Radiation of Turnstile Antennas Above a Conducting Ground Plane

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
(September 18, 2008)

1 Problem

A "turnstile" antenna [1, 2] consists of a pair of linear dipole antennas oriented at 90° to each other, and driven 90° out of phase, as shown in Fig. 1.

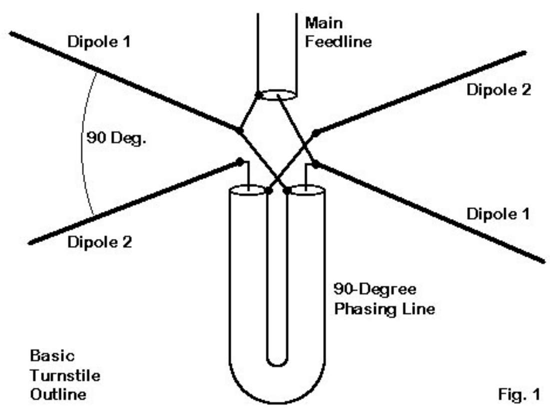

Figure 1: A "turnstile" antenna. From [2].

The linear antennas could be either dipoles as shown in the figure, or simply monopoles.

Consider the case that the length of the linear antennas is small compared to a wavelength, so that it suffices to characterize each antenna by its electric dipole $\mathbf{p}_{1,2}\, e^{-i\omega t}$, where the magnitudes p_1 and p_2 are equal but their phases differ by 90°, the directions of the two moment differs by 90°, *i.e.*, $\mathbf{p}_1 \cdot \mathbf{p}_2 = 0$, and ω is the angular frequency.

Discuss the angular distribution and the polarization of radiation by turnstile antennas in various configurations. The two antennas may or may not be at the same point in space, and they may or may not be above a conducting ground plane.

2 Solution

2.1 The Basic Turnstile Antenna

We first consider a basic turnstile antenna whose component antennas lie in the x-y plane at a common point. Then, we can write the total electric dipole moment of the antenna system as,

$$\mathbf{p} = \mathbf{p}_0\, e^{-i\omega t} = p_0(\hat{\mathbf{x}} + i\hat{\mathbf{y}}\, e^{-i\omega t}, \tag{1}$$

The electromagnetic fields in the far zone are then,

$$\mathbf{B} = k^2 \frac{e^{i(kr-\omega t)}}{r} \hat{\mathbf{r}} \times \mathbf{p}_0, \qquad \mathbf{E} = \mathbf{B} \times \hat{\mathbf{r}}, \tag{2}$$

whose components in spherical coordinates are

$$E_r = B_r = \hat{\mathbf{r}} \cdot \mathbf{B} = 0, \tag{3}$$

$$E_\theta = B_\phi = p_0 k^2 \frac{e^{i(kr-\omega t)}}{r} \cos\theta(\cos\phi + i\sin\phi), \tag{4}$$

$$E_\phi = -B_\theta = -p_0 k^2 \frac{e^{i(kr-\omega t)}}{r} (\sin\phi - i\cos\phi), \tag{5}$$

noting that $\hat{\mathbf{r}} \times \hat{\mathbf{x}} = \sin\phi\,\hat{\boldsymbol{\theta}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\phi}}$, and $\hat{\mathbf{r}} \times \hat{\mathbf{y}} = -\cos\phi\,\hat{\boldsymbol{\theta}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\phi}}$. In the plane of the antenna, $\theta = 90°$, the electric field has no θ component, and hence no z component; the turnstile radiation in the horizontal plane is horizontally polarized. In the vertical direction, $\theta = 0°$ or $180°$, the radiation is circularly polarized. For intermediate angles θ the radiation is elliptically polarized.

