Vector Gravity

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
(December 4, 1996)

As early as 1836, Faraday considered a possible relation between electromagnetism and gravity [1], and noted a key difference is that gravity is always attractive while electricity can be both attractive and repulsive. In 1850 [2], and again in 1859 [3], he made experimental searches for electromotive forces induced in massive circuits that fell through the slightly nonuniform gravitational field at the Earth's surface (with negative results).[1]

In sec. 82 of his great 1864 paper *A Dynamical Theory of the Electromagnetic Field* [5], J.C. Maxwell considered whether Newtonian gravity could be described by a vector field theory. He was dissatisfied with his results because the potential energy of a static gravitational configuration is always negative but he felt this should be re-expressible as an integral over field-energy density which, being the square of the gravitational field, is positive. Another way of stating the issue is that in a consistent Lagrangian theory of vector fields the force between like charges is repulsive [6]. In this note, we neglect this fundamental issue, and consider other aspects of a possible vector theory of gravity that is attractive for like charges (masses) [7].

It has often been noted that magnetism can largely be understood as the consequence of electrostatics plus special relativity [8]. Similarly, static gravity plus special relativity implies phenomena often called "magnetic gravity" [9]-[15]. These simple arguments advise us that some manifestation of "magnetic gravity" must be contained in any valid theory of gravity. The first recorded discussion of "magnetic gravity" appears to be by Oliver Heaviside in his Maxwellian vector theory of gravity (1883)[16], which we review here.

The static gravitational force on a mass m due to a mass density ρ can be written in close analogy to electrostatics as,

$$\mathbf{F} = m\mathbf{g},$$

where the static gravitational field $\mathbf{g}$ obeys

$$\nabla \cdot \mathbf{g} = -4\pi G\rho, \qquad \nabla \times \mathbf{g} = 0,$$

where G is Newton's gravitational constant.

Heaviside argued that in analogy to the remaining Maxwell equations we should expect a gravitomagnetic field $\mathbf{h}$ that obeys,

$$\nabla \cdot \mathbf{h} = 0, \qquad \nabla \times \mathbf{h} = -4\pi H\rho\mathbf{v},$$

where $\mathbf{v}$ is the velocity of the mass that causes field $\mathbf{h}$ and H is a constant (that should be called Heaviside's constant) which characterizes the strength of the gravitomagnetic interaction. The force on mass m is now,

$$\mathbf{F} = m\mathbf{g} + m\mathbf{v} \times \mathbf{h},$$

---

[1] See also [4].

where $\mathbf{v}$ is the velocity of mass m in this expression, in analogy to the Lorentz force law (which latter was first clearly stated by Heaviside in 1886 [18, 19]).

If the constant H (and also G) were larger there might have been an experimental measurement of its value. Then, it would have been noted that,

$$\sqrt{\frac{G}{H}} = c,$$

the speed of light!

In the preceding I have chosen different units for the field $\mathbf{h}$ than those recommended by Heaviside to emphasize how, if observed, the field $\mathbf{h}$ might have been interpreted initially as quite distinct from the field $\mathbf{g}$ and having nothing to do with the speed of light.

Lacking evidence of gravitomagnetostatic effects, Heaviside proceed by analogy to the full Maxwell equations and inferred the time-dependent equations of the gravitational field would be,

$$\nabla \cdot \mathbf{g} = -4\pi G\rho, \qquad \nabla \times \mathbf{g} = -\frac{\partial \mathbf{h}}{\partial t},$$

and,

$$\nabla \cdot \mathbf{h} = 0, \qquad \nabla \times \mathbf{h} = -4\pi H\rho\mathbf{v} + \frac{H}{G}\frac{\partial \mathbf{g}}{\partial t}.$$

Heaviside then noted that there should be gravitational waves which propagate with velocity,

$$v = \sqrt{\frac{G}{H}},$$

As an example Heaviside considered that the propagation velocity might well be the speed of light. From this assumption the constant H has the value 7.3×10^{-28} m/kg.

