Transient Voltage in a Long Circuit

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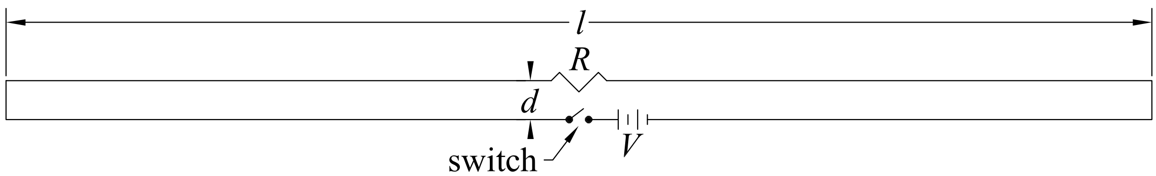
Several contentious videos[1] have discussed when a voltage can be detected in a load resistor R (perhaps a light bulb) in a "long" circuit, powered by a "battery" of voltage V, after a switch in the "other" wire, near the resistor, is closed, as sketched in the figure below. The circuit is rectangular, with length l and width $d \ll l$. The load resistance R is large compared to that in the long wires of the circuit.



The discussion in the videos was marred by the lack of a definition of what it means to "detect" a voltage, and also by a lack of awareness of the lore of antennas, including possible reflections of transient waves off a "ground" plane near the antenna.

Since electromagnetic effects propagate at the speed c of light, some voltage/current arises in the resistor at time $t = d/c$ after the switch is closed. But, the tiny initial voltage may not be detectable above the noise voltage in the circuit.

We suppose the voltage across the load resistor is detectable once it exceeds $V_{\rm detect}$, and define the fraction $f_{\rm detect} = V_{\rm detect}/V$.

In Sec. 2 below we present empirical evidence that in the absence of "interference" effects from the environment of the circuit, the voltage across the load resistor rises roughly linearly between time $t = d/c$ and time $t = l/c$ when the waves that propagate away from the switch and reflect of the ends of the circuit (at time $t - l/2c$) arrive at the load resistor. If so,

$$V_{\rm load}(t) \approx V \frac{t - d/c}{l/c - d/c}, \qquad f(t) = \frac{V_{\rm load}(t)}{V} \approx \frac{t - d/c}{l/c - d/c}. \tag{1}$$

Then, the load voltage is detectable at time $t_{\rm detect}$ related by

$$V_{\rm load}(t_{\rm detect}) = V_{\rm detect} \approx V \frac{t_{\rm detect} - d/c}{l/c - d/c}, \qquad t_{\rm detect} \approx \frac{d}{c} + f_{\rm detect} \frac{l - d}{c}. \tag{2}$$

If $f_{\rm detect} l \gg d$, we have[2]

$$t_{\rm detect} \approx f_{\rm detect} \frac{l}{c}. \tag{3}$$

The sections below provide various additional comments on this topic.

[1]The first of these is by "Veritasium" (Derek Muller),
https://www.youtube.com/watch?v=bHIhgxav9LY

[2]A viewer of the video linked in footnote 1 was offered 4 choices as to $t_{\rm detect}$, but eq. (3) was not among them.

1 Ordinary Circuit Analysis

The speed of electromagnetic waves is taken to be infinite in ordinary circuit analysis. Rather, the transient effect in a simple circuit with a battery and a resistor is related to the self inductance L of the circuit.

For the circuit shown on p. 1 above, we follow Maxwell[3] in noting that the self inductance is related by $\Phi = LI$ where Φ is the magnetic flux linked by the circuit when it carries (steady) current I. A reasonable approximation is to consider only the magnetic flux due to the current in the long wires (of length l) in the circuit, and to suppose that the (azimuthal) magnetic field $\mathbf{B}$ due to the current I falls off with transverse distance r from the (long) wires according to $B \approx \mu_0 I/2\pi r$, in SI units. Then, for wires of radius a, the magnetic flux through the circuit is, approximately,

$$\Phi \approx 2 \int_a^d \frac{\mu_0\, I}{2\pi r}\, dr = \frac{\mu_0\, Il}{\pi} \ln \frac{d}{a}, \qquad L = \frac{\Phi}{I} \approx \frac{\mu_0\, l}{\pi} \ln \frac{d}{a}. \tag{4}$$

The ordinary circuit equation is (for $t > 0$ after the switch is closed),

$$V = L\dot{I} + IR, \tag{5}$$

whose solution is

$$I(t > 0) = \frac{V}{R}\left(1 - e^{-Rt/L}\right), \qquad V_{\rm load}(t > 0) = I(t > 0)R = V\left(1 - e^{-Rt/L}\right). \tag{6}$$

In this approximation, the voltage across the load resistor rises from zero to its steady-state value of V with time constant $\tau = L/R$.