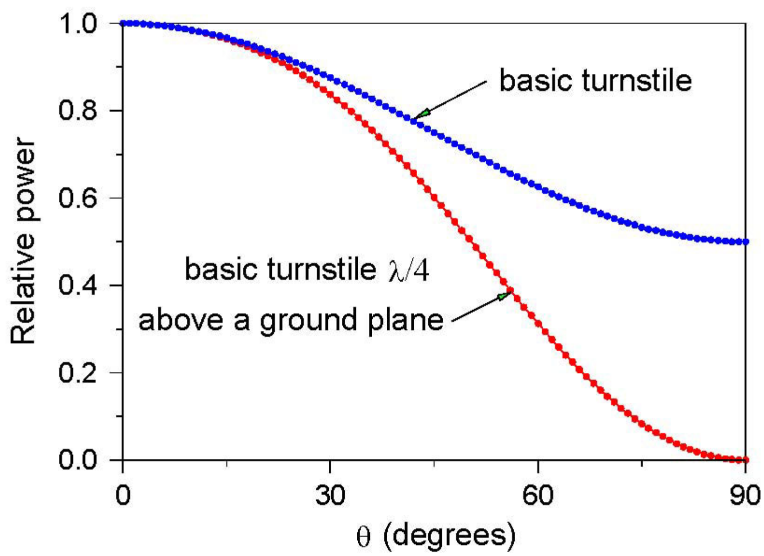

Figure 2: The relative radiation pattern of a basic turnstile antenna, and of a basic turnstile antenna at height $\lambda/4$ above a conducting ground plane.

The magnitudes of the fields are,

$$E = B = \frac{p_0 k^2}{r} \sqrt{1 + \cos^2\theta}, \tag{6}$$

so the time-averaged radiation pattern is,

$$\frac{dP}{d\Omega} = \frac{cr^2}{8\pi} B^2 = \frac{p_0^2 \omega^4}{8\pi c^3} (1 + \cos^2\theta). \tag{7}$$

The intensity of the radiation varies by a factor of 2 over the sphere, that is, by 3 dB, as shown in Fig. 2. Compared to other simple antennas, this pattern is remarkably isotropic. The radiated power is greatest for $\theta = 0$ or 180° in which directions the polarization is purely circular.

2.2 Separated Elements but No Ground Plane

If the first antennas is at the origin while the second is at position (x_2, y_2, z_2), then the path difference d from the two antennas to a distant observer in direction (θ, ϕ) is,

$$d = x_2 \sin\theta \cos\phi + y_2 \sin\theta \sin\phi + z_2 \cos\theta, \tag{8}$$

which introduces a phase difference kd (in radians) between the fields from the two antennas, where $k = \omega/c = 2\pi/\lambda$ is the wave number.

For greater generality, we also suppose that the second antenna is driven at phase δ with respect to the first. That is, we write the total electric dipole moment as,

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 = p_0(\hat{\mathbf{x}} + e^{i\delta}\hat{\mathbf{y}})e^{-i\omega t}, \tag{9}$$

The electromagnetic fields in the far zone are then,

$$\mathbf{B} = k^2 \frac{e^{i(kr-\omega t)}}{r} \hat{\mathbf{r}} \times (\mathbf{p}_1 + e^{-ikd}\mathbf{p}_2) = p_0 k^2 \frac{e^{i(kr-\omega t)}}{r} \hat{\mathbf{r}} \times (\hat{\mathbf{x}} + e^{i(\delta - kd)}\, \hat{\mathbf{y}}), \qquad \mathbf{E} = \mathbf{B} \times \hat{\mathbf{r}}, \tag{10}$$

whose components in spherical coordinates are,

$$E_r = B_r \;=\; \hat{\mathbf{r}} \cdot \mathbf{B} = 0, \tag{11}$$

$$E_\theta = B_\phi \;=\; p_0 k^2 \frac{e^{i(kr-\omega t)}}{r} \cos\theta(\cos\phi + e^{i(\delta-kd)} \sin\phi), \tag{12}$$

$$E_\phi = -B_\theta \;=\; -p_0 k^2 \frac{e^{i(kr-\omega t)}}{r} (\sin\phi - e^{i(\delta-kd)} \cos\phi). \tag{13}$$

The radiation is circularly polarized in the vertical direction ($\theta = 0$) whenever the phase factor obeys $\delta - kz_2 = (2n+1)\pi/2$ for some integer n. For example, the two antennas could be driven in phase ($\delta = 0$) and placed height $\Delta z = \lambda/4$ apart to achieve circular polarization.

The time-averaged radiation pattern is,

$$\frac{dP}{d\Omega} = \frac{cr^2}{8\pi} B^2 = \frac{p_0^2 \omega^4}{8\pi c^3} [1 + \cos^2\theta - (1 - \cos\theta) \sin 2\phi \cos(\delta - kd(\theta, \phi))]. \tag{14}$$

For example, if the two antennas are both along the z-axis and are driven in phase, then the strength of the radiation in the horizontal plane (for which $d = 0$) varies as $1 - \sin 2\phi$.