Heaviside then noted that the gravitational field of the Sun, taken as moving relative to the "ether" defined by the fixed stars, would be modified by terms in $(v_{\mathrm{Sun}}/c)^2$ exactly as is the case for the field of a rapidly moving electric charge (which result he had been the first to derive correctly to all orders in v/c in 1888 [19]). He then calculated the resulting precession of the Earth's orbit around the Sun and concluded this effect was small enough to have gone unnoticed thus far, and therefore offered no contradiction to the hypothesis that gravitational effects propagate at the speed of light.

Heaviside also considered the effect of the dipole gravitomagnetic field of the rotating Sun, finding the dipole moment to be $-H\mathbf{L}/2$ where $\mathbf{L}$ is the angular momentum of the Sun. However, the effect of this moment on the precession of a planet's orbit has the opposite sign to the observed effect, and is too small in magnitude by a factor $L_{\mathrm{Sun}}/L_{\mathrm{orbital,\ planet}}$. (Surprisingly, Heaviside seemed to be unaware of the long history of measurements of the precession of Mercury's orbit.)

It appears that the first confrontation between experiment and new predictions of gravitational field theory occurred some 20 years before Einstein's celebrated work.

From Heaviside's habit of recording the date on which sections of his book first appeared as short articles in *The Electrician* magazine, I infer that gravitation occupied his attention for only three weeks in 1893 and that he never returned to the subject.

Heaviside's work could be called a low-velocity, weak-field approximation to general relativity. This topic was revived in an interesting paper by Forward [20], that is perhaps insufficiently well-known due in part to its place of publication. See also [21]-[23].

The precession of planetary orbits is not a good test of gravitomagnetism; that precession is due to corrections of order v^2/c^2 to the field $\mathbf{g}$ that are "post-Maxwellian". (The term "post-Newtonian" typically used in the literature is perhaps not sufficiently precise in this regard.). However, gravitomagnetism is a useful insight for understanding the precession of orbiting gyroscopes that will hopefully be observed in experiments now under construction. [24].

References

[1] G. Cantor, *Faraday's search for the gravelectric effect*, Phys. Ed. **26**, 289 (1991), `kirkmcd.princeton.edu/examples/GR/cantor_pe_26_289_91.pdf`

[2] M. Faraday, *Experimental Researches in Electricity.- Twenty-fourth Series*, Phil. Trans. Roy. Soc. London **141**, 1 (1851), `http://kirkmcd.princeton.edu/examples/EM/faraday_ptrsl_141_1_51.pdf`

[3] M. Faraday, *The Force of Gravitation*, Lecture at the Royal Institution (1859), `http://kirkmcd.princeton.edu/examples/EM/faraday_lectures_59.pdf`

[4] M. Faraday, *On the Conservation of Force*, Phil. Mag. **13**, 225 (1857), `http://kirkmcd.princeton.edu/examples/EM/faraday_pm_13_225_57.pdf`

[5] J.C Maxwell, *A Dynamical Theory of the Electromagnetic Field* Phil. Trans. Roy. Soc. London **155**, 459 (1865), `kirkmcd.princeton.edu/examples/EM/maxwell_ptrsl_155_459_65.pdf`

[6] See, for example, Chaps. 7 and 8 of F.E. Low, *Classical Field Theory* (Wiley, 1997), `http://kirkmcd.princeton.edu/examples/EM/low_97.pdf`

[7] O.D. Jefimenko in his books *Causality, Electromagnetic Induction and Gravitation*, and *Gravitation and Cogravitation* (Electret Scientific Company, Star City, 2000, 2006) ignored the issue of negative gravitational field energy, and gave extensive discussion of analogies between electromagnetism and gravitation.
`http://kirkmcd.princeton.edu/examples/EM/jefimenko_00.pdf`
`http://kirkmcd.princeton.edu/examples/GR/jefimenko_06.pdf`

[8] See, for example, Chap. 3 of M. Schwartz, *Principles of Electrodynamics* (McGraw-Hill, 1972), `http://kirkmcd.princeton.edu/examples/EM/schwartz_72.pdf`

[9] D. Bedford and P. Krumm, *On relativistic gravitation*, Am. J. Phys. **53**, 889, (1985), `kirkmcd.princeton.edu/examples/GR/bedford_ajp_53_889_85.pdf`

