K. McDonald
Princeton U.
Mar. 5, 1996
BaBar TNDC-96-23

An Analysis of Gas Flow in the BaBar Drift Chamber

Abstract

- An analysis based on the Green's function for the diffusion equation indicates that the BaBar drift chamber could be purged of air down to 1 ppm in 8 hours with a purge gas velocity in the chamber of 0.043 cm/s, corresponding to one volume change every 2 hours. Anecdotal evidence from CLEO and ALEPH supports this claim.
- It is proposed that the gas be fed into the chamber through 64 holes of 1/8″ ID on the rear endplate at the lattice points of the electronics card-cage superstructure, and the gas be taken out through 64 corresponding holes in the front endplate. The average flow velocity in these tubes during purging would be 180 cm/s.
- An analysis based on Landau's jet solution to the Navier-Stokes equation indicates that the gas flow near the endplate between feed holes will be sufficient to insure good purging there.
- Pressure drops in the gas feed system are the order of 0.1″ of water.
- The leakage of chamber gas out of (and air into) a feedthrough hole during a 5-minute wire repair would be only 20 cm^3 on the rear endplate, and 130 cm^3 on the front endplate.
- The endplates will be surrounded by plena flushed with dry nitrogen at a flow rate of about one volume change per hour.

1 Purging Time

The maximum gas-flow rate that the chamber must sustain is determined by specifications on purging. We take this specification in its most basic form to be that the chamber should not be operated with more than 1 part per million of air, and that this condition is to be reached in some time T after the chamber is completely filled with air.

If the drift chamber had no leaks and no materials that outgas, it could run almost indefinitely with zero gas flow. The original spark chamber built at Princeton by Cronin in 1962 with metal plates in a glass bell jar has been in continuous operation with a cosmic-ray trigger for 34 years with no gas change!

1.1 Classic Lore

A classic criterion for purging is derived as follows. Suppose that during each volume change the old and new gas completely mix. Then to a good approximation the concentration of the original gas (*i.e.*, air) drops by half. Then after N volume changes the concentration of the original gas is 2^{-N}. If we want this concentration to be 10^{-6} we find that 19.93 gas changes are required, in agreement with the 'lore' that chambers don't work well until after 20 gas changes.

1.2 A One-Dimensional Model

However, the assumption that the old and new gases mix completely is not very accurate, since the rate of mixing is governed by the relatively slow process of mutual diffusion. On p. 263 of *The Mathematical Theory of Non-Uniform Gases* by S. Chapman and T.G. Cowlings (Cambridge U. Press, 1970) I read that the mutual diffusion coefficient D for He-O_2 mixtures is 0.626 cm^2/sec, and 0.607 for He-N_2 mixtures. In general, a light-heavy gas mixture has diffusion coefficient near 2/3 (while heavy-heavy mixtures have coefficients in the range 0.1-0.2).

Unfortunately, analytic solutions to the time dependence of diffusing gas mixtures are sparse. Indeed, there appears to be exactly one known solution, but of a useful character. It applies to one-dimensional diffusion, where the density $n(x, t)$ of one of the species considered as a perturbation on a near-uniform background of the other obeys the differential equation

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2}.$$

A solution to this equation is known for the initial condition that the concentration of the first gas is a delta function about $x = 0$ at time $t = 0$:

$$n(x, t) = \frac{n_0}{2\sqrt{\pi D t}} e^{-x^2/4Dt},$$

This solution can be used as a Green's function to generate a formal solution to the case that the initial density $n(x, 0)$ is known:

$$n(x, t) = \frac{1}{2\sqrt{\pi D t}} \int_{-\infty}^{\infty} e^{-(x-x')^2/4Dt} n(x', 0) dx', \qquad (t > 0).$$

I use this relation to make an estimate of the purging problem as follows. Approximate the chamber as one dimensional and covering the interval $0 \leq x \leq L$, with a gas flow velocity v in the $+x$ direction. At $t = 0$ there is a sharp transition between the air at $x > 0$, and the chamber gas at $x < 0$ that is about to flow into the chamber to replace the air. Now when $v \ll v_{\text{sound}}$ we invoke the principal of relativity: the problem of flowing gas in which we desire the concentration of air at $x = L$ is equivalent to the problem with no gas flow where we examine the gas concentration at $x = L - vt$.

