Counterintuitive Performance of Land and Sea Yachts

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1 Introduction

Certain land and sea "yachts", with no internal power source, are capable of "sailing" directly into to the wind, and also "sailing" directly downwind faster than the wind.

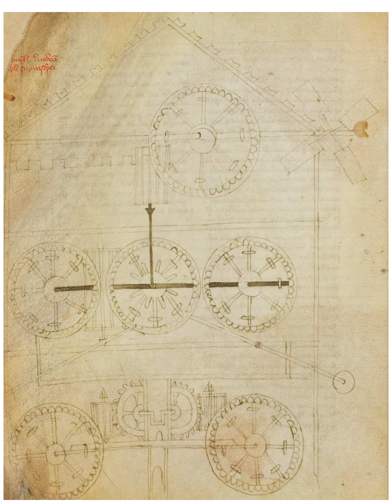
Such behavior is not possible for "yachts" that are propelled only by the force of the wind on "fixed sails" attached to the "yachts",[1] which leads many people to assume that no variant of a "yacht" could exhibit the stated behavior.

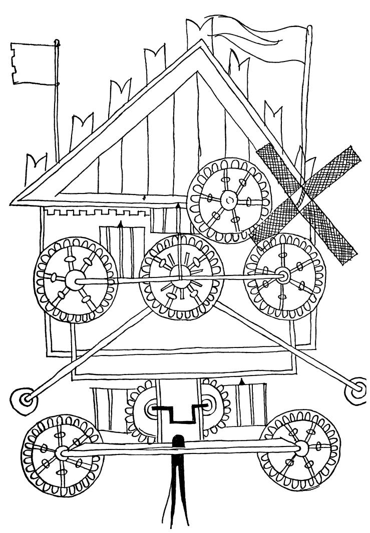
However, the "yachts" that achieve this counterintuitive performance do so with the aid of wind-powered propellers (turbines) rather than classic sails.

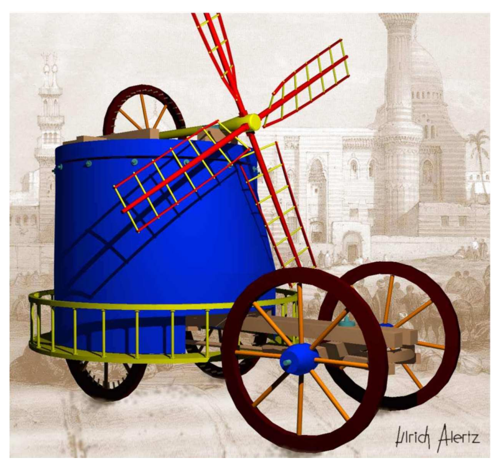
There exist numerous YouTube videos [1]-[42] showing aspects of this performance, but skepticism as to postings on "social media" is such that these are doubted by many people.[2]

2 Some History

A concept of a windmill-driven land "yacht" was sketched by **Guido de Vigevano** in 1335(!), p. 53 of [43], as shown on the left below. The middle figure is from [51], and the right figure is a recent interpretation by **Ulrich Alertz** [74]. In the latter, the horizontal axis of the windmill can be rotated about the vertical, which permits the "yacht" to "sail" both directly downwind and upwind (by keeping the same "face" of the windmill into the wind in both cases).



