Tokyo Drift

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1 Problem

In the movie *The Fast and the Furious: Tokyo Drift*,[a] the bad guy "drifts" a Nissan 350Z at constant speed up a spiral ramp in a parking garage. The radius of the spiral is 11 m, and the slope of the spiral is 7.5°. The coefficient of static friction between the tires and the road is $\mu_s = 0.48$, and the coefficient of kinetic friction is $\mu_k = 0.42$.

[a] At about 3:00 in https://www.youtube.com/watch?v=Rv1ODNBw4Zw

a. What is the maximum speed that the car could drive in a horizontal circle of radius $r = 11$ m without "drifting"? That is, the car's wheels roll without slipping in this part.

b. How fast could the car "drift" in a horizontal circle of radius 11 m? When "drifting", all four wheels slip with respect to the road.

c. How fast could the car drive up the spiral (helical) ramp, at constant speed v, without "drifting?"

   Hint: The spiral motion of the car can be decomposed into motion in a horizontal circle at constant speed v_h plus vertical motion at constant speed v_y.

   The slope of 7.5° is always in the direction of motion of the car (and not perpendicular to this direction as for a banked curve).

d. How fast could the car drive up the spiral (helical) ramp, at constant speed v, when "drifting"?

2 Solution

a. The horizontal circle is on a horizontal plane. There is no tilting or banking in parts a and b.

For rolling without slipping, the friction is static friction, and the frictional force is perpendicular to the velocity vector $\mathbf{v}$. For motion in a circle, the frictional force $F_{\rm fr}$ provides the needed centripetal acceleration, $F_{\rm fr} = mv^2/r$, where m is the mass of the car. For motion in a horizontal plane, the normal force is vertical and has magnitude $N = mg$. Hence, the maximum speed v of the circular motion is related by,

$$\frac{mv^2}{r} = F_{\rm fr} = \mu_s N = \mu_s mg, \tag{1}$$

and

$$v = \sqrt{\mu_s gr} = \sqrt{0.48 \cdot 9.8 \cdot 11} = 7.1 \text{ m/s} = 25.9 \text{ km/hr}. \tag{2}$$

b. The analysis is the same as for part a, except that now the friction is kinetic rather than static. Hence,

$$v = \sqrt{\mu_s gr} = \sqrt{0.42 \cdot 9.8 \cdot 11} = 6.73 \text{ m/s} = 24.2 \text{ km/hr}. \tag{3}$$

While "drifting" is very cinematic, it is actually slower that driving without "drifting".

The assumption in part b is that sliding friction provides the needed centripetal force, although the direction of this force is at right angles to the direction of motion. Usually, the direction of sliding friction is opposite to the direction of motion. Great skill is required to drive the car so that the assumption is valid.

c. The slope of 7.5° of the spiral ramp is always in the direction of motion of the car (and not perpendicular to this direction as for a banked curve). The normal force makes angle 7.5° to the vertical, and the force of (static) friction is the plane perpendicular to the normal force.

Because the car is driving up the slope, with no acceleration along the direction of the normal force, the latter is now only,

$$N = mg\cos 7.5^\circ. \tag{4}$$

For the car to be able to drive uphill at constant speed, there must be a component of friction that points uphill to counteract the component of the force of gravity downhill,

$$F_{\rm fr,uphill} = mg\sin 7.5^\circ. \tag{5}$$

The force of (static) friction also provides the centripetal force, which is horizontal and perpendicular to the slope. The maximum force of (static) friction is,

$$F_{\rm fr} = \mu_s N = \mu_s mg\cos 7.5^\circ. \tag{6}$$

Hence, the maximum centripetal force is given by,

$$F_{\text{cent}} = \sqrt{F_{\text{fr}}^2 - F_{\text{fr,uphill}}^2} = mg\sqrt{\mu_s^2 \cos^2 7.5^\circ - \sin^2 7.5^\circ}. \tag{7}$$

The horizontal component of the velocity $\mathbf{v}$ is $v_h = v \cos 7.5^\circ$. The analysis of the horizontal circular motion is $mv_h^2/r = F_{\text{cent}}$, so that,

$$v_h = \sqrt{gr\sqrt{\mu_s^2 \cos^2 7.5^\circ - \sin^2 7.5^\circ}}. \tag{8}$$

Thus,

$$v = \frac{v_h}{\cos 7.5^\circ} = \frac{\sqrt{gr\sqrt{\mu_s^2 \cos^2 7.5^\circ - \sin^2 7.5^\circ}}}{\cos 7.5^\circ} = \frac{\sqrt{9.8 \cdot 11 \cdot 0.458}}{0.991} = 7.09 \text{ m/s}. \tag{9}$$

As expected, eq. (9) reduces to eq. (2) as the slope of the spiral ramp goes to zero.

IF the problem had been one of motion on a horizontal banked curve whose slope is 7.5°, *the maximum velocity would be obtained when the force of (static) friction points down the slope with magnitude* $\mu_s N$,

$$v_{\max} = \sqrt{gr\frac{\sin 7.5^\circ + \mu_s \cos 7.5^\circ}{\cos 7.5^\circ - \mu_s \sin 7.5^\circ}} = 8.4 \ m/s. \tag{10}$$

d. Again, the analysis for “drifting” is the same as without “drifting”, but with the coefficient of friction being kinetic rather that static.

$$v = \frac{v_h}{\cos 7.5^\circ} = \frac{\sqrt{gr\sqrt{\mu_k^2 \cos^2 7.5^\circ - \sin^2 7.5^\circ}}}{\cos 7.5^\circ} = \frac{\sqrt{9.8 \cdot 11 \cdot 0.395}}{0.991} = 6.59 \text{ m/s}. \tag{11}$$