I claim this approach will actually overestimate the residual concentration of air in the chamber; in our model all the air originally at $x > 0$ retains some probability of diffusing into the oncoming chamber gas, while in practice once the air exits the chamber at $x = L$

is has very little probability of diffusing back into the chamber due to the much higher flow rate in the exhaust tube(s).

The air concentration in the chamber $x = L$ is thus estimated to be

$$n(L,t) = \frac{1}{2\sqrt{\pi D t}} \int_0^\infty e^{-(L-vt-x')^2/4Dt} dx' = \frac{1}{2}\mathrm{erfc}\left(\frac{vt-L}{\sqrt{4Dt}}\right),$$

taking $n(x, 0)$ to be 1 for $x > 0$ and 0 for $x < 0$, where

$$\mathrm{erfc}(z) \equiv \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-z^2} dz \approx \frac{e^{-z^2}}{\sqrt{\pi} z} \text{ for large } z$$

is the complement of the error function.

We desire that the remaining concentration of air at $x = L$ be only 10^{-6}, which implies that the argument of the erfc is about 3.4.

To complete the model we must chose the time at which we desire the concentration of air to drop to 10^{-6}. I take this as

$$T = 8 \text{ hours } = 28800 \text{ sec},$$

so the chamber could be operational only one shift after being completely filled with air. Then using $D \approx 2/3$ cm^2/sec as noted above I find the gas velocity in the chamber must be

$$v = 0.043 \text{ cm/sec}$$

during purging.

The time to change a chamber volume is then L/v which is about 7000 sec $\approx$ 2 hours for chamber length $L = 300$ cm. During the 8-hour purge the chamber undergoes only about 4 volume exchanges, much less than suggested by the 'lore'.

1.3 Stagnant Regions?

The one-dimensional model assumes that the gas flows axially only. But in practice the gas will enter through a finite number of holes in the endplate. The regions close to the endplate between the holes will be somewhat stagnant and might, in effect, remain as pockets of air that will contaminate the chamber for a long time.

If pockets exist where the flow rate is zero they would be quite a problem as the following estimate shows. In the absence of flow, only diffusion can remove the air. Suppose the characteristic size of a stagnant region is x. Then the concentration of air molecules originally in the center of that region drops in time as $1/\sqrt{4\pi D t}$ using the delta-function solution to the diffusion equation. We want the concentration over a region of length x to have dropped to 10^{-6}, which implies that the time required for this is $10^{12} x^2/4\pi D$ sec. With x of order 10 cm and $D \approx 1$ cm^2/s we would need about 10^6 years for the chamber to purge!

This reinforces the theme of the previous subsection that flow not diffusion is the mechanism of purging.

Therefore we inquire as to the likely flow pattern of gas that enters the chamber through a small hole. There are only about five analytic solutions to problems of viscous flow, including falling spheres (Stokes), flow in a circular tube (Poiseuille/Stokes) and flow out of the tip of a tiny nozzle into open space (Landau). I will use the latter result as the basis for an estimate. See sec. 23 of *Fluid Mechanics*, 2nd. ed., Vol. 6 of the Landau & Lifshitz Course of Theoretical Physics (Pergamon Press, 1987).