[10] D. Bedford and P. Krumm, *The gravitational Poynting vector and energy transfer*, Am. J. Phys. **55**, 362 (1987), `kirkmcd.princeton.edu/examples/GR/krumm_ajp_55_352_87.pdf`

[11] H. Kolbenstvedt, *Gravomagnetism in special relativity*, Am. J. Phys. **56**, 523 (1988), `kirkmcd.princeton.edu/examples/GR/kolbenstvedt_56_523_88.pdf`

[12] E. G. Harris, *Analogy between general relativity and electromagnetism for slowly moving particles in weak gravitational fields*, Am. J. Phys. **59**, 421 (1991), `kirkmcd.princeton.edu/examples/GR/harris_ajp_59_421_91.pdf`

[13] B. Mashoon, *Gravitoelectromagnetism: A Brief Review* (Apr. 17, 2008), `https://arxiv.org/abs/gr-qc/0311030`

[14] F. Hermann and M. Pohlig, *Gravitoelectromagnetism: Removing action-at-a-distancein teaching physics*, Am. J. Phys. **90**, 410 (2022), `http://kirkmcd.princeton.edu/examples/GR/hermann_ajp_90_410_22.pdf`

[15] V.M. Vasyliūnas, *How energy is conserved in Newtonian gravity*, Am. J. Phys. **90**, 416 (2022), `http://kirkmcd.princeton.edu/examples/GR/vasyliunas_ajp_80_416_22.pdf`

[16] O. Heaviside, *A Gravitational and Electromagnetic Analogy*, Electrician **31**, 281, 359 (1993), `http://kirkmcd.princeton.edu/examples/EM/heaviside_electrician_31_281_93.pdf` `http://kirkmcd.princeton.edu/examples/EM/heaviside_electrician_31_359_93.pdf`
Also on pp. 455-46 of [17].

[17] *Electromagnetic Theory*, Vol. 1 ("The Electrician" Printing and Publishing Co., 1894), `kirkmcd.princeton.edu/examples/EM/heaviside_electromagnetic_theory_1.pdf`

[18] O. Heaviside, *Electromagnetic Induction and Its Propagation*, part 22, Electrician **16**, 186 (1886), `http://kirkmcd.princeton.edu/examples/EM/heaviside_electrician_16_186_86.pdf`
Also on p. 542 of [17].

[19] O. Heaviside, *On the Electromagnetic Effects due to the Motion of Electrification through a Dielectric*, Phil. Mag. **27**, 324 (1889), `http://kirkmcd.princeton.edu/examples/EM/heaviside_pm_27_324_89.pdf`

[20] R.L. Forward, *General Relativity for the Experimentalist*, Proc. Inst. Radio Eng. **49**, 892 (1961), `kirkmcd.princeton.edu/examples/GR/forward_pire_49_892_61.pdf`

[21] V.B. Braginsky, C.M. Caves and K.S. Thorne, *Laboratory Experiments to Test Relativistic Gravity*, Phys. Rev. D **15**, 2047 (1977), sec. III, `kirkmcd.princeton.edu/examples/GR/braginsky_prd_15_2047_77.pdf`

[22] K.S. Thorne, *Gravitomagnetism, Jets in Quasars and the Stanford Gyroscope Experiment*, in *Near Zero: New Frontiers of Physics*, ed. by J.D. Fairbank *et al.* (W.H. Freeman, 1988). `http://kirkmcd.princeton.edu/examples/GR/thorne_88.pdf`

[23] C.M. Will, *The Confrontation between General Relativity and Experiment*, Living Rev. Rel. **17**, 4 (2014), `http://kirkmcd.princeton.edu/examples/GR/will_lrr_17_4_14.pdf`

[24] Barely significant results were reported in C.W.F. Everitt *et al.*, *Gravity Probe B: Final Results of a Space Experiment to Test General Relativity*, Phys. Rev. Lett **106**, 221101 (2011), `http://kirkmcd.princeton.edu/examples/GR/everitt_prl_106_221101_11.pdf`