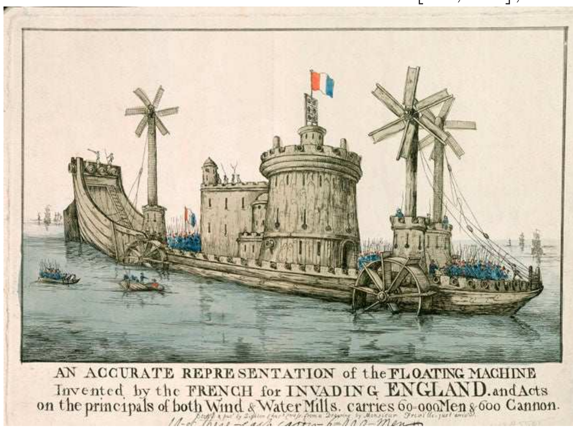


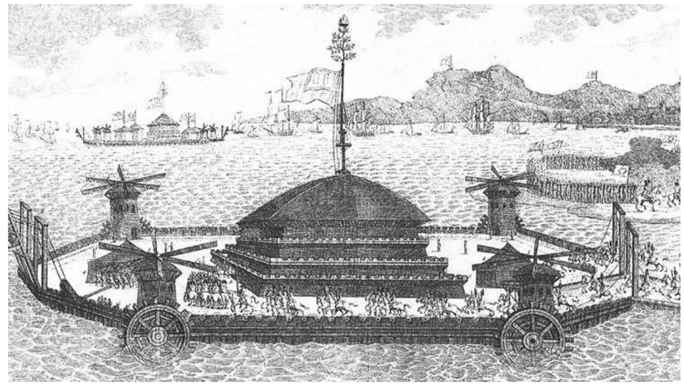


[1]For a review of the physics of sailing (of boats with classic sails), see [78]. For videos, see [26, 36].

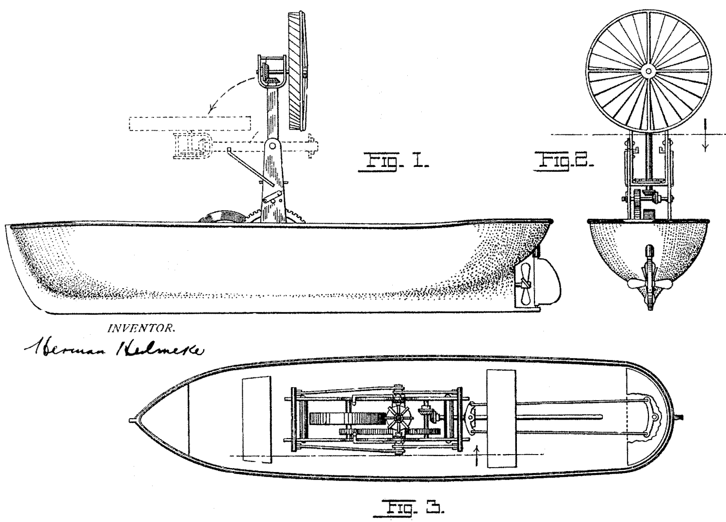
[2]Many of these videos claim that "physicists" say this behavior is "impossible", whereas all papers published by physicists (and nonphysicists) on this topic say that it is possible.

Fears of a French invasion, led by Napoleon, of England in 1798 led to “fake news” of windmill-driven “sea castles” [92, 98], two versions of which are shown below.

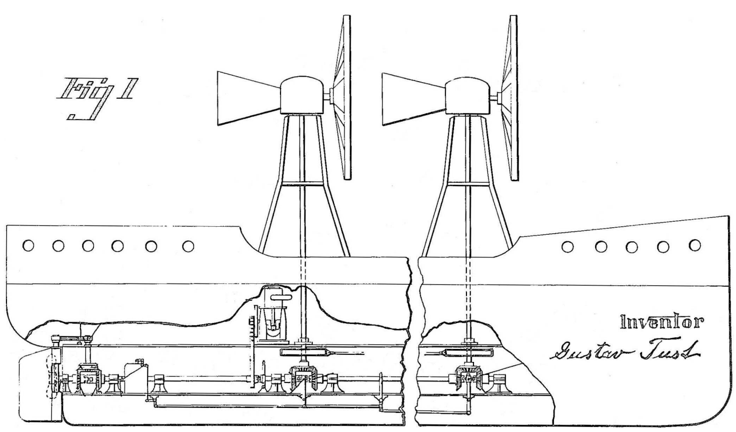




In 1904, **H. Helmeke** filed a patent for a propeller-driven boat [44], sketched below. The horizontal axis of the propeller could be rotated about the vertical, permitting “sailing” both downwind and upwind.

