Cornell University, Ithaca, NY 14850
ruina@cornell.edu
June, 2000