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Analytic Stress Analysis of a Stepped Endplate

1 Introduction

The present plan for the BaBar Drift Chamber front endplate is to have a thickness of 24 mm out to a radius of 47 cm, and then a thickness of 12 mm at larger radii. Sec. 10.7 of *Roark's Formulas for Stress and Strain* (W.C. Young, 6th ed., McGraw-Hill, 1989) encourages the reader to combine the equations in Table 24 to construct an analytic stress analysis of an annular plate with loads and/or thicknesses that vary with radius. I have written a FORTRAN program following this suggestion that permits quick calculation of deflections and stresses in the endplate, which can be conveniently partitioned into 51 annular regions: 40 wire layers plus 11 gap regions around the superlayers where there are no wires.

This calculation is readily extended to simulate the process of stringing the chamber layer by layer. The resulting predictions as to the deflections of the plates as a function of the number of completed layers can be used to determine the overtensioning to the wires, described in note Princeton/BaBar/TNDC-96-42.

2 Method

In the analytic method of stress analysis presented in Table 24 of *Roark* the mechanical state of the plate at some radius r is described by four variables that we write as components of a vector $\vec{V}$:

$V_{,1} = y =$ deflection of the plate;
$V_{,2} = \theta =$ angle of the plate;
$V_{,3} = M_r =$ radial bending moment per unit length of circumference;
$V_{,4} = Q =$ shear force per unit length of circumference.

Other quantities of interest can be deduced from these four. Thus the tangential bending moment per unit length of circumference is

$$M_t = \frac{\theta D(1-\nu^2)}{r} + \nu M_r,$$

where ν is Poisson's ratio (taken as 0.3 for aluminum) and D is the plate constant

$$D = \frac{Et^3}{12(1-\nu^2)},$$

where E is the Young's modulus (taken as 7×10^{10} Pa for aluminum) and t is the thickness of the plate (in the region of interest). The radial and tangential stresses are then related by

$$\sigma_r = \frac{6M_r}{t^2}, \qquad \text{and} \qquad \sigma_t = \frac{6M_r}{t^2}.$$

The values $\vec{V}(r)$ are related to the values $\vec{V}(b)$ at the inner radius b of an annular region by the matrix transformation

$$\vec{V}(r) = \overleftrightarrow{M}(r,b) \cdot \vec{V}(b) + \vec{C}(r,b),$$

where the matrix $\overleftrightarrow{M}$ and vector $\vec{C}$ are given in terms of analytic functions listed on pp. 398-399 and p. 404 of *Roark*.

We now consider a plate divided into n annular regions with inner radii b_i, $i = 1, ..., n$. We desire the transformation

$$\vec{V}(r) = \overleftrightarrow{M}(r,b_1) \cdot \vec{V}(b_1) + \vec{C}(r,b_1)$$

between properties at radius r and those at the innermost radius b_1. This can be built iteratively out of transformations valid only within region i,

$$\vec{V}(b_{i+1}) = \overleftrightarrow{M}_i(b_{i+1},b_i) \cdot \vec{V}(b_i) + \vec{C}_i(b_{i+1},b_i),$$

according to

$$\vec{C}(b_{i+1},b_1) = \overleftrightarrow{M}_i(b_{i+1},b_i) \cdot \vec{C}(b_i,b_1) + \vec{C}_i(b_{i+1},b_i),$$

$$\overleftrightarrow{M}(b_{i+1},b_1) = \overleftrightarrow{M}_i(b_{i+1},b_i) \cdot \overleftrightarrow{M}(b_i,b_1).$$

To complete a solution, we must also have the values $\vec{V}(b_1)$ at the inner radius. However, these are typically not completely known. Rather some mixtures of values at the inner and outer radius of the plate are specified by the boundary conditions. For example, Case 2c describes the situation that the inner and outer radii of the plate are simply supported; there are no radial bending moments at these radii. We define the deflections to be zero at the inner and outer radii,

$$V_{,1}(b_1) = 0 = V_{,1}(b_{n+1}),$$

and set the radial bending moments equal to zero there also,

$$V_{,3}(b_1) = 0 = V_{,3}(b_{n+1}),$$

labelling the outer radius of the plate as b_{n+1}. The two unknowns, $V_{,2}(b_1)$ and $V_{,4}(b_1)$, are readily determined from the transformation between inner and outer radii:

$$0 = V_{,1}(b_{n+1}) = M_{,1,2}(b_{n+1},b_1)V_{,2}(b_1) + M_{,1,4}(b_{n+1},b_1)V_{,4}(b_1) + C_{,1}(b_{n+1},b_1),$$

and

$$0 = V_{,3}(b_{n+1}) = M_{,3,2}(b_{n+1}, b_1)V_{,2}(b_1) + M_{,3,4}(b_{n+1}, b_1)V_{,4}(b_1) + C_{,3}(b_{n+1}, b_1),$$

Cases for other forms of boundary conditions are also readily constructed. The program `case2_step.for` that implements this procedure can be found in the Princeton Technical Notes Web page.