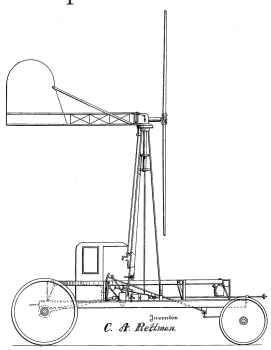


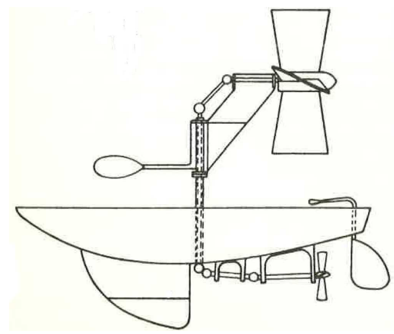
In March 1922, **G. Tust** filed a patent [45] for a propeller-driven boat, sketched below. The pitch of the propellers could be altered by 90°, permitting “sailing” both downwind and upwind.



The earliest physics report of a "yacht" with a propeller in air linked to another under water that "sailed" (without actual sails) directly against the wind is from Oct. 1922 [46], by **J. Constantin** and **J.-L. Breton**. This report was only qualitative, with no numbers, equations or figures. It was claimed that work on this concept started in 1910.

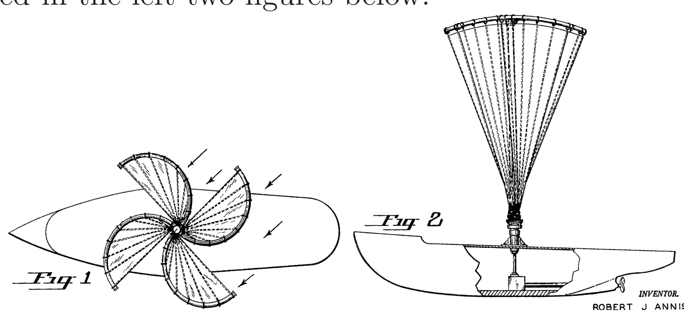
In 1935, **C.A. Rettman** filed a patent for a wind-propelled tractor [47], sketched on the left below, which may be the first "modern" proposal for a propeller-driven land yacht. The horizontal axis of the propeller could be rotated about the vertical, permitting "sailing" both downwind and upwind.

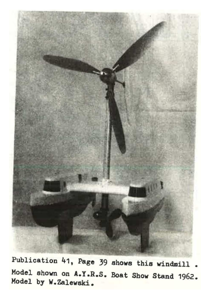




In 1949, **D. Schatz** patented a version of a propeller-driven sea yacht [48], sketched on the right above. See also pp. 74-76 of [58] (which mentioned Tust on pp. 66-68) and [67].

In 1952, **R.J. Annis** filed a patent for a boat driven by a propeller with a vertical axis, as sketched in the left two figures below.





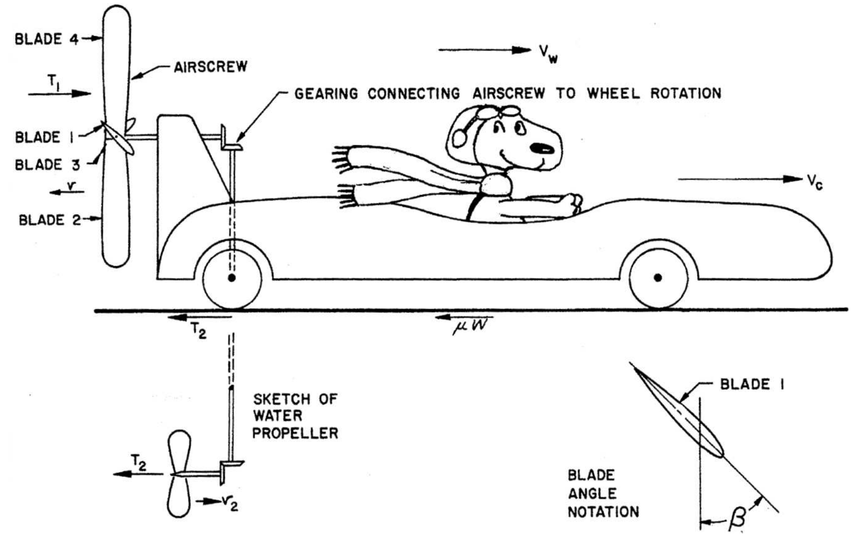
In 1960, **W. Zalewski** file a patent for a wind-propelled boat [50]. He proposed that the propeller mounting be rotatable about a vertical mast, to permit "sailing" both downwind or upwind. The figure on the right above shows a model of this from 1962 [52] (also p. 22 of [68]).