We illustrate this for the example of the circuit studied in a second video of Veritasium[4] in which $d = 1$ m, $l = 20$ m, and the load resistor had value $R = 1.1$ kΩ. For this case, the estimate (4) of the self inductance is, for radius $a = 1$ mm of the wire, $L \approx 5.5 \times 10^{-4}$ Henries, and the time constant is $\tau = L/R \approx 5 \times 10^{-7}$s = 500 ns. This is almost ten times $l/c = 66$ ns, the time for the electromagnetic wave, emitted at the switch, to be reflected by the far ends of the circuit and arrive at the load resistor.

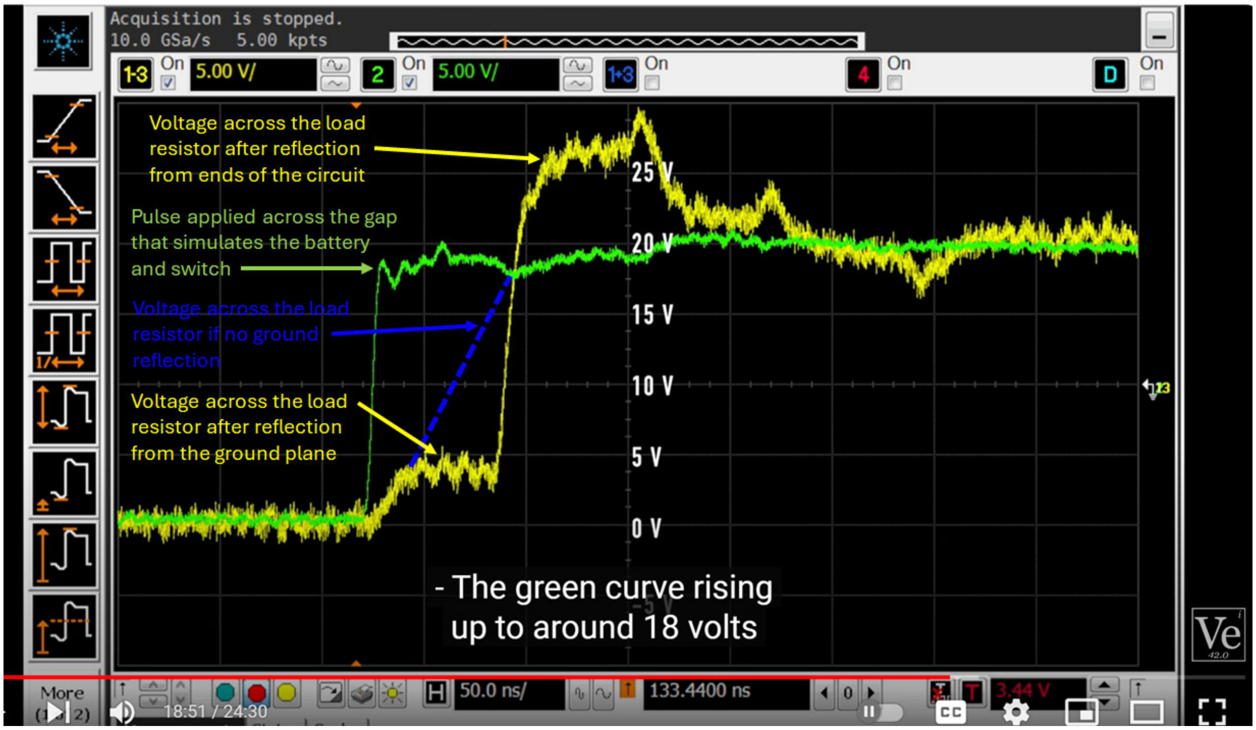
A better circuit-analysis approximation for a long circuit is that the long, parallel wires form a transmission line, described by a sequence of capacitors between the wires and inductors along them. This is often called the "lumped circuit" approximation, and is mentioned in the video of footnote 4 around time 16:50. If the "lumped circuit" consisted of 20 (square) cells, each $1_{\rm cell} = 1$ m in length, the estimate of the self inductance of each cell would be 1/10 of that for the whole circuit. Hence, the estimated rise of the voltage across load resistor (which is in the central cell of the "lumped circuit") would be 1/10 of the above estimate (taking the central cell to be an underdamped R-L-C circuit), *i.e.*, about 50 ns. Then, nearly full voltage V across the load resistor would be achieved by time $l/c \approx 66$ ns. In this model, some voltage drop across the load resistor would exist "instantaneously" after the switch is closed.

[3]See Art. 685 of Maxwell's *Treatise*, http://kirkmcd.princeton.edu/examples/EM/maxwell_treatise_v2_73.pdf

[4]https://www.youtube.com/watch?v=oI_X2cMHNe0

2 Veritasium's Second Video and the Effect of a Ground Plane

The figure below is from time 18:51 of the video by Veritasium linked in footnote 4.



