Can a Static Transmission Line Be a Source of Infinite Power?

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(July 9, 2022)

1 Problem

Consider a transmission line, unterminated at its ends, and charged to some static potential difference V between its two conductors. The static field of the transmission line can be thought of as the sum of two electromagnetic waves traveling along the line, and reflecting off its ends. Each of these waves is associated with electric currents I inside the conductors of the transmission line, so each generates power at the rate I^2R where R is the (small but nonzero) electrical resistance between the two ends of the line.

So long as the transmission line remains charged, it should be generating power according to the above argument, which eventually can be arbitrarily large. Some of this power would be continously radiated out of the transmission line at its two ends [1].

Can this be so?

2 Solution

The principle of conservation of energy indicates that the above argument is wrong, and the static field of the charged transmission line is not truly equivalent to two counterpropagating electromagnetic waves with equal and opposite electric currents; otherwise we would have a solution to the "energy crisis".

Yet, there are examples in which it is useful to note that a static field associated with a transmission line, when perturbed, is equivalent to the sum of two traveling waves.

2.1 Discharge of a Transmission-Line Capacitor

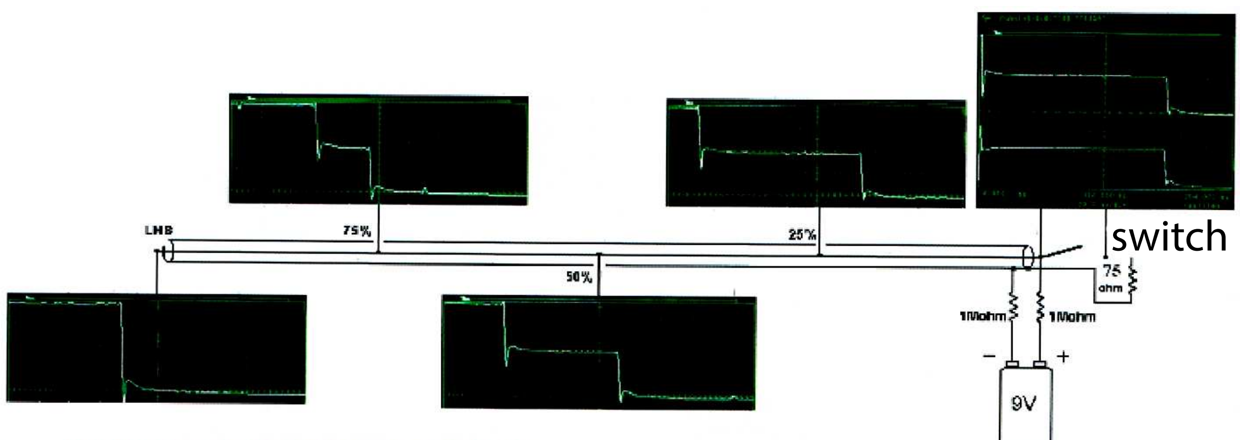


Figure 6: Waveforms seen at relevant points in the charged cable

An older pulse generator, Tektronix model 109 [2], used an external coaxial cable (transmission line) of length l as its "charging line," in which the speed of electromagnetic waves is v. First, this cable was charged to twice to voltage of the desired pulse voltage, and then discharged into a load (via an output transmission line of the same impedance as the "charging line") to generate a pulse whose length $2l/v$ was twice that of the transit time l/v of electromagnetic waves along the "charging line".

This procedure is illustrated in the figure on the previous page, from [3].[1] The lower trace in the top-right photo shows the nominal pulse, as also seen in the enlargement below.

The top trace in Fig. 1 below was observed at the end of the "charging line" next to the switch that initiates the pulse. As noted above, the pulse voltage is half the charging voltage.

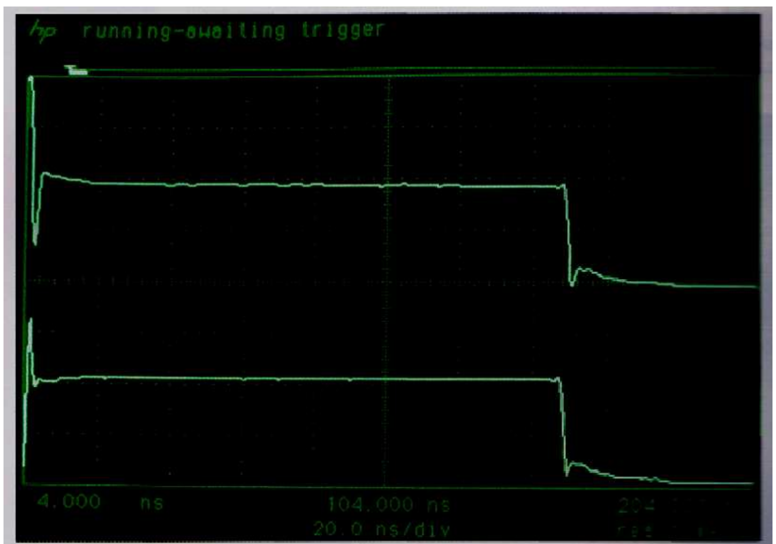


Figure 1: Bottom Trace: The bottom trace probe is across the 75Ω terminator used as a trigger. It shows a pulse of the half battery voltage (actually 8V after the 2 x 1MΩ resistors and the probe loading);
Top trace: Left-hand side of the reed switch. The trace immediately drops from 8V to 4V

This is easily understood by considering the static field in the charging line (prior to closing the switch) as the sum of two counterpropagating waves, each with half of the charging voltage.