In 1969, **Andrew Bauer** [53, 55] made successful trials of a land yacht traveling both directly against the wind, and downwind faster than the wind; see also the video at [19]. A sketch of the concepts of both land and sea "yachts" was provided, shown on the left below, with a photo of his land yacht on the right. Bauer gave an extensive mathematical analysis of the performance in [53, 55], and later published several other papers on wind turbine propulsion of vehicles [56, 59].[3] He claimed to have learned of the concept from **D.L. Elder**

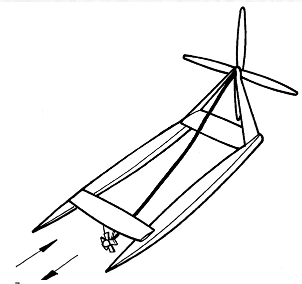
[3] Bauer used a propeller with adjustable pitch angle, β in the figure on the above. For "sailing" downwind he started with $\beta = 45°$ and reduced this to $20°$ as the yacht gained speed. For "sailing" upwind he started with $\beta = -90°$ and reduced this to $-130°$ as the yacht gained speed.

Most subsequent propeller-powered yachts used fixed-pitch-angle propellers, with $\beta \approx 45°$, and reversed the linkage between the propeller and the wheels (or second propeller) for downwind *vs.* upwind "sailing".

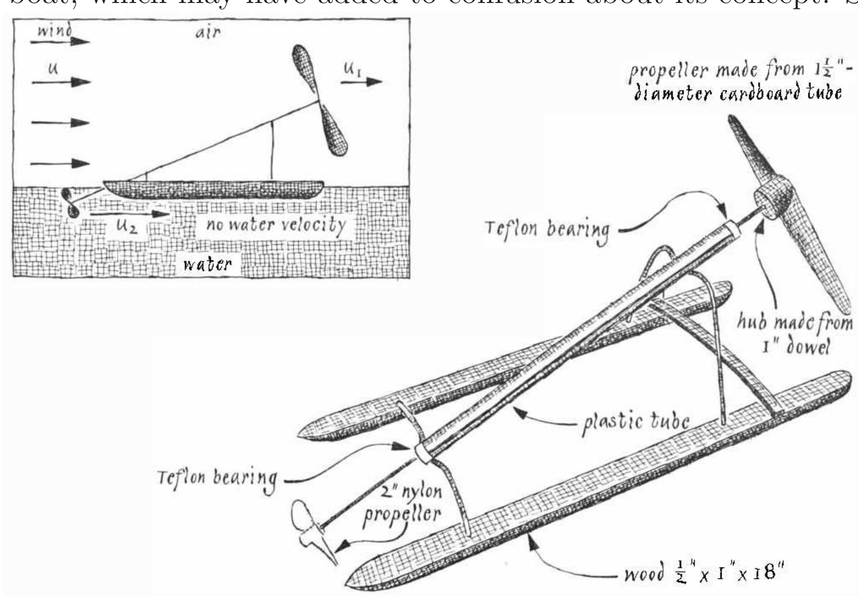
some 20 years earlier.



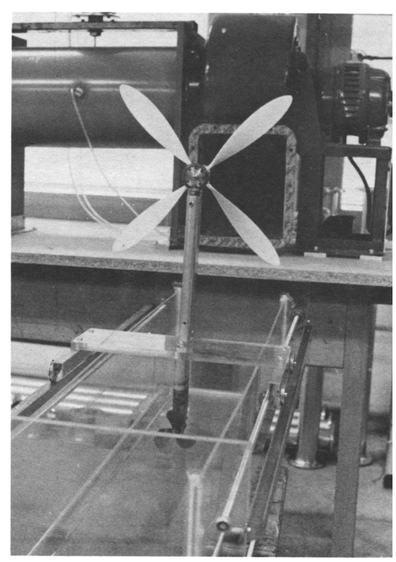
In 1971, **F.N. Potter** filed a patent for a toy boat with two propellers, one in air and the other underwater, as sketched to the right. He noted that the boat could sail either upwind or downwind by reversing the linkage between the two propellers.