3 Corrections for Holes

The analytic calculation described above assumes the plate is homogeneous within each annular region. In particular, there are no holes. To a good approximation the effect of small holes drilled in the plate is twofold:

1. The modulus is reduced. Both our ALGOR finite-element analysis and John Hodgson's ANSYS calculation (June 27, 1996) indicate that the effective modulus is 1/1.5 times the nominal 7×10^{10} Pa for aluminum.

2. The peak stress around the holes is greater than the stress in a plate with no holes. Table 37, Cases 6 and 7 of *Roark* discusses analytic calculations of the stress-concentration factor around holes for various idealized patterns of bending; this factor varies between 2 and 3. Our ALGOR finite-element analysis indicates a factor of 2.2 for the geometry of the BaBar endplate, while Hodgson's result indicate that a factor of 4.1 holds. This discrepancy is not understood. The present version of the program `case2_step.for` uses a value of 4.1.

4 Results

Many of the parameters describing the 51 regions used in the stress analysis are listed in Table 1. The wire tensions are taken as 34, 86 and 182 gm for the 20-, 80- and 120-μm wires, respectively, following TNDC-96-43 by C. Hearty. The total wire load is 3502 kg for the 7104 sense wires, 7104 clearing wires and 14560 field wires. The endplate simulated in Table 1 has thickness $t = 24$ mm out to a radius of 47 cm, and thickness 12 mm at larger radii. Table 2 summarizes some of the results of the stress analysis based on the parameters listed in Table 1.

Other endplates are simulated in Tables 3 and 4. In particular, Table 4 summarizes the stress in endplates that are clamped at their inner radius (slope constrained to be zero there).

5 Conclusions

1. We could reduce the peak stress by about 25% by using a step.

2. There is only a slight difference in the peak stress for a step at 47 cm compared to one at 63 cm.

Table 1: Input parameters for a stress analysis of a stepped endplate.

Region	Inner Radius (cm)	No. of Sense Wires	No. of Clear Wires	No. of Field Wires	Load Pressure (Pa)	Modulus Factor	Hole Stress Factor	t (mm)	Plate Constant (N)
1	23.5	0	0	0	0.0	1.00	1.0	24.	90495.
2	25.3	96	192	288	32.9	0.67	4.1	24.	60330.
3	26.6	96	0	192	18.4	0.67	4.1	24.	60330.
4	27.8	96	0	192	17.8	0.67	4.1	24.	60330.
5	29.0	96	192	192	24.7	0.67	4.1	24.	60330.
6	30.2	0	0	0	0.0	1.00	1.0	24.	90495.
7	31.2	112	224	224	26.3	0.67	4.1	24.	60330.
8	32.4	112	0	224	17.8	0.67	4.1	24.	60330.
9	33.6	112	0	224	17.2	0.67	4.1	24.	60330.
10	34.8	112	224	224	24.1	0.67	4.1	24.	60330.
11	36.0	0	0	0	0.0	1.00	1.0	24.	90495.
12	36.5	128	256	256	26.0	0.67	4.1	24.	60330.
13	37.6	128	0	256	17.6	0.67	4.1	24.	60330.
14	38.8	128	0	256	17.1	0.67	4.1	24.	60330.
15	40.0	128	256	256	23.8	0.67	4.1	24.	60330.
16	41.2	0	0	0	0.0	1.00	1.0	24.	90495.
17	41.7	144	288	288	25.7	0.67	4.1	24.	60330.
18	42.9	144	0	288	17.4	0.67	4.1	24.	60330.
19	44.0	144	0	288	17.0	0.67	4.1	24.	60330.
20	45.2	144	288	288	23.9	0.67	4.1	24.	60330.
21	46.4	0	0	0	0.0	1.00	1.0	24.	90495.
22	47.5	176	352	352	27.1	0.67	4.1	12.	7541.
23	48.7	176	0	352	19.0	0.67	4.1	12.	7541.
24	49.8	176	0	352	18.5	0.67	4.1	12.	7541.
25	51.0	176	352	352	25.7	0.67	4.1	12.	7541.
26	52.2	0	0	0	0.0	1.00	1.0	12.	11312.
27	52.7	192	384	384	27.2	0.67	4.1	12.	7541.
28	53.9	192	0	384	18.5	0.67	4.1	12.	7541.
29	55.1	192	0	384	18.2	0.67	4.1	12.	7541.
30	56.2	192	384	384	25.5	0.67	4.1	12.	7541.
31	57.4	0	0	0	0.0	1.00	1.0	12.	11312.
32	57.9	208	416	416	26.8	0.67	4.1	12.	7541.
33	59.1	208	0	416	18.3	0.67	4.1	12.	7541.
34	60.3	208	0	416	18.0	0.67	4.1	12.	7541.
35	61.5	208	416	416	25.5	0.67	4.1	12.	7541.
36	62.6	0	0	0	0.0	1.00	1.0	12.	11312.
37	63.7	224	448	448	26.0	0.67	4.1	12.	7541.
38	64.9	224	0	448	18.2	0.67	4.1	12.	7541.
39	66.1	224	0	448	17.7	0.67	4.1	12.	7541.
40	67.2	224	448	448	24.9	0.67	4.1	12.	7541.
41	68.4	0	0	0	0.0	1.00	1.0	12.	11312.
42	68.9	240	480	480	26.0	0.67	4.1	12.	7541.
43	70.1	240	0	480	17.9	0.67	4.1	12.	7541.
44	71.3	240	0	480	17.6	0.67	4.1	12.	7541.
45	72.5	240	480	480	24.8	0.67	4.1	12.	7541.
46	73.6	0	0	0	0.0	1.00	1.0	12.	11312.
47	74.1	256	512	512	25.8	0.67	4.1	12.	7541.
48	75.3	256	0	512	17.7	0.67	4.1	12.	7541.
49	76.5	256	0	512	17.5	0.67	4.1	12.	7541.
50	77.7	256	512	768	29.0	0.67	4.1	12.	7541.
51	79.0	0	0	0	0.0	1.00	1.0	12.	11312.

