Single-Bubble Sonoluminescence

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(February 2, 1995)

1 Problem

In the phenomenon of single-bubble sonoluminescence, a water bubble of initial radius of 40 μm is observed to emit light when its radius collapses to about 0.5 μm under one atmosphere pressure. Approximately 6×10^6 photons are emitted in the energy range 1-6 eV, with a bremsstrahlung-like spectrum of the form $dN \propto dE/E$ where E is the photon energy. (Water is opaque to photons above about 6 eV.)

In this problem the bubble can assumed to contain vacuum.

a) Suppose that all the kinetic energy of the collapsing bubble is converted to photons with the spectrum $dN \propto dE/E$. What would be the maximum photon energy emitted?

b) At what radius does the velocity of the inner surface of the bubble reach the speed of sound in water, 1,500 m/s?

2 Solution

This problem is based on measurements by Putterman *et al.*[1], and references therein.

a) The bubble is a spherical cavity of initial radius R_0 in an infinite reservoir of (incompressible) water at pressure $P = 1$ atm $= 10^5$ N/m^2 far from the bubble. As the bubble collapses, the pressure does work on the decreasing volume, which energy goes to increasing the kinetic energy of the moving fluid [2].

When the bubble has collapsed to radius R its volume has changed by amount,

$$\Delta V = \frac{4\pi}{3}(R_0^3 - R^3), \tag{1}$$

so the work done is,

$$KE = W = P\Delta V = \frac{4\pi P}{3}(R_0^3 - R^3) \approx \frac{4\pi R_0^3 P}{3}\,, \tag{2}$$

where the approximation holds once $R \ll R_0$.

For example, if a 40-μm-radius water bubble collapses under 1 atmosphere pressure, the maximum kinetic energy of the system is (in MKSA units),

$$KE_{\rm max} \approx \frac{4\pi(4 \times 10^{-5})^3 10^5}{3} \approx 2.5 \times 10^{-8} \text{ J} \approx 1.5 \times 10^{11} \text{ eV}. \tag{3}$$

Suppose that all the energy is dissipated in sonoluminescence with a photon spectrum $dN \propto d\nu/\nu$, and that at frequencies above 1 eV but below the water transmission cutoff (6 eV) a total of 6×10^6 photons are observed.

We write the photon spectrum as $dN = a dE/E$. Then, the observed number of photons determines parameter a via,

$$6 \times 10^6 = a \int_1^6 \frac{dE}{E} = a \ln 6 = 1.8a, \tag{4}$$

so $a = 3.3 \times 10^6$.

The maximum photon energy, $E_{\max}$, is now determined by,

$$KE = 1.5 \times 10^{11} \text{ eV} = \int_0^{E_{\max}} E \, dN = a \int_0^{E_{\max}} dE = aE_{\max} = 3.3 \times 10^6 E_{\max}. \tag{5}$$

Thus $E_{\max} = 45,000$ eV $= 45$ keV.

b) If the surface of the bubble has (radial) velocity $v(R)$, then the fluid velocity at a radius $r > R$ is also radial, with a value that follows from the continuity equation,

$$0 = \nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{dr^2 v}{dr}. \tag{6}$$

[Use Gauss' law if don't remember the divergence in spherical coords.] Hence, $r^2 v$ is constant throughout the fluid at any given time, and so,

$$v(r) = \frac{R^2}{r^2} v(R). \tag{7}$$

The total kinetic energy of the moving fluid, of density ρ, is,

$$KE = \int_R^\infty 4\pi r^2 \, dr \frac{\rho v^2(r)}{2} = 2\pi \rho v^2(R) R^4 \int_R^\infty \frac{dr}{r^2} = 2\pi \rho v^2(R) R^3. \tag{8}$$

Equating (2) and (8) we find that,

$$v^2(R) = \frac{2P}{3\rho} \left(\frac{R_0^3}{R^3} - 1 \right) \approx \frac{2P}{3\rho} \frac{R_0^3}{R^3}, \tag{9}$$

where the approximation holds once $R \ll R_0$

The inner surface of a water bubble of initial radius $R_0 = 40$ μm reaches velocity $v_{\text{sound}} \approx 1,500$ m/s at radius,

$$R = R_0 \sqrt[3]{\frac{2P}{3\rho v_{\text{sound}}^2}} = 40 \ \mu\text{m} \ \sqrt[3]{\frac{2 \times 10^5}{3 \cdot 10^3 \cdot (1.5 \times 10^3)^2}} = 1.4 \ \mu\text{m}. \tag{10}$$

This radius is about three times larger than that at which the sonoluminescence is observed to occur. Since the water velocity is hypersonic for $R < 1.4$ μm it is likely that other dissipative mechanisms besides sonoluminescence are involved, and that maximum photon energy is less than 45 keV as found in part a).

References

[1] K.R. Weninger, B.P. Barber and S.J. Putterman, *Pulsed Mie Scattering Measurements of the Collapse of a Sonoluminescing Bubble*, Phys. Rev. Lett. **78**, 1799 (1997), http://kirkmcd.princeton.edu/examples/fluids/weninger_prl_78_1799_97.pdf

[2] Lord Rayleigh, *On the Pressure developed in a Liquid during the Collapse of a Spherical Cavity*, Phil. Mag. **34**, 94 (1917), http://kirkmcd.princeton.edu/examples/fluids/rayleigh_pm_34_94_17.pdf