In 1975, the *Amateur Scientist* column of *Scientific American* [60] quoted a description by **S. Martin** of a dual-propeller (model) boat, attributed to **P. Kauffman** and **E. Lindahl**, that successfully "sailed" directly against the wind. This was called a "push-me-pull-you" boat, which may have added to confusion about its concept. See the left figure below.

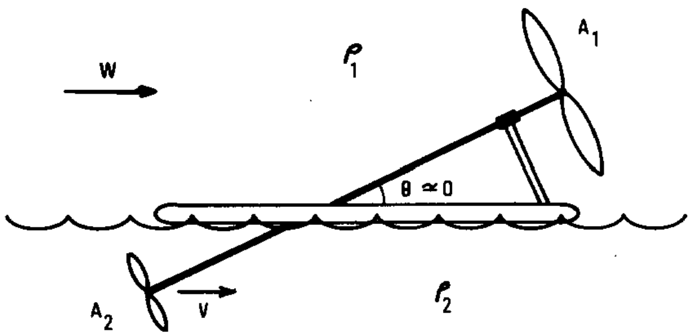


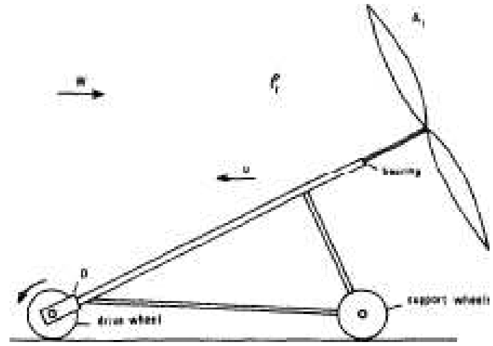
"Push-me-pull-you" boat designed by Peter Kauffman and Eric Lindahl



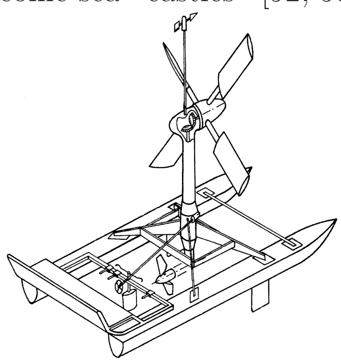
In 1977, **R. Flatt** reported a successful model boat that could sail directly against the wind, as shown in the right figure above. He cited [46] and [53, 59], but not [60].

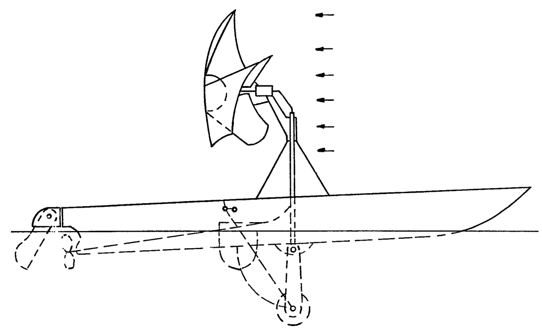
In 1978 and 1981, **B.L. Blackford** [62, 66] discussed the physics of a "push-me pull-you" boat, and also of a land vehicle, and briefly reported successful operation of models of both types of vehicles (sketched in the figures below).[4] Blackford cited [60]. See also [71, 72].





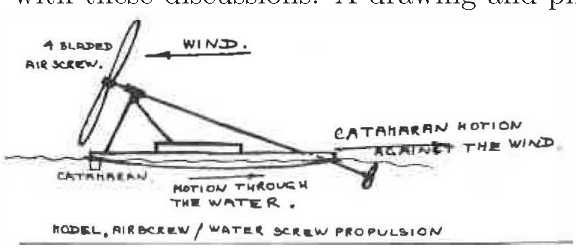
In 1980, **J.-P. Vidal** filed a patent for a propeller-driven sea yacht [65] to "sail" directly against the wind, as well as downwind faster than the wind, as sketched on the left below He claimed that this concept dates back to the 18th century, perhaps referring to the supposed Napoleonic sea "castles" [92, 98].