The green curve shows the voltage pulse applied to a small gap in the center of line not containing the load resistor (which gap simulates the switch and battery), with the horizontal time scale of 50 ns per large division and 10 ns per small division. The rise time of this pulse was about 10 ns, which is greater than the transit time $d/c \approx 3$ ns between the switch and the load resistor. This complicates the interpretation as to the time delay before the voltage drop across the load resistor, shown by the yellow curve, is "detectable".

It seems (to me) that the voltage across the load resistor was "detectable" about 5 ns after the start of the applied pulse.

The voltage drop across the load resistor seemed to rise linearly with time for $5 < t < 20$ ns (taking $t = 0$ at the start of the applied pulse). The observed voltage leveled off for $20 < t < 66$ ns, and "rang" with period of 20 ns. However, if the linear rise had continued, the load-resistor voltage would have reached the full applied voltage at time $t \approx 66$ ns $= l/c$.

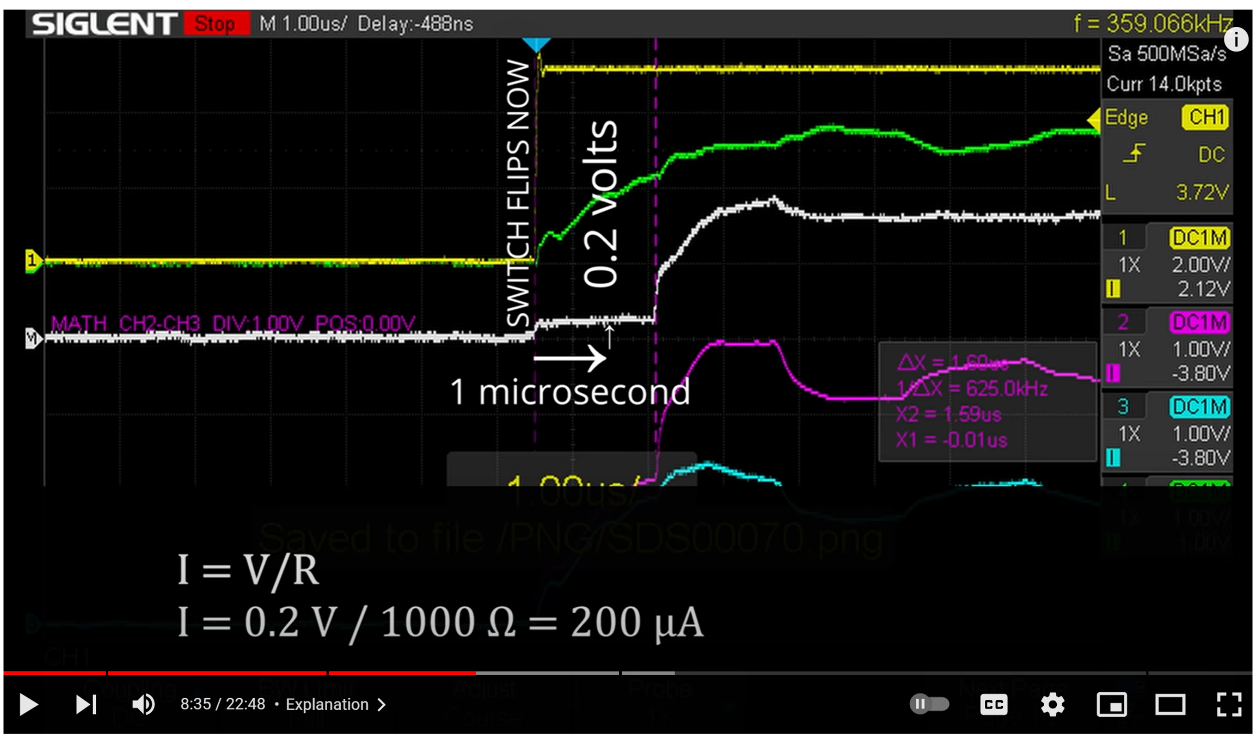
The inference is that some environmental effect, associated with a time scale of 20 ns, affected the voltage drop across the load resistor. Most likely the "ground", some $h = 10$ ft below the circuit, was a good conductor, and waves moving down from the circuit reflected off it, back up onto the circuit. The period for such reflections would be $2h/c \approx 20$ ns, recalling that the speed of light is close to 1 ft per ns. Such "ground" effects are common in antenna systems. In the present example, the paved surface (visible around 1:00 in the video) on which the circuit was mounted probably contained iron reinforcing rod (rebar), and so was a good mirror for the electromagnetic waves emitted at the small gap.