In addition to the usual condition of incompressible flow, $\nabla \cdot \mathbf{v} = 0$, jet flow in a viscous gas conserves momentum (but not energy). Then it turns out that all properties of the jet can be related to a single dimensionless parameter a, where the momentum flux is

$$\Pi = \frac{16\pi\eta^2}{\rho} f(a), \qquad \text{where} \qquad f(a) = a\left\{1 + \frac{a}{3(a^2-1)} - \frac{a}{2}\ln\frac{a+1}{a-1}\right\},$$

η is the viscosity, ρ is the density of the gas, and parameter a varies from 1 to ∞ as the momentum flux varies from ∞ to 0. The flow velocity in spherical coordinates is

$$v_r = \frac{2\eta}{\rho r}\left\{\frac{a^2-1}{(a-\cos\theta)^2} - 1\right\}, \qquad v_\theta = -\frac{2\eta\sin\theta}{\rho r(a-\cos\theta)}.$$

We will use limiting forms of these expressions in subsequent arguments.

In the *CRC Handbook of Chemistry and Physics* I find the viscosity η of helium as 194 μpois at STP, that of propane as 80 μpois and pentane as 68 μpois. I estimate the viscosity of isobutane as 74 μpois, and that of a 90/10 He-isobutane mixture as 182 μpois. I believe the pois is the cgs unit of viscosity.

Some care is required in using Landau's 'free' jet solution in a situation where walls are present. His solution obeys both the continuity equation (conservation of molecules) and momentum conservation. Of these two only the continuity equation holds exactly in our situation; the chamber walls can absorb momentum that would still be contained in a 'free' jet.

If the gas flows into the chamber through $N = 64$ holes, the equation of continuity tells us that the average velocity $\overline{v}$ in the feed tubes obeys

$$\overline{v} = v\frac{A_{\text{chamber}}}{NA_{\text{tube}}} = v\frac{r_2^2 - r_1^2}{Nr_0^2} = 0.043\frac{81^2 - 24^2}{64(0.15)^2} = 180 \text{ cm/s},$$

supposing the feed tubes have 1/4″ OD and 1/8″ ID so $r_0 = 0.15$ cm, and that the chamber radii are $r_1 = 24$ cm and $r_2 = 81$ cm. This is a modest velocity.

The flow pattern of gas in small tubes obeys Poiseuille's law that the radial dependence of the velocity is

$$v(r) = 4\overline{v}\left(1 - \frac{r^2}{r_0^2}\right), \qquad (0 \le r \le r_0).$$

The momentum density in the tube is ρv, so the momentum flux is

$$\Pi_{\text{tube}} = \int_0^{r_0} \rho v^2 \; 2\pi r dr = \frac{16}{3}\rho\overline{v}^2 A_{\text{tube}}.$$

A 90/10 He/isobutane gas mixture has density $\rho = 0.00042$ g/cm^3. Then $\Pi_{\text{tube}} = 5.13$ g-cm/s^2, and the total momentum flux in 64 tube is 328 g-cm/s^2.

For comparison, the momentum flux in the chamber is $\Pi_{\text{chamber}} = \rho v^2 A = \pi\rho v^2(r_2^2 - r_1^2) = 0.0146$ g-cm/s^2. Essentially all of the momentum flux in the inlet tubes is lost somewhere onto the chamber walls, causing a force of 328 dynes $\approx 1/3$ gram weight, which is negligible in practice. Hence Landau's solution cannot be correct in detail for our chamber, but we wish to use it to derive insight into the behavior of the gas flow near an inlet hole.

If we proceed using $\Pi_{\text{tube}} = 5$ g-cm/s^2 we will soon find jet parameters that imply a flow velocity near the hole many times 180 cm/s. This seems implausible to me, so instead I choose a value for the jet momentum flux that corresponds to axial flow velocities of $4\overline{v} = 720$ cm/s (the peak velocity in the feed tube) at a distance r_0 from the hole. Using a result from p. 83 of Landau and Lifshitz I therefore take $\Pi_{\text{tube}} = 8\pi\eta r_0(4\overline{v})/3 = 2.67\pi(0.000182)(0.15)(720) = 0.164$ g-cm/s^2.