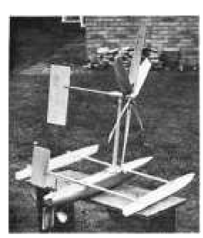


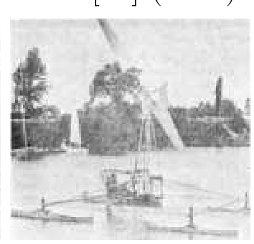


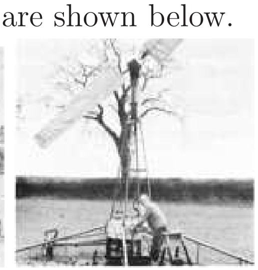
In 1981, **B. Jöst** filed for a German patent [67] (and filed for a US patent in 1984) of a propeller-driven boat that "sailed" directly against the wind, as sketched on the right above.

The topic of "sailing" directly upwind, and downwind faster than the wind is discussed in many documents published by the Amateur Yachting Research Society (UK), 1962-2007, https://www.ayrs.org/, including [52, 64, 68, 70, 73, 75]-[87, 99]. The video [1] is associated with these discussions. A drawing and photos from pp. 6-9 of [64] (1979) are shown below.









In 2010, **R. Cavallaro** published a note on his tests of full-scale land yachts [88]. His efforts are recorded in many videos, starting in 2008, including [4, 5, 6, 7, 8, 9, 10, 12, 13, 16, 17, 18, 22, 25, 28]. See also the photos on the next page. The linkage between the propeller and wheels was reversed for "sailing" downwind and upwind.

[4]For unpublished comments on [62], see [63].

Other published discussions of this topic include [89]-[97].

3 "Sailing" Directly Downwind Faster than the Wind

We now consider wheeled land yachts, sketched on pp. 3-5 above, which contain a propeller mounted on a (nearly) horizontal axis attached to the yacht. The linkage between the propeller and (some of) the wheels of the yacht is such than when the yacht moves "forwards" (to the right in the sketches above), the propeller blows air "backwards".

Such yachts can start from rest (with respect to the ground), with the wind "behind" them, as the (initial) drag force of the wind on the body of the yacht (and on its propeller) pushes the vehicle "forwards".

When the yacht's groundspeed v_y reaches the groundspeed v_w of the wind, the latter no longer pushes the vehicle forward. However, the wheels of the yacht are turning at this time, which turns the propeller, which pushes air "backwards", and propels/accelerates the yacht "forwards" to speeds greater than the groundspeed of the wind.

As the yacht accelerates "forward", the angular velocity of its wheels increases, which requires a torque about the center of each wheel. This torque is due to the horizontal (frictional) forces of contact, $\mathbf{F}_{\rm contact}$, of the ground on the wheels, and these contact forces point "backwards". In particular, these contact forces do NOT propel the yacht "forward" (in the frame where the ground is at rest). Rather, the propeller propels the yacht.

For yacht groundspeeds greater than that of the wind, the air drag opposes the "forward" motion of the yacht, such that there exists a maximum (terminal) velocity in the "forward" direction. In practice, maximum "forward" speeds of about 2.5 times the windspeed (with respect to the ground) have been achieved [16, 18].

3.1 Energetics

The above discussion clarifies how there can be a net "forward" force on the wheeled land yacht even when its groundspeed exceeds that of the wind.

In this case, the kinetic energy of the yacht increases, and by conservation of energy, the kinetic energy of the air must decrease (in the rest frame of the ground).

We now focus on a time when the groundspeed v_y equals the groundspeed v_w of the wind.

Then, in the rest frame of the (body of the) yacht (the $\star$ frame), during a small time $dt^\star$ ($= dt$) the propeller sucks in a mass dm of air in "front" (to the right in the figures on pp. 3-5) with characteristic velocity $-v^\star_{\rm front}$, and propels it to the "back" (to the left in the figures on pp. 1-2) and gives this air characteristic velocity $-v^\star_{\rm back}$, where $v^\star_{\rm back} > v^\star_{\rm front}$.