Table 2: Deflections and stress for the plate of Table 1, simply supported at both inner and outer radii.

Region	Mid Radius (cm)	Δz (mm)	θ (mrad)	M_r (kN)	M_t (kN)	Load (kg)	σ_r (MPa)	σ_t (MPa)
1	24.4	0.067	-7.4	3.3	-2453.7	1609.	0.0	-25.6
2	26.0	0.182	-7.3	37.2	-1494.1	1593.	1.6	-63.8
3	27.2	0.272	-7.1	84.5	-1384.3	1545.	3.6	-59.1
4	28.4	0.356	-7.0	126.7	-1288.6	1509.	5.4	-55.0
5	29.6	0.438	-6.9	166.5	-1200.5	1465.	7.1	-51.3
6	30.7	0.515	-6.8	191.4	-1721.4	1441.	2.0	-17.9
7	31.8	0.592	-6.7	216.8	-1056.7	1409.	9.3	-45.1
8	33.0	0.670	-6.6	251.1	-984.4	1356.	10.7	-42.0
9	34.2	0.747	-6.4	282.8	-917.1	1313.	12.1	-39.2
10	35.4	0.822	-6.3	312.0	-854.3	1261.	13.3	-36.5
11	36.2	0.874	-6.2	327.7	-1275.0	1242.	3.4	-13.3
12	37.1	0.926	-6.1	342.9	-775.2	1195.	14.6	-33.1
13	38.2	0.997	-6.0	366.1	-721.5	1134.	15.6	-30.8
14	39.4	1.067	-5.9	386.9	-671.3	1085.	16.5	-28.7
15	40.6	1.135	-5.7	405.4	-624.2	1024.	17.3	-26.7
16	41.4	1.183	-5.6	414.8	-959.6	996.	4.3	-10.0
17	42.3	1.230	-5.5	423.5	-565.3	948.	18.1	-24.1
18	43.5	1.295	-5.4	436.3	-525.4	880.	18.6	-22.4
19	44.6	1.358	-5.3	447.0	-488.0	824.	19.1	-20.8
20	45.8	1.419	-5.1	455.6	-453.3	755.	19.5	-19.4
21	46.9	1.476	-5.0	458.2	-713.3	715.	4.8	-7.4
22	48.1	1.532	-4.6	466.8	89.8	665.	79.7	15.3
23	49.3	1.582	-3.8	481.5	106.8	581.	82.3	18.2
24	50.4	1.622	-3.0	493.0	122.2	512.	84.2	20.9
25	51.6	1.653	-2.2	501.8	136.4	428.	85.7	23.3
26	52.4	1.669	-1.7	505.9	133.8	381.	21.1	5.6
27	53.3	1.682	-1.2	509.2	152.8	325.	87.0	26.1
28	54.5	1.691	-0.4	511.0	163.7	233.	87.3	28.0
29	55.6	1.691	0.4	510.2	173.2	157.	87.2	29.6
30	56.8	1.682	1.2	507.1	181.4	65.	86.6	31.0
31	57.7	1.670	1.7	503.1	195.5	11.	21.0	8.1
32	58.5	1.654	2.2	498.4	189.5	-47.	85.1	32.4
33	59.7	1.623	2.9	489.2	194.5	-147.	83.6	33.2
34	60.9	1.584	3.7	477.8	198.2	-229.	81.6	33.9
35	62.0	1.537	4.4	464.3	200.7	-329.	79.3	34.3
36	63.2	1.484	4.9	449.4	226.9	-388.	18.7	9.5
37	64.3	1.426	5.5	434.0	200.2	-451.	74.1	34.2
38	65.5	1.358	6.1	415.3	199.5	-559.	71.0	34.1
39	66.7	1.282	6.7	394.8	197.6	-647.	67.4	33.8
40	67.8	1.200	7.2	372.3	194.6	-754.	63.6	33.2
41	68.7	1.138	7.6	354.9	228.3	-822.	14.8	9.5
42	69.5	1.073	7.9	337.0	187.5	-886.	57.6	32.0
43	70.7	0.977	8.4	309.6	181.6	-1001.	52.9	31.0
44	71.9	0.876	8.8	280.4	174.5	-1096.	47.9	29.8
45	73.0	0.770	9.1	249.4	166.3	-1211.	42.6	28.4
46	73.9	0.693	9.3	226.1	202.0	-1285.	9.4	8.4
47	74.7	0.614	9.5	202.4	152.4	-1352.	34.6	26.0
48	75.9	0.500	9.8	166.9	141.4	-1475.	28.5	24.2
49	77.1	0.384	9.9	129.7	129.4	-1576.	22.2	22.1
50	78.3	0.259	10.1	88.5	115.5	-1721.	15.1	19.7
51	80.0	0.096	10.1	32.4	138.1	-1810.	1.3	5.8

