Forces on a Siphon

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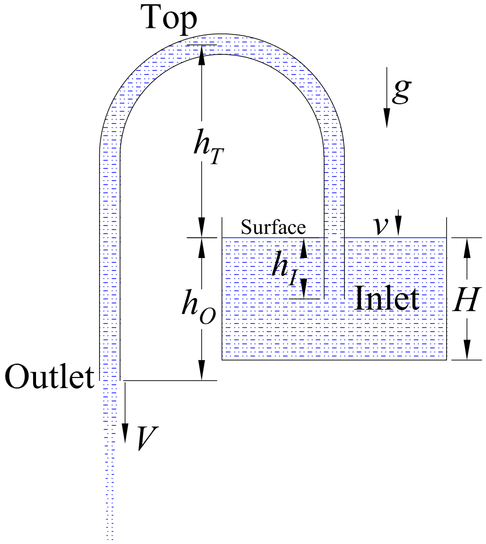
1 Problem

Discuss the forces on the fluid in a siphon, a version of which is sketched below. The reservoir that holds the fluid supply, and the siphon tube, are held at rest.



2 Solution

We restrict our analysis to the fluid in the reservoir and in the tube.[1]

The action of a siphon is driven by the force of gravity. Other forces on the fluid include air pressure, the normal force of the reservoir and tube on the water, and the tensile strength of water (acting at the surface of the outlet of the tube). We ignore viscosity and surface tension (capillary forces).

Already by 1881, classroom demonstrations of the operation of a siphon in vacuum were available (see pp. 166-167 of [1]). It was inferred from such demonstrations (see p. 211 of the 1905 edition of [1], quoted in English in [2]) that air pressure does not drive a siphon, but rather it is driven by gravity (particularly the force of gravity on the fluid in the siphon tube below the upper surface level of the fluid in the reservoir).[2]

Thus, a siphon has much in common with a "chain fountain" in which a chain/rope/cable can rise off a coil/pile on some surface and move in an arc to a location below that surface

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[1] This fluid constitutes the "control volume" in the language of fluid engineering.

[2] A siphon can be operated with a constant level of the fluid in the reservoir, such that air pressure cannot do any work on the fluid [3]. Siphons operated at altitudes between 0 and 40,000′ above sea level have the same flow rate [4], which reinforces that air pressure is not the driving force in a siphon.

given appropriate initial conditions. See, for example, [5, 6]. As discussed in Sec. 2.2.3 of [6], the arc of the chain reaches a maximum height H above the coil/pile of chain related to the speed V of the arc of the chain by $V^2 \approx 7gH/6$.

In contrast, the speed V of the fluid in the tube (and at the outlet) of a siphon is reasonably well approximated by assuming conservation of energy in the motion of the fluid in the siphon, which implies that

$$V^2 \approx 2gh_O, \tag{1}$$

where h_O is the difference between the heights of the top surface of the fluid in the reservoir and the outlet O of the siphon tube. This argument is often stated as an application of Bernoulli's law to the (quasisteady) flow of liquid in the siphon.

Turning to some details of the motion of the fluid in the siphon, we first note that for an incompressible fluid,

$$Va = v(A - a) \approx vA, \tag{2}$$

where $a \ll A$ is the cross sectional area of the siphon tube (assumed here to be uniform along the tube, although this need not be so), A is the area of the bottom surface of the reservoir, v is the downward velocity of the top surface of the fluid in the reservoir,

$$v = -\frac{dH}{dt} = -\dot{H} \approx V\frac{a}{A}\,, \tag{3}$$

and H is the height of the fluid in the reservoir.

We now consider the force $\mathbf{F}$ on the fluid inside the reservoir and siphon tube, ignoring the tiny forces due to atmospheric pressure on the top of, and on the hole in the bottom of, the fluid in the system, as well as effects of friction/viscosity and surface tension, and write

$$\mathbf{F} = m\mathbf{g} + \mathbf{N} + \mathbf{F}_{\text{tensile}} = \frac{d\mathbf{p}}{dt}\,, \tag{4}$$

where m is the mass and $\mathbf{p}$ is the momentum of the fluid in the reservoir plus siphon tube (*i.e.*, the control volume), and $\mathbf{g}$ is the acceleration due to gravity.

We evaluate the total force $\mathbf{F} = d\mathbf{p}/dt$ (according to Newton's 2nd law) in the approximation that the fluid in the reservoir, excluding that inside the siphon tube, has the same (downward) velocity v given by eq. (3), and that momentum, velocity and forces $\mathbf{F}$, $m\mathbf{g}$ and $\mathbf{F}_{\text{tensile}}$ are positive when downward, while the normal force $\mathbf{N}$ is positive when upwards. Then, the vector equation (4) can be written as the scalar equation,

$$F = mg - N + F_{\text{tensile}} = \frac{dp}{dt}. \tag{5}$$

The mass of the fluid in the reservoir plus siphon tube is

$$m = \rho[AH + a(l - h_I)], \tag{6}$$

where ρ is the mass density of the fluid, l is the length of the siphon tube, and h_I is the height of the upper surface of the fluid in the reservoir above the inlet to the siphon tube.