2.3 Both Elements Close to a Ground Plane

In the case that the x-y plane is a conducting ground plane and the antenna is just above this plane, an electric dipole $\mathbf{p} = \mathbf{p}_\perp + \mathbf{p}_\parallel$ has an image dipole $\mathbf{p}' = \mathbf{p}_\perp - \mathbf{p}_\parallel$, where $\perp$ and $\parallel$ label vector components perpendicular and parallel to the ground plane, respectively. The total electric dipole moment is $\mathbf{p}_{\text{total}} = 2\mathbf{p}_\perp$. Hence, if the antenna is close to the ground plane, such that phase differences between radiation from $\mathbf{p}$ and its image $\mathbf{p}'$ can be ignored, there is nonzero radiation only if the component $\mathbf{p}_\perp$ is nonzero. And, the polarization of this radiation is purely vertical. **No circular or elliptical polarization can be generated by any electrical dipole antenna all of whose elements are close to a conducting ground plane.**

In particular, the basic turnstile antenna considered in sec. 2.1 emits no radiation if placed close to a conducting ground plane.

If the elements of the turnstile antenna are rotated so the they lie in, say, the x-z plane, radiation is emitted, but the antenna is in effect a single vertical dipole antenna.

2.4 The Antenna Elements Need Not Be Close to the Ground Plane

The general case of a system of two linear dipole antennas above a ground plane can be described by two electric dipole moments, $\mathbf{p}_1$ at $(0, 0, z_1)$ and $\mathbf{p_2}e^{i\delta}$ at (x_2, y_2, z_2), and their image moments $\mathbf{p}'_1 = \mathbf{p}_{1\perp} - \mathbf{p}_{1\|}$ at $(0, 0, -z_1)$ and $\mathbf{p}'_2 = \mathbf{p}_{2\perp}e^{i\delta} - \mathbf{p}_{2\|}e^{i\delta}$ at $(x_2, y_2, -z_2)$. The fields and the radiation pattern of this antenna system can be deduced by a small generalization of the methods of secs. 2.1 and 2.2, the but results are lengthy.

We first consider the special case of a basic turnstile antenna whose elements are parallel to the x-y plane, but at a height z_a above that plane. Then, the dipole moment of the physical antenna can be written as in eq. (1), and the dipole moment of the image antenna is the negative of this. However, there is a path difference $2z_a cos\theta$ between the physical and the image antennas to an observer at angle θ to the vertical. This implies that the fields of the total antenna system are those of eqs. (3)-(5) multiplied by,

$$1 - e^{-2ikz_a \cos\theta} \tag{15}$$

(which vanishes for $z_a = 0$ so that there is no radiation emitted in this case). The polarization of the radiation is the same as in sec. 2.1, so when nonzero radiation is emitted in the vertical direction, it is purely circularly polarized.

The radiated power is given by eq. (7), multiplied by the absolute square of eq. (15),

$$\frac{dP}{d\Omega} = \frac{p_0^2\omega^4}{4\pi c^3}(1 + \cos^2\theta)[1 - \cos(2kz_a\cos\theta)]. \tag{16}$$

For height $z_a = \lambda/4$ the radiation pattern peaks at $\theta = 0$ and vanishes in the horizontal plane, as shown in Fig. 2. Hence, placement of a basic turnstile antenna at this height above a conducting ground plane improves the relative amount of power that goes into circularly polarized radiation.

We now consider whether it is possible to arrange that circularly polarized radiation be emitted in a direction parallel to the ground plane. The “mirror” symmetry between the physical antenna and the image antenna precludes this, because if the physical antenna emits right-handed circularly polarized radiation parallel to the ground plane, then the image antenna emits left-handed circularly polarized radiation in the same direction, such that the sum of these two is linearly polarized radiation parallel to the ground plane.

Acknowledgment

Thanks to Jacques Audet for suggesting this problem, and to Bruce Cragin and others of the antennex discussion group for clarifications of its solution.

References

[1] G.H. Brown, *The "Turnstile" Antenna*, Electronics **9**, 15 (April, 1936),
http://kirkmcd.princeton.edu/examples/EM/brown_electronics_9_14_36.pdf

[2] L.B. Cebik, *The Turnstile Antenna. An Omni-Directional Horizontally Polarized Antenna*, http://www.antentop.org/w4rnl.001/turns.html
http://kirkmcd.princeton.edu/examples/EM/cebik_turnstile.pdf