Such a jet is reasonably 'strong' according to Landau, and its flow pattern is characterized by angle θ_0 where

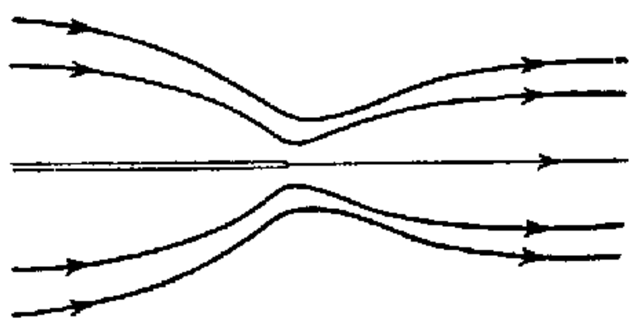
$$\theta_0^2 = \frac{64\pi\eta^2}{3\rho\Pi} = 0.032, \qquad \text{so} \qquad \theta_0 = 0.18 \text{ rad } = 10^\circ.$$

At small angles the velocity follows

$$v_r = \frac{8\eta\theta_0^2}{\rho r(\theta^2 + \theta_0^2)^2}, \qquad v_\theta = -\frac{4\eta\theta}{\rho r(\theta^2 + \theta_0^2)},$$

and $v_r(r, 0) = 4\overline{v}r_0/r = 108/r$ cm/s. At large angles the behavior is independent of the precise value of the jet momentum flux Π provided it is 'large':

$$v_r = -\frac{2\eta}{\rho r} = -\frac{0.87}{r} \text{ cm/s}, \qquad v_\theta = -\frac{2\eta}{\rho r}\cot\frac{\theta}{2}.$$



The figure shows streamlines of the jet.

Of greatest interest to us is the result $v_r = -0.87/r$ cm/s at large angles to the jet, near the surface of the endplate. The jet sucks in gas from the chamber and spews it forward along with the gas entering the chamber from the tube. Naïvely, Bernoulli's equation tells us that the high-velocity region of the jet is at low pressure, so gas outside the jet will be sucked into the side of the jet. This action is excellent for purging the gas near the endplate; even at $r = 20$ cm from a hole the induced (transverse) flow velocity is 0.043 cm/s, equal to the average axial velocity over the whole chamber.

The preceding argument assumes the gas flows into the chamber at the endplate. But the gas must also flow out somewhere – into holes in the front endplate in my view. Because the Navier-Stokes equation involves energy dissipation its solutions are not time-reversal invariant in general. In particular, Landau's jet solution does not apply to flow disappearing into a small hole. However, we can use the continuity equation and Bernoulli's equation to gain qualitative insight. As the gas nears a small-diameter exit hole its velocity must increase; the high-velocity region will be at lower pressure and again gas will be sucked into the side of the 'sink' flow. As before, the induced transverse flow will purge the regions near the endplates but between the exit holes.

I conclude that there will be no stagnant regions in the chamber.

1.4 CLEO and ALEPH

Anecdotal reports from ALEPH and CLEO indicate that indeed they achieve operating conditions within about 8 hours of purging with one volume change avery 2-3 hours. The anecdotes further claim this good performance depends on clever placement of the gas inlet and outlet tubes; namely bring gas in at the top of the chamber and take it out at the both somehow gaining a gravitational boost. I doubt this explanation is relevant, and also doubt the placement of the inlets and outlets matters at all so long as they are not too far from one another.

Instead, we have shown that when the gas flow rate is high enough the air does not diffuse into the oncoming chamber gas to any significant extent, and the purging cycle can be much faster the expected from 'classic lore'.

I infer that is it sensible to retain the suggestion that the gas enter the chamber through a number of holes in the rear endplate, and exit through holes in the front endplate.

2 Pressure Differentials

Poiseuille's law also tells us that an average flow velocity $\overline{v}$ in a tube of radius r and length L requires a pressure drop $\Delta P = 8\eta L\overline{v}/r^2$.

For example, the purging flow of $v = 0.043$ cm/s in the chamber requires a pressure drop of 3×10^{-6} dynes/cm^2 $= 3\times10^{-12}$ atmospheres. This does not appear to cause any problem.