The water pressure P at the bottom of the reservoir is approximately uniform, with value $P \approx \rho g(H + h_T)$, where h_T is the height of the top of the siphon above the upper surface of the fluid in the reservoir. Then, for a first approximation, the upward normal force is

$$N \approx PA \approx \rho g\,(H + h_T)A. \tag{7}$$

In this first approximation, the momentum of the fluid in the reservoir (outside the siphon tube) is vertical and downward, with

$$p_{\rm reservoir} \approx \rho A H v - \rho a h_I V = \rho a H V - \rho a h_I V = \rho a V(H - h_I), \tag{8}$$

recalling eq. (2). The momentum of the fluid in the siphon tube is

$$p_{\rm tube} \approx \rho a(h_O - h_I)V, \tag{9}$$

so the momentum of the fluid in the reservoir plus siphon tube is

$$p = p_{\rm reservoir} + p_{\rm tube} \approx \rho a(H + h_O - 2h_I)V \approx \rho a(H + h_O - 2h_I)\sqrt{2gh_O}, \tag{10}$$

recalling eq. (1).

The rate of change of the momentum p of water inside the tank is the sum of $\partial p/\partial t$ and the rate $dp_{\rm leaving}/dt$ of momentum of water leaving the tank,

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + \frac{dp_{\rm leaving}}{dt}\,. \tag{11}$$

For the latter, we note that the mass of the fluid leaving the siphon tube (with velocity V through the hole of area a) during time dt is $dm = \rho\, aV\,dt$, so the momentum that leaves the tank during time dt is,

$$dp_{\rm leaving} = dm\,V = \rho\, aV^2\,dt, \qquad \frac{dp_{\rm leaving}}{dt} = \rho\, aV^2 \approx 2\rho a g h_0, \tag{12}$$

recalling eq. (1).

From eq. (10), noting that $\dot H = \dot h_I = \dot h_0$,

$$\frac{\partial p}{\partial t} = \rho a(H + h_O - 2h_I)\frac{dV}{dt}\,. \tag{13}$$

For a tube of small area $a \ll A$, the time rate of change dV/dt, of order a. Then, $\partial p/\partial t$ is of order a^2, so we can neglect this term in the first approximation, and we have that

$$\begin{aligned}\frac{dp}{dt} &\approx \frac{dp_{\rm leaving}}{dt} \approx 2\rho a g h_0 = F = mg - N + F_{\rm tensile}\\ &\approx \rho g[AH + a(l - h_I)] - \rho g A(H + h_T) + F_{\rm tensile}.\end{aligned} \tag{14}$$

and hence,

$$F_{\rm tensile} \approx \rho g[2ah_0 + Ah_T - a(l - h_I)] \approx \rho g A h_T, \tag{15}$$

where F_{tensile} acts at (or near) the (imaginary) surface of the outlet of the siphon tube. For large enough h_T, the tensile strength of the fluid will not be sufficient to prevent a bubble forming at the top of the siphon tube, after which the siphon action will cease.

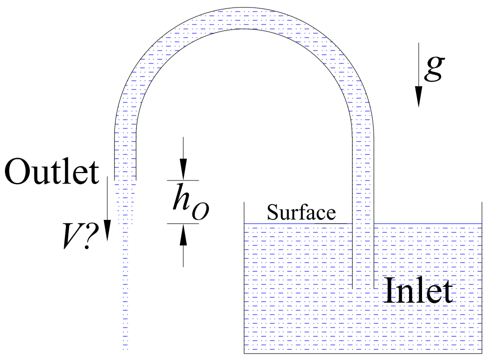
It is sometimes claimed that the limiting h_T is related by atmospheric pressure, $\rho g h_{T,\text{max}} \overset{?}{=} P_{\text{atmosphere}}$. However, with care, siphons can operate with tube height larger than this supposed limit. This was demonstrated by Duane already in 1902 [7]. See also [8, 9].[3]

Other papers on the physics of siphons include [19]-[37].

2.1 Can a Siphon Operate with its Outlet above the Fluid Level in the Reservoir? (Oct. 29, 2024)

We have argued that the column of fluid below the outlet of the siphon pulls downward on the fluid in the siphon. This suggests that it might be possible for a siphon to operate with its outlet above the level of the fluid in the reservoir, as sketched below.

In contrast, the usual argument the the flow velocity a the outlet is $V = \sqrt{2gh_O}$ for h_O positive as in the figure on p. 1, suggests that there will be no flow for h_O as in the figure below.



In a simple experiment with a siphon from whose outlet a vertical stream of water could flow undisturbed downward for about 75 cm, the siphon ceased to flow when the outlet was raised to the level of the water in the reservoir.[4] The uncertainty in the crude height measurement was about 1 mm, so the tensile force on the water just outside the outlet due to the falling water below must be very small when $h_O \approx 0$.

The water below the outlet of the siphon is approximately in "free fall", where it is not plausible that a large internal tensile force exists. It seems that the tensile force in the water must drop from a significant nonzero value in the siphon tube near the outlet to near zero at a height of order $\sqrt{a}$ (*i.e.*, of the radius of the outlet) below the outlet.

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[3] A closely related phenomenon is that a column of liquid in a tube sealed at its upper end (barometer) can have height h, above a reservoir of the fluid, with $\rho g h > P_{\text{atmosphere}}$, as perhaps first demonstrated by Donny in 1846 [10], who claimed that this was anticipated by Laplace. See also [11]-[18].

[4] The siphon flow did continue when the outlet was above the inlet, but below the upper surface of the water in the reservoir.

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