An effect of the waves reflected off the "ground" was to stop the growth of the voltage drop across the load resistor until time $t \approx 66$ ns, when waves emanating from the gap/switch arrived at the load resistor after reflecting off the ends of the circuit. At this time the voltage across the load resistor jumped to greater than the applied-pulse voltage, and "rang" for several periods of 66 ns until stabilizing at a steady voltage equal to the applied-pulse voltage.

In the video of footnote 4, great importance was attached to the load-resistor voltage for time $20 < t < 66$ ns, noting that if the load resistor were a light bulb, that bulb would be perceived as being "on", although somewhat dim compared to the eventual full brightness. However, this behavior was the result of "ground" reflections that would not be present in a more ideal experiment. It remains that the initial linear rise of the voltage drop of the load resistor was "detectable" very soon after the transit time $d/c \approx 3$ ns, when electromagnetic waves first arrived at the load resistor and current started to flow in it.

3 The Video of AlphaPhoenix

Inspired by Veritasium's first video, AlphaPhoenix posted a video[5] soon thereafter (and before Veritasium's second video) that studied a circuit with $l \approx 500$ m, $d \approx 0.5$ m, $R \approx 1.1$ kΩ, driven by a 5-volt pulse. The observed voltage across the load resistor is shown as the white trace in the figure below (at time 8:35 in the video)



Soon after the start of the applied pulse, a small voltage (0.2 V) appeared across the load resistor, and persisted until the wave reflected off the ends of the circuit arrived at the load resistor some 1.6 μs later. This first interval of constant load voltage is (we infer) associated

[5] https://www.youtube.com/watch?v=2Vrhk5OjBP8

with the reflection off the (conducting) water table under the surface of the field where the experiment was performed.

Later in the video, the ends of the circuit (at ± 250 m from the load resistor) were cut, such the currents associated with the reflected wave were opposite to those when the ends of the circuit were connected. As a result, the voltage across the load resistor changed sign at $t \approx 1.6$ μs, and damped to zero after a few periods of "ringing". This can be seen in the figure below, from time 18:21 in the video.

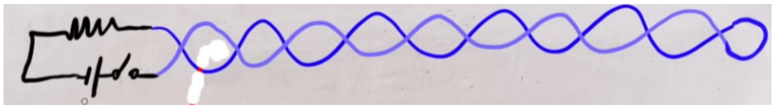
4 Use of a Twisted Pair

Interference from the environment in a long circuit can be greatly suppressed by use of a twisted pair of lines, as is well known in the telephone industry. The long circuit shown on p. 1 above would have been much more immune to effects of reflections off the "ground" plane if the long wires had been twisted. But, if the distance d between the long wires is large (of order 1 m), implementing a twisted pair would require considerable effort.

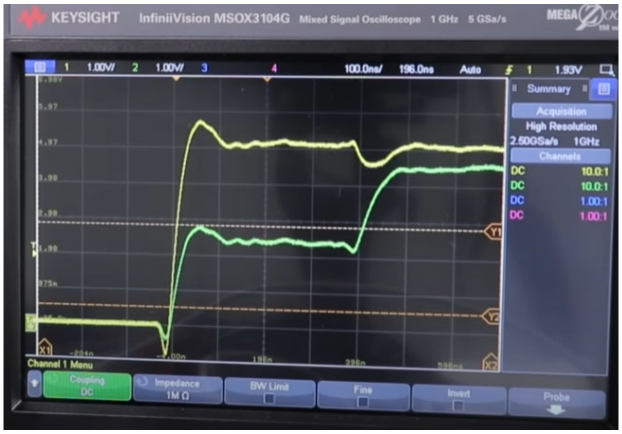
A study using a telephone twisted pair, with $d \approx 1$ mm was made (as a response to Veritasium's first video) by "Electroboom" (Medhi Sadaghdar)[6] using a 40-m long cable in which the propagation speed was about $2c/3$ and the transite time from end to end was about 200 ns.

The circuit configuration is sketched below (from time 10:08 of the video), in which the layout is "one-sided" with respect to the battery/switch. The 40-m-long twisted pair was wound onto a spool with a diamter of a few cm.

[6] https://www.youtube.com/watch?v=9hhcUT947FI



The lower traces in the oscilloscope screenshots below (with 100 ns per division for the horizontal time scale) show the response for the twisted pair "shorted" at is far end (left, time 13:40 of the video) and "open" at the far end (right, time 13:49 of the video).



These data are much "cleaner" than those shown on the previous pages, in which reflections from the environment were important.

When the twisted pair was "shorted" at its far end, the voltage drop across the load resistor was close to that of the applied pulse (upper trace) after only one "ring", and became equal to the applied voltage shortly thereafter.

In contrast, when the far end of the twisted pair was "open", the voltage across the load resistor was very close to its steady state value of zero already by 500 ns after the drive pulse was applied.