Table 3: Summary of peak stresses in various possible endplates simply supported at both inner and outer radii.

r(step) (cm)	Thick Thickness (mm)	Thin Thickness (mm)	Peak Deflection (mm)	Peak Radial Stress (MPa)
–	12	12	3.09	119.2
–	24	24	0.39	29.8
47	24	10	2.40	105.4
47	24	11	2.01	96.0
47	24	12	1.69	87.3
47	24	13	1.43	79.3
47	24	14	1.22	72.1
47	24	15	1.05	65.4
47	24	16	0.91	59.5
63	24	10	1.08	120.4
63	24	11	0.88	102.1
63	24	12	0.75	87.2
63	24	13	0.65	75.4
63	24	14	0.59	65.6

3. The peak deflection is less for a step at 63 cm than one at 46 cm. This effect is more pronounced for simple supports at both inner and outer radii.
4. For a step at 47 cm there is roughly a 15% reduction in peak stress by fixing the slope at the inner radius, as compared to a simple support there.
5. For a step at 63 cm there is a 35% stress reduction in fixing the slope at the inner radius, as compared to a simple support there.
6. For a step at 63 cm down to 14 mm a simply supported plate gives very similar performance to a step down to 10 mm for a plate fixed at the inner radius.
7. A step at 63 cm and a fixed slope at the inner radius offers the greatest advantage in stress reduction and in minimizing deflection.
8. If we choose a step at 47 cm, an inner stiffening ring would be justified on the basis of the reduced deflection more than reduced stress.

Table 4: Summary of peak stresses in various possible endplates clamped at the inner radius and simply supported at the outer radius. The peak stress occurs at the inner radius where the plate is clamped.

r(step) (cm)	Thick Thickness (mm)	Thin Thickness (mm)	Peak Deflection (mm)	Peak Radial Stress (MPa)
–	12	12	1.50	155.0
–	24	24	0.19	38.8
47	24	10	0.82	84.5
47	24	11	0.71	79.8
47	24	12	0.63	75.1
47	24	13	0.56	70.5
47	24	14	0.50	66.1
47	24	15	0.45	61.9
47	24	16	0.40	58.1
63	24	10	0.60	72.5
63	24	11	0.50	64.7
63	24	12	0.43	57.5
63	24	13	0.37	51.1
63	24	14	0.32	45.4