Lift Force on Water Skis

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1 Problem

Relate the lift force F_L on water skis to the drag coefficient C_D of the water skis for water flow perpendicular to the flat surface of the skis, when the skis make angle θ to the surface of the water.

What horsepower is required to pull the skis with velocity v when the mass M of the skis plus rider is, say, $M = 100$ kg?

2 Solution

The motion of water skis through nominally still water is associated with turbulence, as characterized by the Reynolds number,

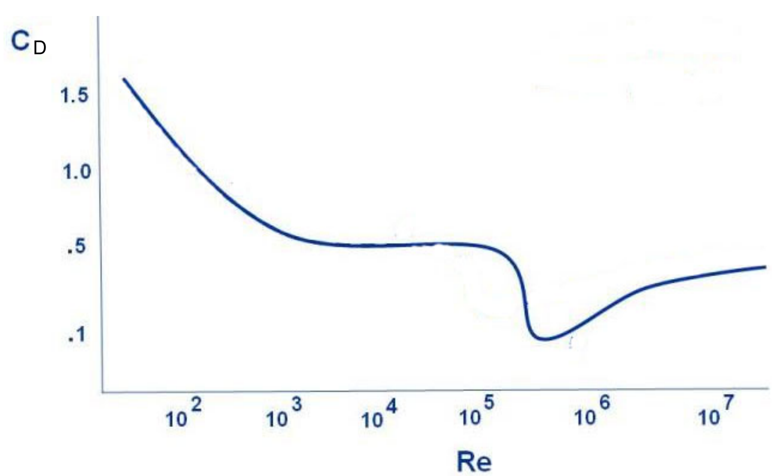
$$\mathsf{Re} = \frac{\rho L v}{\mu} \approx \frac{10^3 \cdot 1 \cdot 10}{10^{-3}} = 10^7, \tag{1}$$

for water of density $\rho = 10^3$ kg/m^2, skis of characteristic length $L = 1$ m, moving with horizontal velocity $v = 10$ m/s on the water, whose dynamic viscosity μ is 10^{-3} SI units.

For such a large Reynolds number, the drag force is almost entirely due to "pressure drag",[1] In this regime, the drag force for flow perpendicular (normal) to the surface of a thin, flat plate has the form,

$$F_{\rm drag} = \frac{1}{2} C_D \rho A v^2, \tag{2}$$

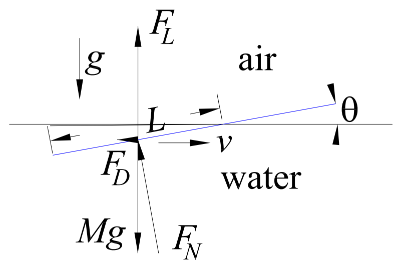
where $A \approx L^2$ is the area of the plate, and $C_D \approx 0.2$ is the drag coefficient for $\mathsf{Re} = 10^7$.



[1] See, for example, https://en.wikipedia.org/wiki/Drag_(physics)

For water skis, of characteristic area $A \approx L^2 = 1$ m^2, that make angle θ to the horizontal surface of the water, the effective area of the skis is $A \sin\theta$, so we estimate the normal force of the water on the skis as,

$$F_N = \frac{1}{2} C_D \rho A \sin\theta\, v^2, \tag{3}$$



The lift force is then,[2,3]

$$F_L = F_N \cos\theta = \frac{1}{2} C_D \rho A v^2 \sin\theta \cos\theta, \tag{4}$$

and the horizontal drag force on the skis is,

$$F_D = F_N \sin\theta = \frac{1}{2} C_D \rho A v^2 \sin^2\theta. \tag{5}$$

For $\sin\theta = 0.1$ $\cos\theta \approx 1$ and the lift force is,

$$F_L = \frac{1}{2} 0.2 \cdot 10^3 \cdot 1 \cdot 10^2 \cdot 0.1 \cdot 1 = 10^3 \text{ N} \tag{6}$$

for the model parameters stated above. This equals the gravitational force $Mg \approx 100 \cdot 10 = 10^3$ N, such that the water skier does not sink. Hence, our model of the lift force on the water skis is reasonable.

The lift force should be independent of the velocity. Hence, $v^2 \sin\theta \cos\theta = 10$, and,

$$\sin 2\theta = \frac{20}{v^2}\,, \tag{7}$$

for v in m/s. Note that this model predicts the skier could not float/glide on the surface of the water for $v < \sqrt{20} \approx 4.5$ m/s.

The horizontal drag force F_D is approximately $\sin\theta$ times the lift force, *i.e.*, only 100 N for the model parameters. The power required to pull the skier is,

$$P = F_D v = \frac{1}{2} C_D \rho A v^3 \sin^2\theta = \frac{C_D \rho A (v^2 \sin\theta \cos\theta)^2}{2v \cos^2\theta} = \frac{1000 \text{ w}}{v \cos^2\theta}\,, \tag{8}$$

which is only ≈ 100 W ($= 1/7$ hp) for the model parameters, with $v = 10$ m/s and $\sin\theta = 0.1$.

[2]Note that the lift force (4) vanishes for $\theta = 90°$, and is maximal for $\theta = 45°$.

[3]A different model is that for the normal force the effective velocity is $v \sin\theta$ and the effective area is A, implying that $F_N = 0.5 C_D \rho A v^2 \sin^2\theta$ and $F_L = 0.5 C_D \rho A v^2 \sin^2\theta \cos\theta$. For $\sin\theta = 0.1$, the lift force in this model would be only 100 N $= mg/10$.