In the ground frame, where the windspeed is v_w, this volume of air has initial velocity $v_w - v^\star_{\rm front}$ and final velocity $v_w - v^\star_{\rm back}$. The change of the kinetic energy of the small mass dm in the ground frame, as it passes by the propeller, is,

$$\Delta {\rm KE}_{\rm air} = \frac{dm}{2}(v_w - v^\star_{\rm back})^2 - \frac{dm}{2}(v_w - v^\star_{\rm front})^2$$

$$= \frac{dm}{2}(-2v_w v^\star_{\rm back} + -2v_w v^\star_{\rm front} + v^{\star 2}_{\rm back} - v^{\star 2}_{\rm front)} = -\frac{dm}{2}(v^\star_{\rm back} - v^\star_{\rm front})(2v_m - v^\star_{\rm back} - v^\star_{\rm front}). \quad (1)$$

Hence, the air (in the ground frame) loses kinetic energy at the time when $v_y = v_w$ if $2v_m > v^\star_{\rm back} + v^\star_{\rm front}$. Thus, the gain in kinetic energy of the yacht (with respect to the ground), as it accelerates to speeds greater than the speed of the wind, comes from the loss of kinetic energy of the air.[5] This condition is met by use of a relatively large propeller rotating at relatively low angular velocity when $v_y = v_w$.

3.1.1 Analysis in the Rest Frame of the Wind

If we consider the (inertial) frame (the $\star$ frame) in which the speed of the wind is zero, in the absence of the yacht, the air has no kinetic energy (in the absence of the yacht).[6]

Note that the ground has speed $-v_w$ in this frame.

When the yacht is moving "forwards", its wheels are accelerating due to the torque associated with the force of contact of the wheels with the ground.

As noted above, the horizontal force of contact $\mathbf{F}_{\rm contact} = \mathbf{F}^\star_{\rm contact}$ of the ground on the wheels of the yacht is "backwards". In the rest frame of the wind, the point of contact (of the ground with the wheels of the yacht) moves "backwards" with velocity $\mathbf{v}^\star_{\rm contact}$, so the contact force adds positive power, $\mathbf{F}^\star_{\rm contact} \cdot \mathbf{v}^\star_{\rm contact}$, to the yacht, increasing its kinetic energy. An observer on the yacht would say that the (moving) ground provides the energy to accelerate the yacht.[7]

The story here is somewhat paradoxical, in that (in the rest frame of the wind), the ground force on the yacht provides the energy to accelerate the yacht "forwards", but the ground force points "backwards". This paradox has no doubt contributed to the skepticism that the yacht cannot move directly downwind faster than the wind.

4 "Sailing" Directly against the Wind

If the land yacht is to "sail" directly against the wind, some aspect of the linkage between the propeller and the wheels must be reversed from the case of "sailing downwind". For

[5]The essence of this argument was given in [53], and also in [86].

[6]A related puzzler involving moving frames is whether a jet could take off from a treadmill [33, 79].

[7]This view is mentioned at the end of [87].

Rettman's yacht [47], the horizontal axis of the propeller was rotated by 180° about the vertical; for Bauer's yacht [53], the pitch of the propeller blades was changed by $\approx 90°$; for Cavallaro's yacht [88] the linkage between the directions of rotation of the propeller and the wheels was reversed.

As the yacht picks up speed, against the wind, the windspeed relative to the propeller increases, and the propeller becomes more effective in accelerating the yacht "forward". However, the air drag on the body of the yacht also increases as its groundspeed increases, and the yacht does not accelerate indefinitely, but comes to a constant (terminal) velocity, which in practice can be several time the wind groundspeed.

The most delicate issue in this scenario is starting the yacht from rest when it is subject to a "headwind".

The air drag on the body of the yacht tends to push it "backwards", while the rotation of the propeller by the wind drives the yacht "forward" (with the linkage between the propeller and the wheels as mentioned above). The design of the yacht must minimize the first, "backwards" force, and maximize the second, "forwards" force, which implies a large propeller on an otherwise "slim" yacht. Indeed, the demands of the aerodynamic design are greater for motion of the yacht directly against the wind than for motion directly downwind faster than the wind. In practice, sufficient quality of design has been achieved for propeller-driven land (and sea) yachts to accomplish both types of motion.