The other photos in Fig. 6 show the voltage waveforms observed at several points along the charging line, and are also readily understood by the model that prior to closing the switch there existed two waves moving in opposite directions, each of half the static voltage.[2] Then, the wave that propagates towards the switch is reflected off it when the switch is open, but passes over it when the switch is closed. The wave that propagates towards the open end of the "charging line" (to the left in Fig. 6) reflects off that end to produce a pulse (of the same sign) that then propagates toward the switch and also passes over it. As such, the length of the "output" pulse is twice the transit time of the "charging line", and the pulse amplitude

[1]For an earlier report of this, see [4].

[2]The wave that moves to the left has negative current, associated with a flow of charge to the left. The wave that moves to the right has positive current, associated with a flow of charge to the right. That is, the two currents are equal and opposite.

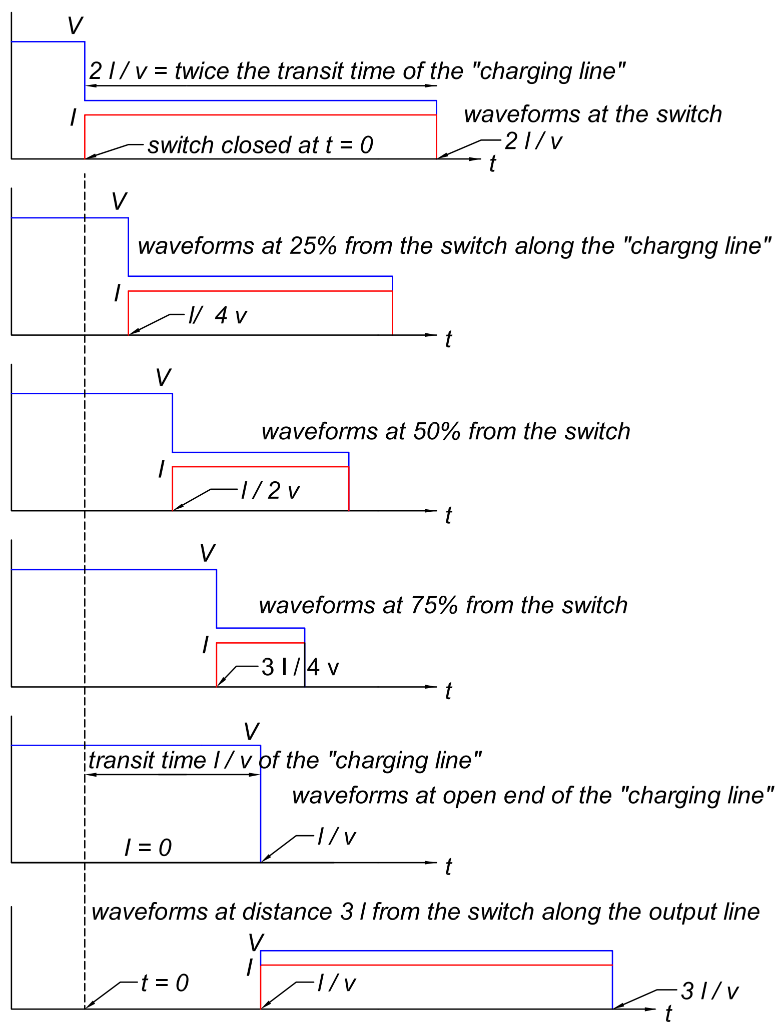
is half the voltage of the static field.

However, it is better not to suppose that these traveling waves actually existed prior to the closure of the switch.

Rather, it is more consistent to consider that the closure of the switch generates a “signal” that propagates away from the switch at the speed of light (in the dielectric of the coaxial cable) which starts currents in the cable corresponding to a wave of half the static voltage, with the wave motion towards the switch.

No currents existed in the cable prior to the closure of the switch. There were no equal and opposite currents in the cable, as would be the case if the static field actually consisted of a pair of counterpropagating waves. That is, electric currents in resistive conductors involve electrons in motion, colliding with almost every ion of the “lattice” of the conductor in the path of the electron. Equal and opposite currents involve electrons moving in both directions, only rarely colliding with other electrons; which is very different from zero electric current.

If current waveforms had been reported in Fig. 6, they would have shown zero current until the moments that the voltage dropped, from its static value to half that value, at the various sampled locations along the “charging line”. These currents would be nonzero, and approximately constant, for the times that the voltage remained half the “charging” voltage, as sketched below.



At any distance greater than $2l$ from the switch along the output transmission line, the waveform is just a pulse of length $\Delta t = 2l/v$, as shown in the bottom figure above for distance $3l$.

Thanks to David Bower for e-discussions of this problem.

References

[1] K.T. McDonald, *Radiation from the Open End of a Coaxial Cable* (Dec. 2, 2009),
http://kirkmcd.princeton.edu/examples/coax_rad.pdf

[2] *Instruction Manual, Tektronix Type 109 Pulse Generator* (1963), pp. 2-2,3,
http://kirkmcd.princeton.edu/examples/EM/tek109_manual.pdf

[3] I. Catt, *The End of the Road?* Electronics World **119**(4), 72 (2013),
http://kirkmcd.princeton.edu/examples/EM/catt_ew_119-4_72_13.pdf

[4] I. Catt, *The Death of Electric Current*, Wireless World **86**(12), 79 (1980),
http://kirkmcd.princeton.edu/examples/EM/catt_ww_86-12_79_80.pdf