If the 1/8$''$ ID tubes that feed the gas into the chamber are 20 cm long between a larger-diameter manifold and the endplate, the pressure drop across these tubes when flowing gas at 180 cm/s would be 2.3×10^{-4} atmospheres $\approx 0.1''$ of water. Again, no problem.

3 Helium Leakage During Wire Repair

The chamber will be operated slightly above atmospheric pressure, perhaps about 0.1$''$ of water $= 2.5\times10^{-4}$ dyne/cm^2. During the extraction of a broken wire this pressure would cause a gas flow out of the 3.2-cm-long feedthrough hole in the rear endplate at a rate $Q = \pi r^4\Delta P/(8\eta L)$ according to Poiseuille. Through a 0.45-cm-diameter sense-wire hole we will have flow rate $Q = 1.4\times10^{-4}$ cm^3/s. If the feedthrough hole is open for, say, 5 minutes during the repair the total leakage would be 0.04 cm^3. If the front endplate is only 0.5 cm thick the leakage there would be 6 times larger.

However, it appears that gas transport due to mutual diffusion will exceed the convective transport just calculated. In sec. 1.2 we noted that the characteristic distance a molecule moves in time t at a boundary with another gas type is $\sqrt{2Dt}$ where D is the diffusion coefficient, about 2/3 for He-O_2 mixtures. The time for a molecule to diffuse down a 3.2-cm-long hole in the rear endplate is $(3.2)^2/1.33 = 7.7$ sec. Thus there is a kind of diffusion velocity of $3.2/7.7 = 0.42$ cm/s in the hole. The volume of gas that passes through the hole in 5 minutes is then $(0.42)(\pi)(0.225)^2(300) = 20$ cm^3.

On the front endplate the diffusion time across 0.5 cm is only 0.19 s, so the diffusion velocity is 2.67 cm/s and the volume transported is 127 cm^3.

Even these larger volumes transported due to diffusion seem quite small, and I doubt that any special precautions need be taken, either for personnel protection or for protection of the DIRC phototubes.

As a corollary, the contamination of the chamber by air during the wire extraction will be the same as the diffusion transport out of the chamber, namely about 150 cm^3 total. Since the total chamber volume is about 5 m^3, the contamination of oxygen during the wire repair is only $150/5/(5 \times 10^6) = 6$ ppm. One could resume chamber operation without purging if necessary, although there would be localized poor performance near the feedthroughs that were repaired for about 5 hours = time for oxygen to have r.m.s diffusion of 150 cm in helium.

4 N_2 Flushing of Endplates

As a precaution against leakage of chamber gas out the many feedthrough holes in the endplates we plan to surround each endplate by a plenum that is flushed with dry nitrogen. The plenum on the read endplate will be about 25 cm thick, and that on the front about 5 cm thick. The corresponding volumes are 500 and 100 liters, respectively.

The remaining issues are the flow rate of nitrogen, and its source. Suppose, for example, we desire one volume exchange per hour, *i.e.*, 600 liters/hour flow rate. To convert to U.S. units, there are 28.3 liters per ft^3, so the desired flow rate would be 21 ft^2/hour = 510 ft^3/day = 14,400 liters/day.

A typical bottle of N_2 contains 200 ft^3, so we would need a lot of bottles! Another alternative is to use the boil-off from a tank of liquid nitrogen. Now one liter of N_2 at STP results from boiling about 1.5 cm^2 of LN_2, so we would need to boil off about 22 liters of LN_2 per day. This is not unreasonable in that SLAC has 300-l and 600-l LN_2 carts available for such purposes.

In case we don't want to bother with the LN_2 logistics, an alternative is a commercial nitrogen generation system such as Whatman model 75-720 (800-343-4048). This produces 20 ft^3/hr of dry (dewpoint = -50°C) nitrogen of about 99% purity (1% residual oxygen) from 100 psi compressed air in a passive process using semipermeable membranes that trap N_2 but let O_2 and H_2O out. Cost with oxygen monitor = \$8500.