5 Terminal Velocity

Estimates of the terminal velocity of land and sea yacht traveling against the wind, and directly downwind, requires models of the air drag on both the body and the propeller(s) of the yachts. Such models are always somewhat doubtful, but are consistent with the terminal velocity being several times the wind groundspeed.

Models of the terminal velocity are discussed in [53, 61, 62, 63, 66, 69, 72, 90, 91, 95].

A Appendix: Analogies

Several people have suggested that certain analogies are helpful in understanding the motion of a yacht directly downwind faster than the wind [31, 37, 38, 63, 75, 77, 81, 82, 96]. It is not clear to this author that these analogies clarify, rather than deflect, one's understanding of this subtle phenomenon.

References

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http://kirkmcd.princeton.edu/examples/mechanics/goodman_catalyst_N23_jan06.pdf

[2] P. Worsley, *R/C boat with windmill drive* (Feb. 4, 2008),
https://www.youtube.com/watch?v=OlqLHRE8ReQ&t=1s

[3] P. Worsley, *Experimental model wind-turbine boat plans are here:*
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(Feb. 9, 2008), https://www.youtube.com/watch?v=oKqC5JsurOk&t=1s

[4] R. Cavallaro, *Direct Downwind Faster Than The Wind (DDWFTTW)* (Nov. 10, 2008),
#1: https://www.youtube.com/watch?v=4pgDs50A-Yw
#2: https://www.youtube.com/watch?v=1BRvYZd81AQ
#3: https://www.youtube.com/watch?v=kWSan2CMgos
#4: https://www.youtube.com/watch?v=LMgDvC5lqsY
#5: https://www.youtube.com/watch?v=1pSYALWQ-nI&t=16s
#6: https://www.youtube.com/watch?v=dgHBDESd38M
#7: https://www.youtube.com/watch?v=7xL8gRJ5F6k

[5] R. Cavallaro, *Downwind Faster than the Wind (DWFTTW) Myth Challenge* (Nov. 15, 2008), https://www.youtube.com/watch?v=xHsXcHoJu-A

[6] R. Cavallaro, *DWFTTW Self-Start* (Nov. 23, 2008),
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[8] R. Cavallaro, *Making of the DDWFTTW cart (3 Parts)* (Dec. 19, 2008),
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https://www.youtube.com/watch?v=gSHNqrF93MU

[9] R. Cavallaro, *DDWFTTW — Physics on a shoestring* (Feb. 6, 2009),
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[10] R. Cavallaro, *DDWFTTW — Prop or Turbine?* (Mar. 12, 2009),
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[11] yachtpals, *Sailing at 126 mph — Greenbird* (Apr. 1, 2009),
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[12] R. Cavallaro, *DDWFTTW — Prop or Turbine* (Mar. 12, 2009),
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[13] R. Cavallaro, *Harold Bricer Test* (Oct. 6, 2009),
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[14] P. Worsley, *Directly into wind test model* (Nov. 29, 2009),
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[15] P. Worsley, *Windmill Driven Cart* (Apr. 11, 2010), https://www.youtube.com/watch?v=s4j-h4vVABo&t=1s

[16] TraderTurok, *Footage of the DDWFTTW vehicle in Ivanpah* (Mar. 28, 2010), https://www.youtube.com/watch?v=86vhmTp7tKU
BUFC second run with full audio (June 7, 2010), https://www.youtube.com/watch?v=EEuAqq8FINw

[17] Piratewoodleg, *Directly Downwind Faster than the Wind on Discovery Channel* (May 5, 2010), https://www.youtube.com/watch?v=DPvGTjmn9y0&t=214s

[18] Windjetproject, *DDW.mov* (July 9, 2010), https://www.youtube.com/watch?v=5CcgmpBGSCI&t=47s

[19] kenf1a, *Andrew Bauer Wind Machine 1969* (Jan. 3, 2012), https://www.youtube.com/watch?v=dAkJ_QVbloQ&t=0s
Version without the soundtrack: https://www.youtube.com/watch?v=yFPomTq_PRU

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